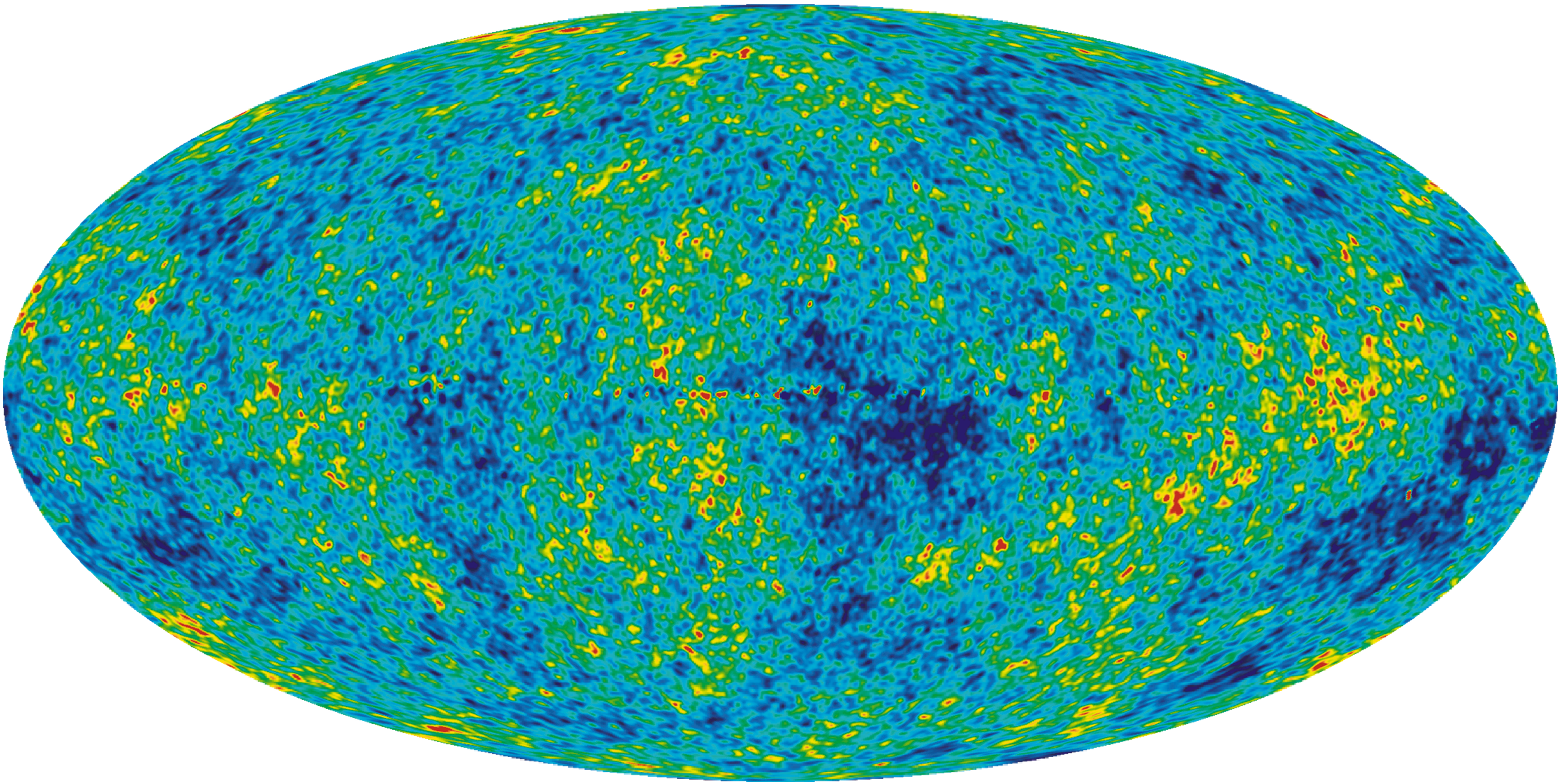
Non-Gaussianities from 2nd Order Temperature Anisotropies

Lotfi Boubekeur

LB, P. Creminelli, J. Noreña, F. Vernizzi, JCAP 0808:028,2008.

LB, P. Creminelli, G. D'Amico, J. Noreña, F. Vernizzi. 0906.0980

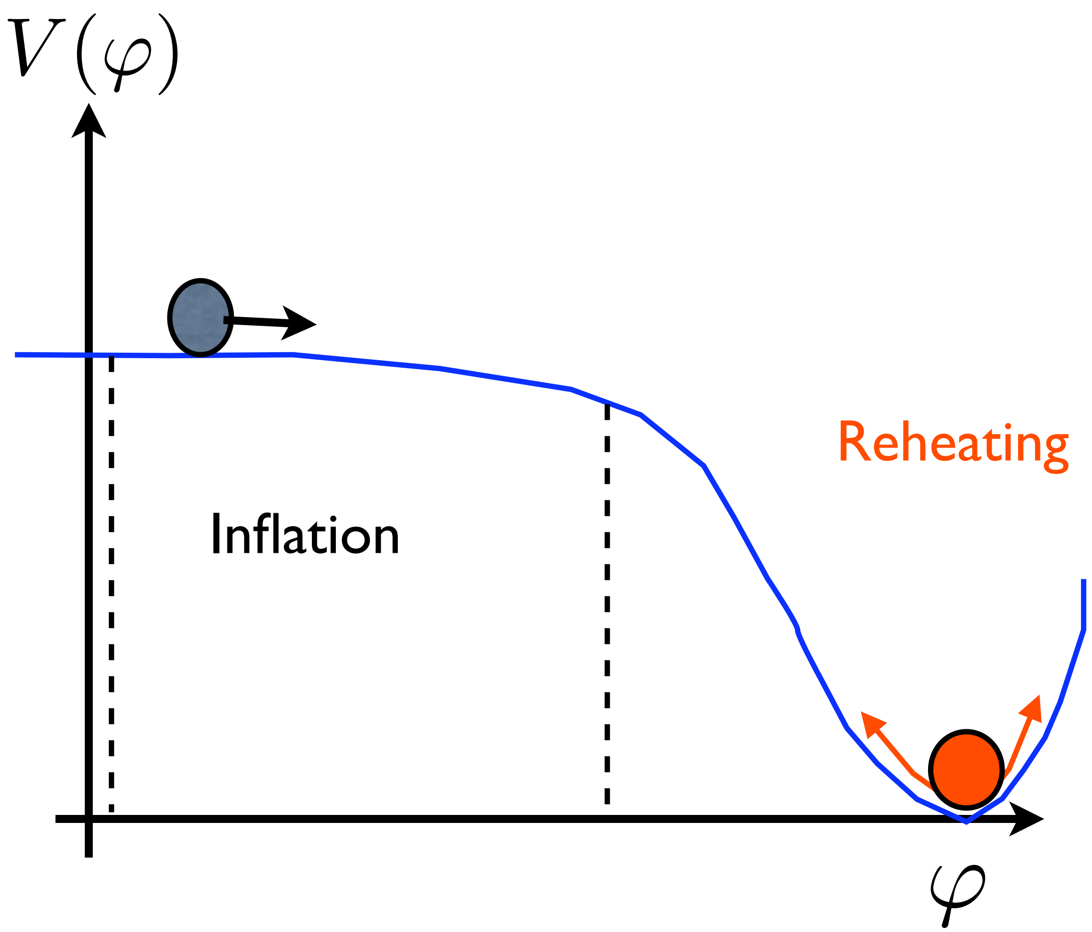


Are there correlations between modes?

Inflation & cosmological perturbations

- Observations: curvature perturbation $\dfrac{\delta T}{T} \sim \phi \sim 10^{-5}$ almost scale-invariant.

  - Slow-roll inflation



$$\ddot{\varphi} + \underline{\underline{3H}}\dot{\varphi} = -V'(\varphi)$$

Slow-roll conditions

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1, \quad \eta \equiv M_P^2 \frac{V''}{V} \ll 1$$

$$\Downarrow$$

==> <u>inflaton is weakly coupled ==> Gaussian.</u>

  - Spectrum of perturbations

$$\zeta \sim \frac{V^{1/2}}{\sqrt{\epsilon} M_P^2} \qquad \text{AND} \qquad |n_S - 1| \sim 1/N_e$$

Non-Gaussianity in the CMB

In cosmology $\dfrac{\delta T}{T} \sim \phi \sim 10^{-5}$ temperature anisotropies.

Correlation functions

$$\langle \phi_{\vec{p}} \phi_{\vec{q}} \rangle \propto \delta^{(3)}(\vec{p} - \vec{q}) P_\phi(p)$$ ⟵ Power spectrum

$$\langle \phi_{\vec{p_1}} \phi_{\vec{p_2}} \phi_{\vec{p_3}} \rangle = 0$$ ⟵ Bispectrum

$$\vdots$$

$$\boxed{\begin{aligned} &\langle \phi_{\vec{p}_1} \cdots \phi_{\vec{p}_{2n}} \rangle \propto \langle \phi^2 \rangle^n \\ &\langle \phi_{\vec{p}_1} \cdots \phi_{\vec{p}_{2n+1}} \rangle = 0 \end{aligned}}$$ ⟵ *Gaussian statistics*

Non-Gaussianity in the CMB

In cosmology $\frac{\delta T}{T} \sim \phi \sim 10^{-5}$ temperature anisotropies.

$$\phi(\vec{x}) = \phi_g(\vec{x}) + f_{\rm NL}(\phi_g^2(\vec{x}) - \langle \phi_g^2 \rangle)$$

Correlation functions

$$\langle \phi_{\vec{p}} \phi_{\vec{q}} \rangle \propto \delta^{(3)}(\vec{p} - \vec{q}) P_\phi(p) \longleftarrow \text{Power spectrum}$$

$$\langle \phi_{\vec{p_1}} \phi_{\vec{p_2}} \phi_{\vec{p_3}} \rangle \propto f_{\rm NL} \longleftarrow \text{Bispectrum}$$

$$\vdots$$

$$\langle \phi_{\vec{p}_1} \cdots \phi_{\vec{p}_{2n}} \rangle \propto \langle \phi^2 \rangle^n$$

$$\boxed{\langle \phi_{\vec{p}_1} \cdots \phi_{\vec{p}_{2n+1}} \rangle \propto f_{\rm NL}^{2n-1}}$$

Non-Gaussian statistics

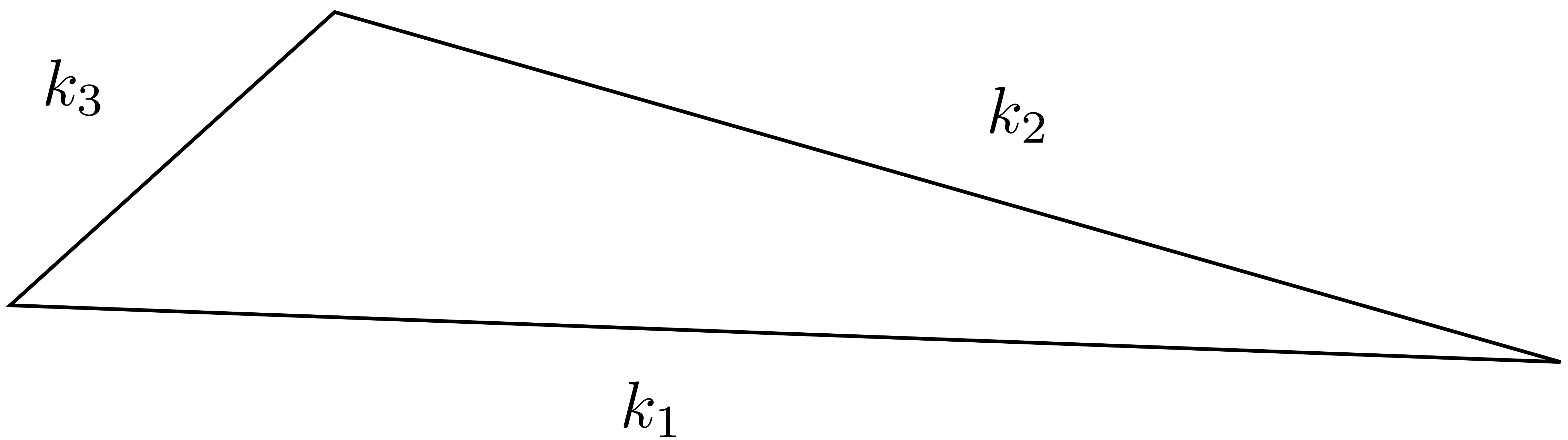
Non-Gaussianity in the CMB

The bispectrum in Fourier space is defined as

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\, B(\vec{k}_1, \vec{k}_2, \vec{k}_3).$$

It scales as

$$B(\lambda\vec{k}_1, \lambda\vec{k}_2, \lambda\vec{k}_3) = \lambda^{-6} B(\vec{k}_1, \vec{k}_2, \vec{k}_3)\,.$$



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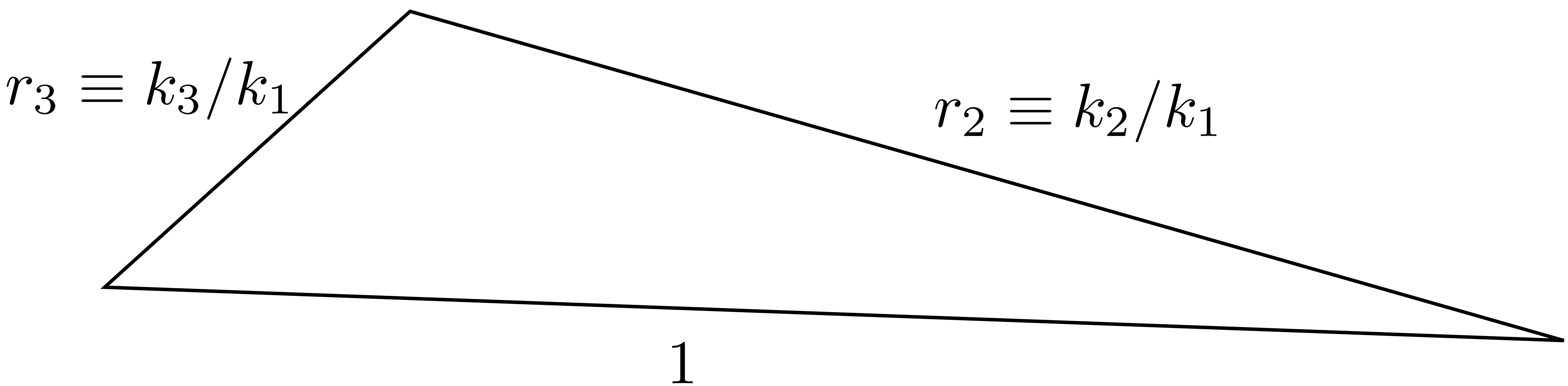

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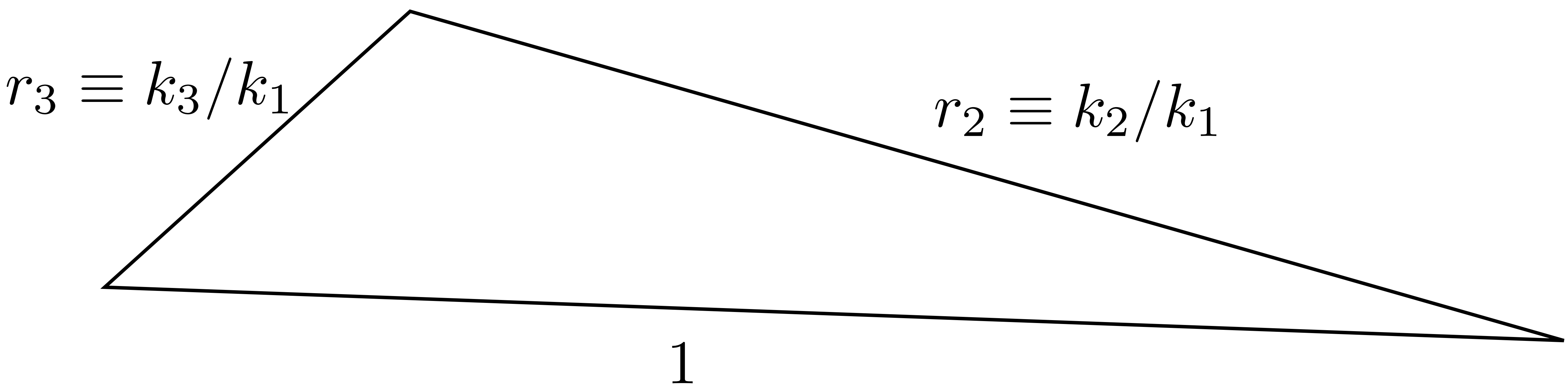

Which shapes are possible?

Non-Gaussianity in the CMB

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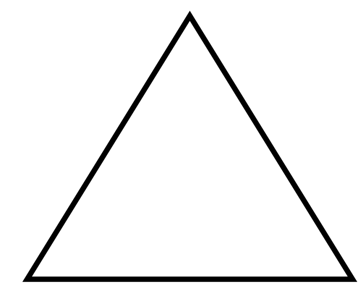
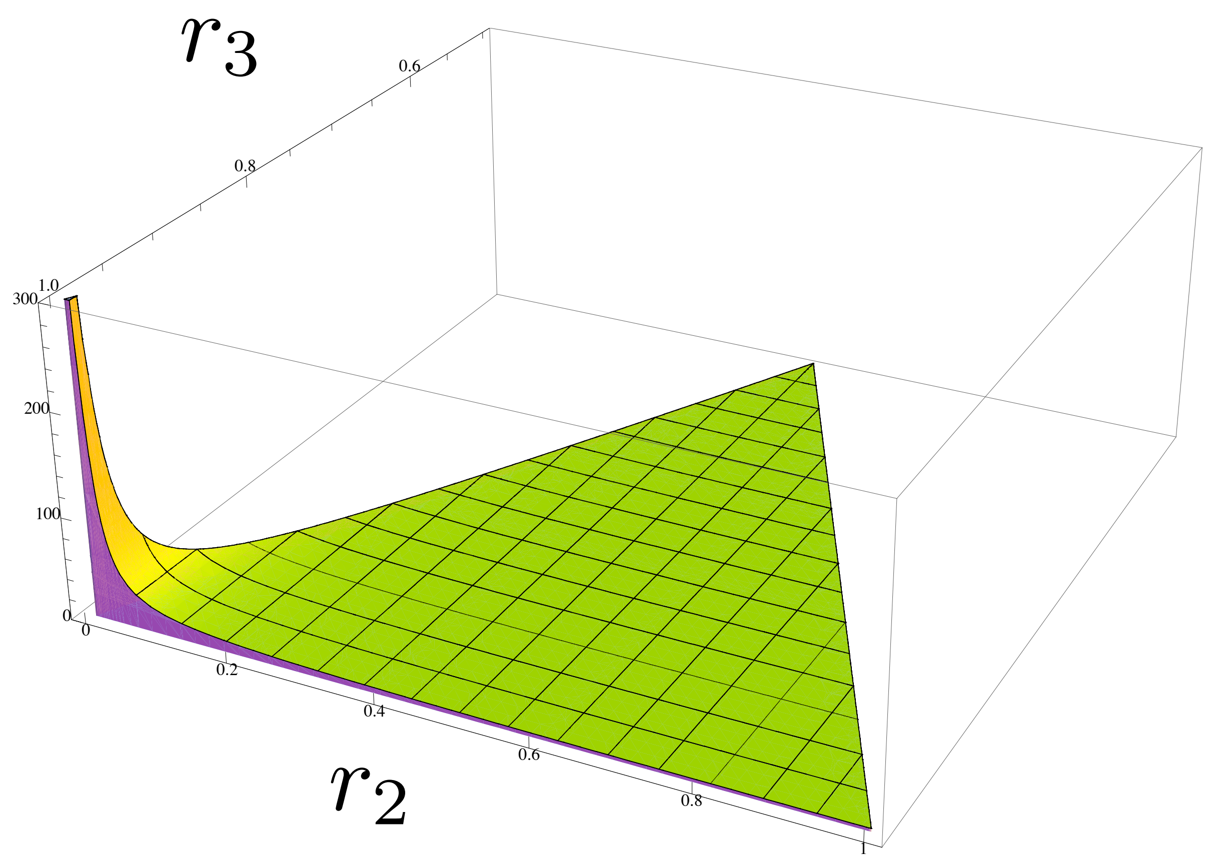
Generally, models predict two types of shapes: local shape & equilateral.

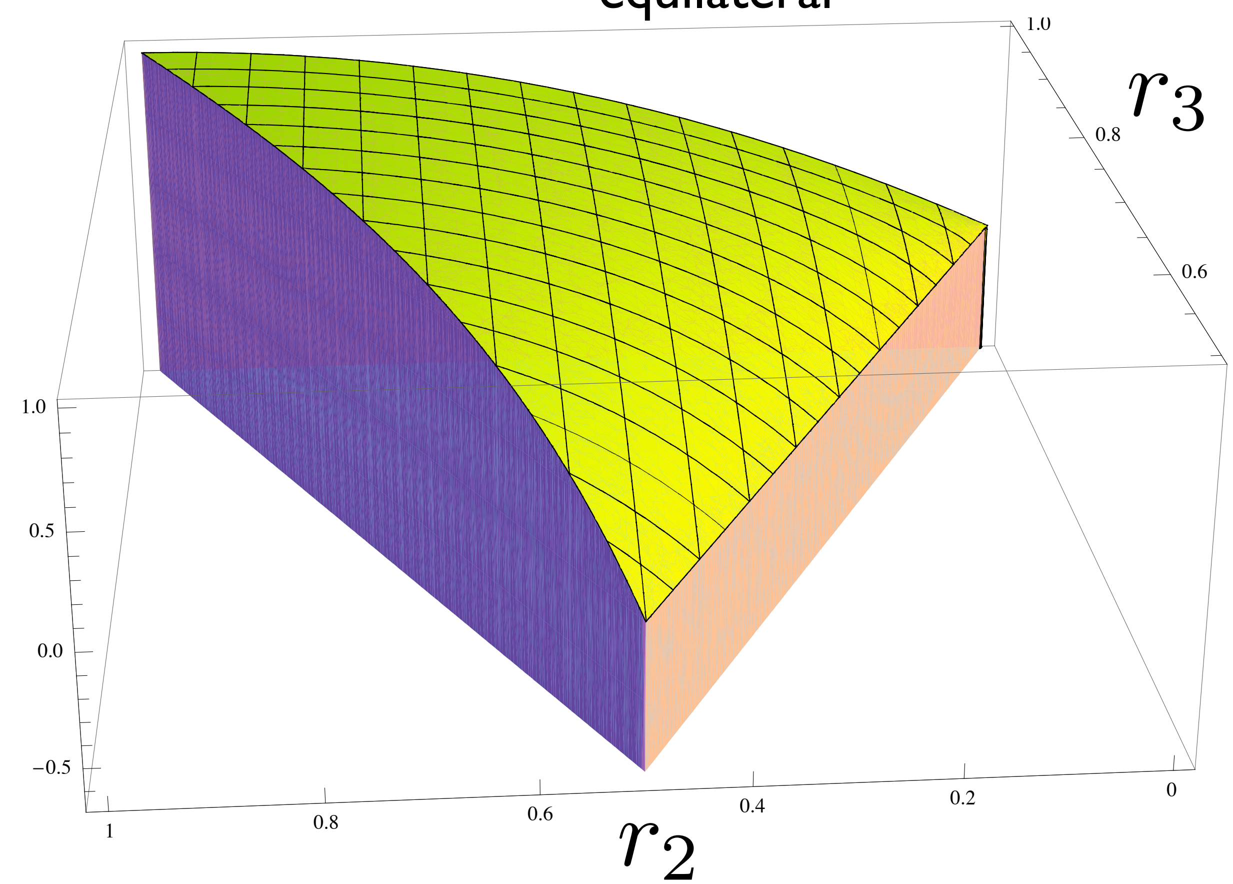
Babich, Creminelli, Zaldarriaga, 04

local



equilateral





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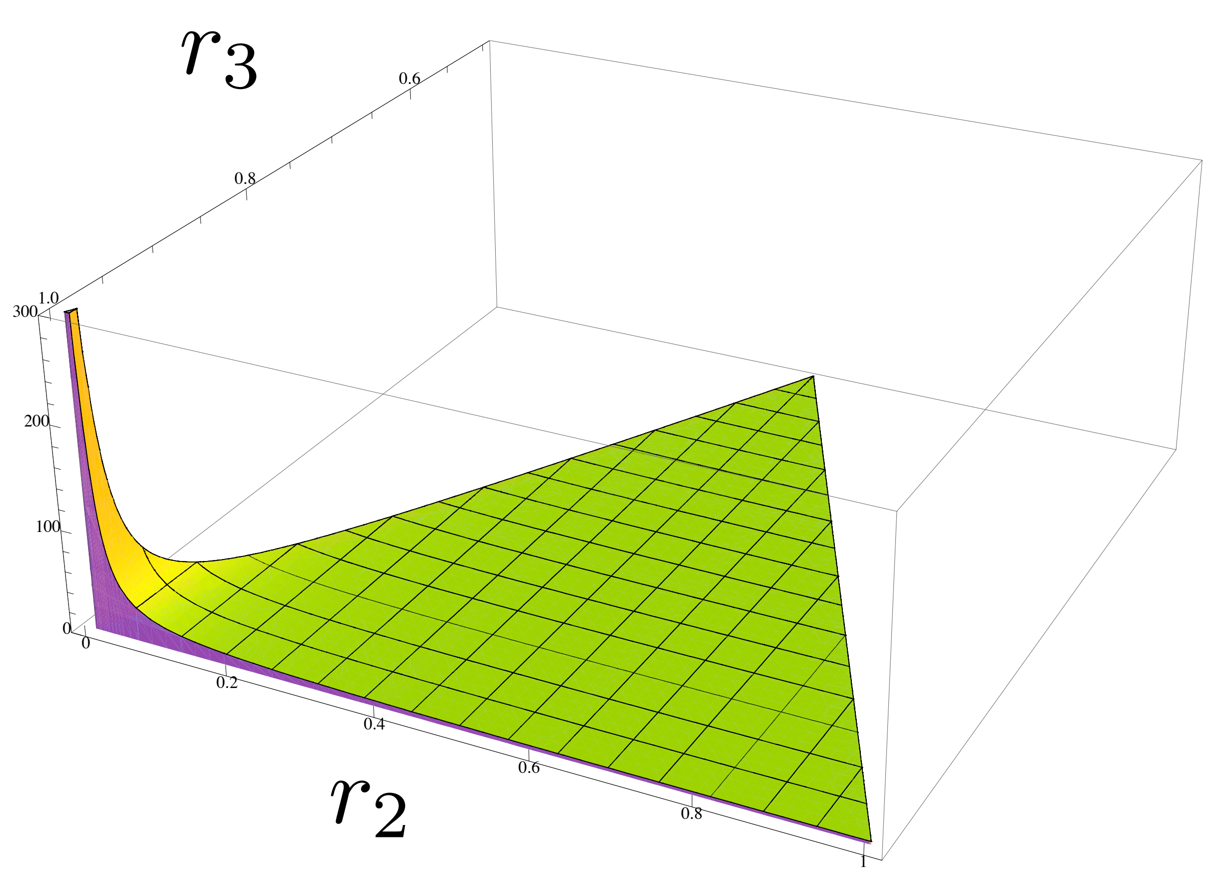
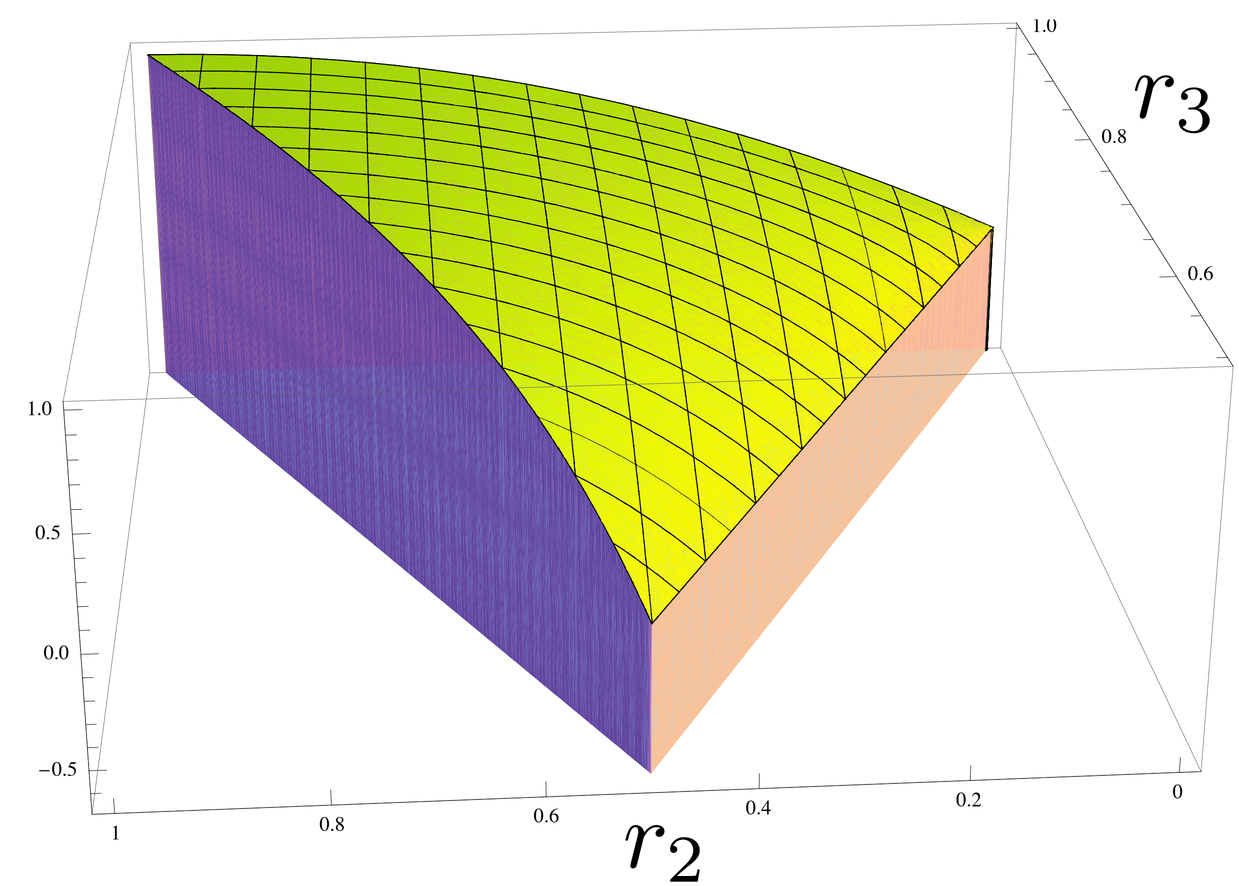
Babich, Creminelli, Zaldarriaga, 04

Single field inflation
Curvaton
inhomogeneous reheating
New ekpyrosis

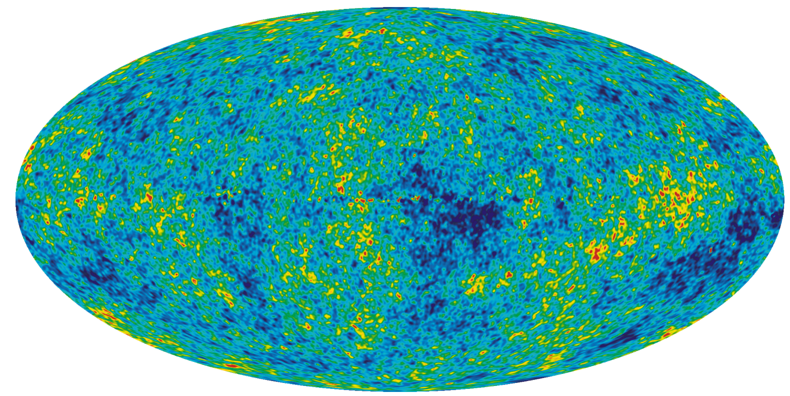
DBI inflation
Higher derivatives,.....





Motivations

Theoretical scenarios



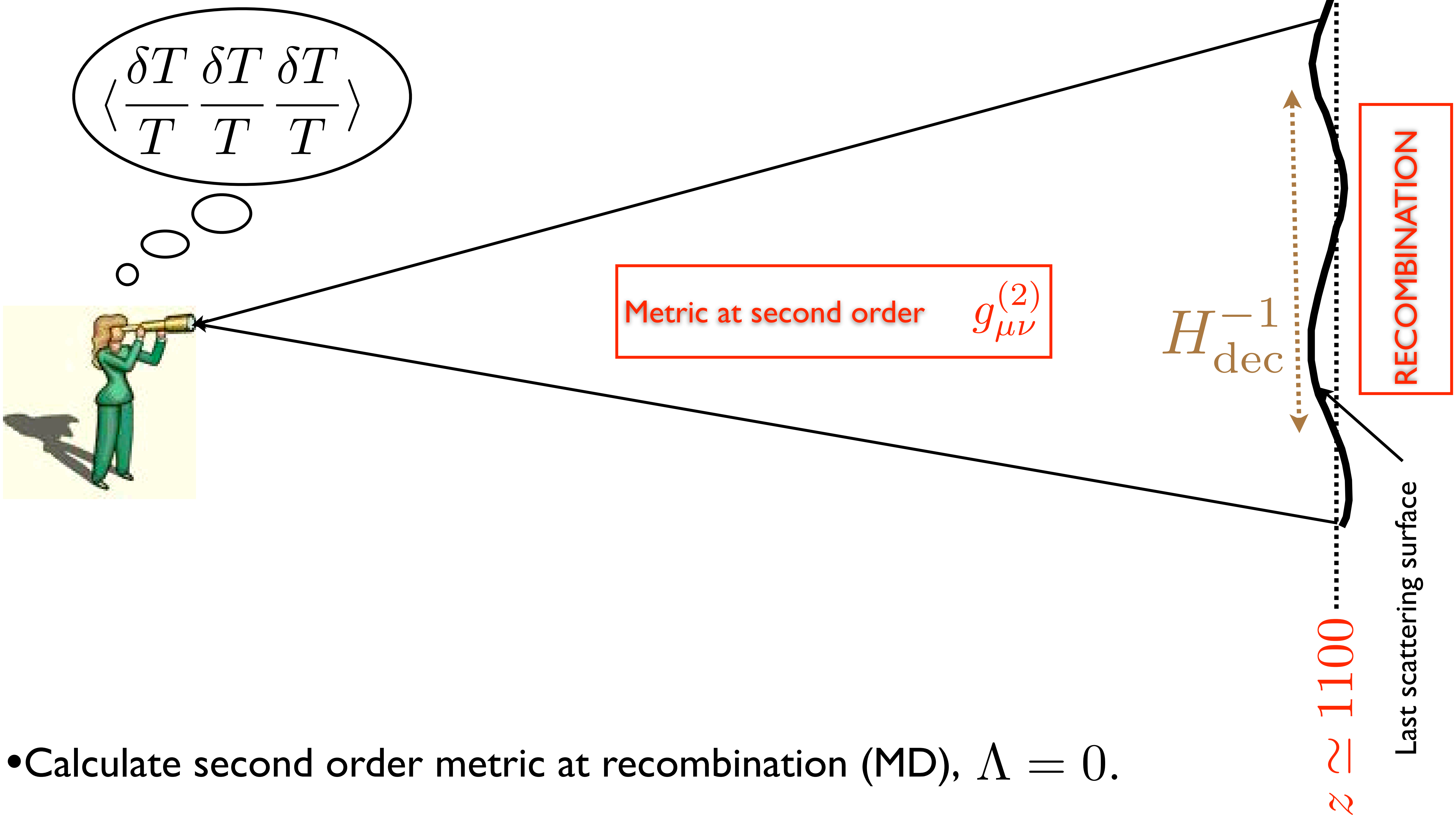
- Inflation $f_{\rm NL} \lesssim 10^{-1}$ J. Maldacena '03
- Curvaton and Inhomogeneous reheating $f_{\rm NL} \sim 100$
- DBI, Ghost inflation, new ekpyrosis, ... $f_{\rm NL} \sim 100$

$$\boxed{-4 < f_{\rm NL}^{\rm local} < 80 \quad \text{at} \;\; 95\% \; \text{CL}}$$

Smith, Senatore & Zaldarriaga, '09

$$\boxed{f_{\rm NL} \sim 5}$$

Expect NG ~O(1) independently of primordial component!



•Calculate second order metric at recombination (MD), $\Lambda = 0$.

•Propagate CMB photon in this metric $\longrightarrow \delta T/T$

•Compute the bispectrum on large scales $\longrightarrow f_{\rm NL}$

Effects neglected

- Primordial non-Gaussianity.
- Plasma effects: Physics of recombination.
  - Non-linear dark matter perturbations.
  - Perturbed recombination.
  - .....

➡ Need to integrate full Boltzmann eqts.

- CC and Radiation not included.
- No tilt.

Fluid description as a scalar

LB, P. Creminelli, J. Noreña, F. Vernizzi, JCAP 0808:028,2008.

- Usual description of a perfect fluid coupled to GR $\Longrightarrow$ energy momentum tensor

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu} \ , \quad p = w\rho$$

- Naive counting $\Longrightarrow$ only one degree of freedom $\Longrightarrow$ scalar field description ?

A. H. Taub '54
B. Shutz '70

- considering $\mathcal{L} = P(X)$, with $X = -\partial_\mu \phi \partial^\mu \phi$ .and requiring perfect fluid eqt. of state

$$\boxed{P(X) = X^{\frac{1+w}{2w}}; \quad w \neq 0}$$

- Energy density $\rho = 2P'X - P$, Pressure $p = P$. Velocity 4-vector: $u_\mu = \dfrac{\partial_\mu \phi}{\sqrt{X}}$.

- Example: Radiation

$$\mathcal{L} = X^2 = (-\partial_\mu \phi \partial^\mu \phi)^2 \ .$$

Fluid description as a scalar

Dubovsky, Gregoire, Nicolis and Rattazzi

- To describe a fluid, we need 3 scalars $\phi^I,\ I = 1,2,3.$
- Usually invariance under $\phi^I \rightarrow D^I_J\ \phi^J,$
- Lagrangian is $\mathcal{L} = F(B); \quad B \equiv \det B^{IJ},$ where $B^{IJ} \equiv \partial^\mu \phi^I \partial_\mu \phi^J$
- Velocity 4-vector

$$u^\mu = \frac{1}{6\sqrt{B}} \epsilon^{\mu\nu\rho\sigma} \epsilon_{IJK}\ \partial_\mu \phi^I \partial_\rho \phi^J \partial_\sigma \phi^K.$$

- Energy density and pressure

$$\rho = -F(B), \quad p = F - 2F'(B)B.$$

- For perfect fluids $F(B) = B^{\frac{1+w}{2}},$
- The 2 pictures are related through a Legendre transformation

$$G(X) - F(B) = 2XG'(X) = -2BF'(B), \quad 2F'(B) = -\frac{1}{2G'(X)}.$$

$$\boxed{G(X) = X^{\frac{1+w}{2w}}}$$

Background dynamics

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,[R+2P(X)]\,, \qquad X \equiv -\partial_\mu\phi\,\partial^\mu\phi$$

C. Armendariz-Picon, T. Damour and V. F. Mukhanov
Seery & Lidsey
X. Chen, M. x. Huang, S. Kachru and G. Shiu

- With $P(X) = X^{\frac{1+w}{2w}}; \quad w \neq 0$ <u>Subtelties for</u> $w = 0.$

- Perform computations and then take $w \longrightarrow 0$ <u>at the end!</u>

- Background Evolution (FRW)

$$\begin{aligned} H^2 &= \frac{1}{3}\left(2XP' - P\right)\,, \\ 2\dot{H} &+ 3H^2 = -P\,, \end{aligned}$$

$$\partial_\mu[\sqrt{g}P'(X)\partial^\mu\phi] = 0 \Longrightarrow \rho \propto \dot{\phi}^{\frac{1+w}{w}} \propto a^{-3(1+w)}\,.$$

It behaves as a perfect fluid.

Cosmological Perturbations

- This description allows to study perturbations à la Maldacena!

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \ .$$

- Solve for N_i and $N \Longrightarrow$ action for physical degrees of freedom.

- The action reads

$$S = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\sqrt{h}\left[N(R^{(3)} + 2P) + N^{-1}\left(E_{ij}E^{ij} - E^2\right)\right] \ ,$$

where
$$E_{ij} \equiv \frac{1}{2}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right) \ .$$

- Momentum and Energy constraint

$$\nabla_i\left[N^{-1}\left(E^i_j - \delta^i_j E\right)\right] = 0 \ ,$$

$$R^{(3)} + 2P - 4XP' - \frac{1}{N^2}\left(E_{ij}E^{ij} - E^2\right) = 0 \ .$$

Cosmological Perturbations (1st order)

Uniform scalar gauge:

$$\delta\phi = 0, \qquad h_{ij} = a^2 e^{2\zeta} \hat{h}_{ij}, \quad \hat{h}_{ij} = \delta_{ij} + \gamma_{ij} + \frac{1}{2}\gamma_{il}\gamma_{lj} + \cdots$$

$$\det \hat{h} = 1, \qquad \gamma_{ii} = 0, \quad \partial_i \gamma_{ij} = 0.$$

- Perturbations at first order $\Longrightarrow N_i$ and N at first order

$$N_1 = \frac{\dot{\zeta}}{H}, \qquad N_{i1} = -\frac{2}{5H}\partial_i \zeta_0$$

- put back in action at 2nd order and solve eqt. of motion.

- metric at first order

$$ds^2 = -dt^2 - \frac{4}{5H}\partial_i \zeta_0 \, dt dx^i + a^2(1 + 2\zeta_0) d\vec{x}^2$$

where $\zeta = \zeta_0(\vec{x})$ is constant wrt. time up to $\mathcal{O}(w) \longrightarrow$ initial condition from inflation.

- On large scales, in the Poisson (Newtonian) gauge $\phi = -\frac{3}{5}\zeta_0$

$$\boxed{ds^2 = a(\tau)^2 \left[-(1 + 2\phi) d\tau^2 + (1 - 2\phi) d\vec{x}^2 \right]}$$

Sachs-Wolfe at 1st order

$$\boxed{ds^2 = a(\tau)^2\left[-(1+2\phi)d\tau^2 + (1-2\phi)d\vec{x}^2\right]}$$

MD ==> $a \propto \tau^2$ and $\dot{\phi} = 0$

Temperature anisotropies @ 1st order

$$\boxed{\frac{\delta T}{T} = -\frac{2\phi_e}{3} + (\phi_e - \cancel{\phi_o}) = \frac{\phi_e}{3}}$$

Sachs-Wolfe (1967).

Intrinsic

gravitational redshift

$$\tilde{\tau} = \tau(1+2\phi)^{1/6}$$

Cosmological Perturbations (2nd order)

Uniform scalar gauge:

$$\delta\phi = 0, \qquad h_{ij} = a^2 e^{2\zeta}\hat{h}_{ij}, \quad \hat{h}_{ij} = \delta_{ij} + \gamma_{ij} + \frac{1}{2}\gamma_{il}\gamma_{lj} + \cdots$$

$$\det \hat{h} = 1, \qquad \gamma_{ii} = 0, \quad \partial_i\gamma_{ij} = 0.$$

- Perturbations at second ordre $\Longrightarrow N_i$ and N at second order

$$\begin{aligned}
g_{00} &= -1 + \frac{4}{25a^2H^2}(\partial_i\zeta)^2 \;, \\
g_{0i} &= -\frac{1}{5H}\partial_i\Big[2\zeta_0 - \partial^{-2}(\partial_j\zeta_0)^2 + 3\partial^{-4}\partial_j\partial_k(\partial_j\zeta_0\partial_k\zeta_0) \\
&\qquad - \frac{4}{5a^2H^2}\partial^{-2}\Big(\frac{3}{7}(\partial^2\zeta_0)^2 + \partial_i\zeta_0\partial_i\partial^2\zeta_0 + \frac{4}{7}(\partial_i\partial_j\zeta_0)^2\Big)\Big] \\
&\quad -\frac{4}{5}\frac{1}{H}\partial^{-2}\left[\partial_i\zeta_0\partial^2\zeta_0 - \frac{\partial_i\partial_k}{\partial^2}\partial_k\zeta_0\partial^2\zeta_0\right] \;, \\
g_{ij} &= a^2\left[1 + 2\zeta_0 + 2\zeta_0^2 - \frac{2}{5a^2H^2}\partial^{-2}\partial_k\partial_l(\partial_k\zeta_0\partial_l\zeta_0)\right]\delta_{ij} + a^2\gamma_{ij} \;.
\end{aligned}$$

Metric @ 2nd order

- In the Poisson gauge, the metric in MD reads

$$ds^2 = a^2(\tau)\left\{-(1+2\Phi)d\tau^2 + 2\omega_i dx^i d\tau + [(1-2\Psi)\delta_{ij} + \gamma_{ij}]dx^i dx^j\right\} ,$$

where

Bartolo, Matarrese & Riotto, '06

LB, P. Creminelli, J. Noreña, F. Vernizzi, '08.

$$\begin{aligned}
\Phi &= \phi + \left[\phi^2 + \partial^{-2}(\partial_j\phi)^2 - 3\partial^{-4}\partial_i\partial_j(\partial_i\phi\partial_j\phi)\right] \\
&\quad \frac{2}{21a^2H^2}\partial^{-2}\left[2(\partial_i\partial_j\phi)^2 + 5(\partial^2\phi)^2 + 7\partial_i\phi\partial_i\partial^2\phi\right] , \\
\Psi &= \phi - \left[\phi^2 + \frac{2}{3}\partial^{-2}(\partial_i\phi)^2 - 2\partial^{-4}\partial_i\partial_j(\partial_i\phi\partial_j\phi)\right] \\
&\quad + \frac{2}{21a^2H^2}\partial^{-2}\left[2(\partial_i\partial_j\phi)^2 + 5(\partial^2\phi)^2 + 7\partial_i\phi\partial_i\partial^2\phi\right] , \\
\omega_i &= -\frac{8}{3aH}\partial^{-2}\left[\partial^2\phi\partial_i\phi - \partial^{-2}\partial_i\partial_j(\partial^2\phi\partial_j\phi)\right] , \\
\gamma_{ij} &= -20\left(\frac{1}{3} - \frac{j_1(k\tau)}{k\tau}\right)\partial^{-2}P^{\mathrm{TT}}_{ij\,kl}\left(\partial_k\phi\partial_l\phi\right) .
\end{aligned}$$

$\frac{\delta T}{T}$ @ 2nd order on large scales

In preparation with P. Creminelli, G. D'Amico, J. Noreña, F. Vernizzi.

- Temperature anisotropies @ 2nd order

1. Temperature related to photon frequency

$$\boxed{T_o(\hat{n}) = \frac{\omega_o}{\omega_e} T_e(\vec{x}_e) \ ,}$$

2. Geodesic equation

$$\boxed{\frac{\omega_o}{\omega_e} = \frac{a_e}{a_o}\sqrt{\frac{1+2\Phi_e}{1+2\Phi_o}}\left[1 + \int_{\tau_e}^{\tau_o} \mathrm{d}\tau \left(\Phi' + \Psi' + \omega_i' \hat{n}^i - \frac{1}{2}\gamma_{ij}' \hat{n}^i \hat{n}^j\right)\right] .}$$

e = emission Last scattering surface.

o = observation.

$\frac{\delta T}{T}$@ 2nd order on large scales

LB, P. Creminelli, G. D'Amico, J. Noreña, F. Vernizzi. '09

- Temperature anisotropies @ 2nd order

$$\boxed{\begin{aligned}\frac{\delta T}{T}(\hat{n}) = &\quad \left[\tfrac{1}{3}\phi + \tfrac{1}{18}\phi^2 + \tfrac{1}{3}\partial^{-2}\Big((\partial_i\phi)^2 - 3\partial^{-2}\partial_i\partial_j(\partial_i\phi\partial_j\phi)\Big)\right]_e \\ + &\quad \int_{\tau_e}^{\tau_o} \mathrm{d}\tau \left(\Phi' + \Psi' + \omega_i'\hat{n}^i - \tfrac{1}{2}\gamma_{ij}'\hat{n}^i\hat{n}^j\right) + \tfrac{1}{3}\vec{\alpha}\cdot\vec{\nabla}_{\hat{n}}\phi_e\,,\end{aligned}}$$

Pyne & Carroll 95, Molerrach & Matarrese 97

- Various contributions that have to be added up. Can't neglect any of them.
  - Intrinsic contribution
  - Integrated contribution
    - ISW or Rees-Sciama
    - Vector
    - Tensor
  - Lensing

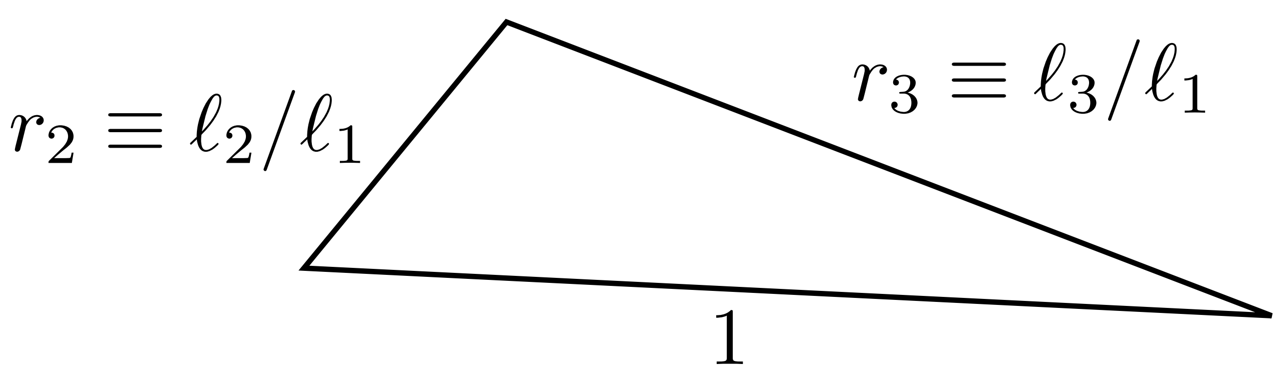
The flat-sky approximation

- For large multipoles, we can use the flat-sky formalism.
- Not accurate on large scales, however expressions are more tractable.
- Pick a small patch of the sky with direction $\hat{z} = (m_x, m_y, 1)$ and expand around it.
- Work with 2d Fourier transform instead of spherical harmonics.

$$a_\ell = \int d^2\vec{m}\, \frac{\delta T}{T}(\hat{n})\, e^{-i\vec{\ell}\cdot\vec{m}}\,, \qquad \frac{\delta T}{T}(\hat{n}) = \int \frac{d^2\ell}{(2\pi)^2}\, a_\ell\, e^{i\vec{\ell}\cdot\vec{m}}$$

- The bispectrum reads
$$\langle a_{\ell_1} a_{\ell_2} a_{\ell_3} \rangle = (2\pi)^2 \delta^{(2)}(\vec{\ell}_1 + \vec{\ell}_2 + \vec{\ell}_3) B(\ell_1, \ell_2, \ell_3)$$

- It scales as
$$B(\lambda\ell_1, \lambda\ell_2, \lambda\ell_3) = \lambda^{-4} B(\ell_1, \ell_2, \ell_3).$$



Intrinsic Contribution

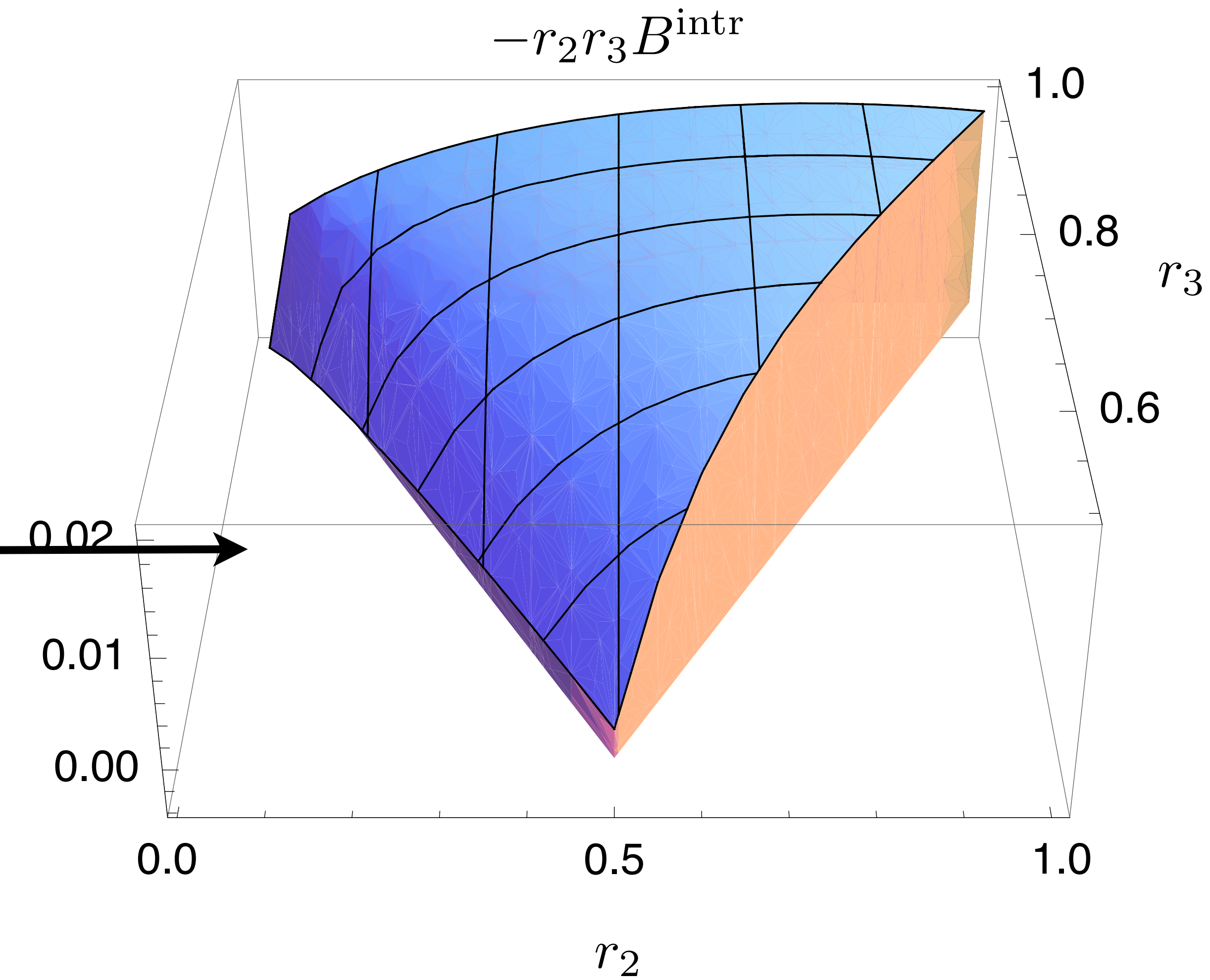
$$\frac{\delta T}{T}(\hat{n}) = \left[\tfrac{1}{3}\phi + \boxed{\tfrac{1}{18}\phi^2 + \tfrac{1}{3}\partial^{-2}\Big((\partial_i\phi)^2 - 3\partial^{-2}\partial_i\partial_j(\partial_i\phi\partial_j\phi)\Big)}\right]_e$$
$$+ \int_{\tau_e}^{\tau_o} \mathrm{d}\tau \left(\Phi' + \Psi' + \omega_i'\hat{n}^i - \tfrac{1}{2}\gamma_{ij}'\hat{n}^i\hat{n}^j\right) + \tfrac{1}{3}\vec{\alpha}\cdot\vec{\nabla}_{\hat{n}}\phi_e\,,$$

- k-independent is of local type

$$f_{\rm NL}^{\rm local} = -1/6$$

- k-dependent is of equilateral type

$$f_{\rm NL}^{\rm equil} = 1.21$$



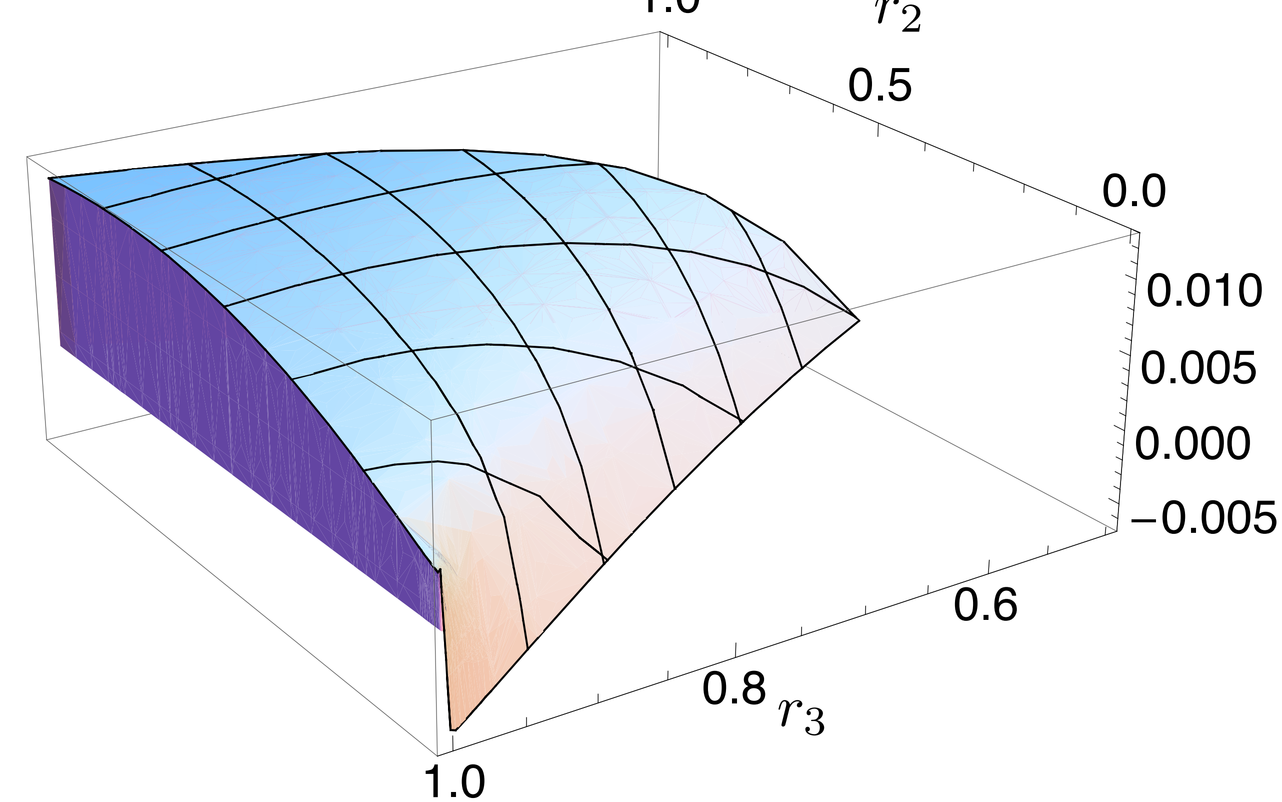
Rees-Sciama Contribution

$$\frac{\delta T}{T}(\hat{n}) = \left[\tfrac{1}{3}\phi + \tfrac{1}{18}\phi^2 + \tfrac{1}{3}\partial^{-2}\big((\partial_i\phi)^2 - 3\partial^{-2}\partial_i\partial_j(\partial_i\phi\partial_j\phi)\big)\right]_e$$

$$+ \int_{\tau_e}^{\tau_o} \mathrm{d}\tau \left(\boxed{\Phi' + \Psi'} + \omega_i'\hat{n}^i - \tfrac{1}{2}\gamma_{ij}'\hat{n}^i\hat{n}^j\right) + \tfrac{1}{3}\vec{\alpha}\cdot\vec{\nabla}_{\hat{n}}\phi_e\,,$$

RS bispectrum is of the equilateral type.

$$f_{\mathrm{NL}}^{\mathrm{equil}} = 0.74$$

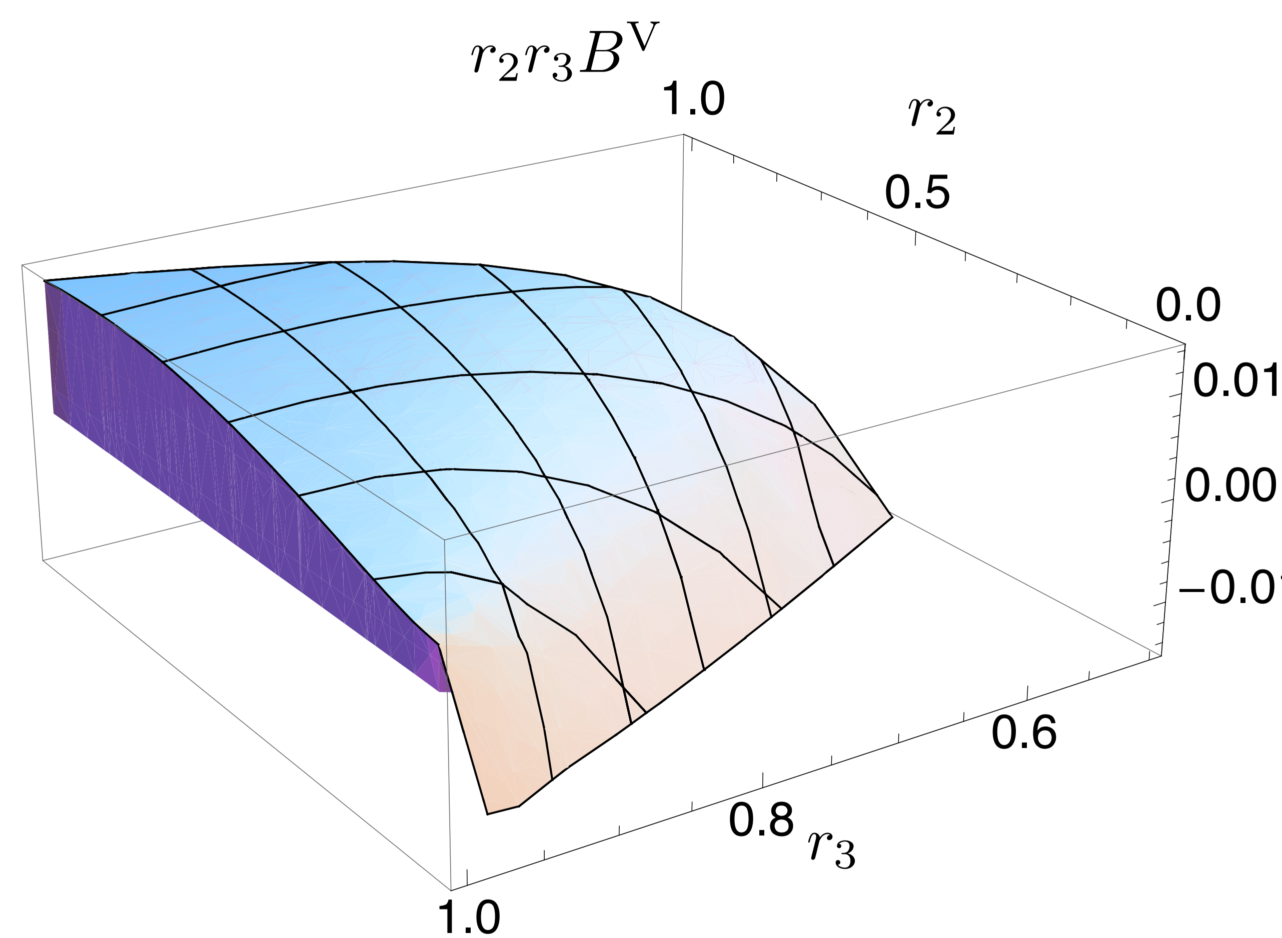


Integrated Vector Contribution

$$\frac{\delta T}{T}(\hat{n}) = \left[\tfrac{1}{3}\phi + \tfrac{1}{18}\phi^2 + \tfrac{1}{3}\partial^{-2}\Big((\partial_i\phi)^2 - 3\partial^{-2}\partial_i\partial_j(\partial_i\phi\partial_j\phi)\Big)\right]_e$$
$$+ \int_{\tau_e}^{\tau_o} \mathrm{d}\tau \left(\Phi' + \Psi' + \boxed{\omega_i'\hat{n}^i} - \tfrac{1}{2}\gamma_{ij}'\hat{n}^i\hat{n}^j\right) + \tfrac{1}{3}\vec{\alpha}\cdot\vec{\nabla}_{\hat{n}}\phi_e\,,$$

Vector bispectrum is of the equilateral type.

$$f_{\rm NL}^{\rm equil} = -0.84$$



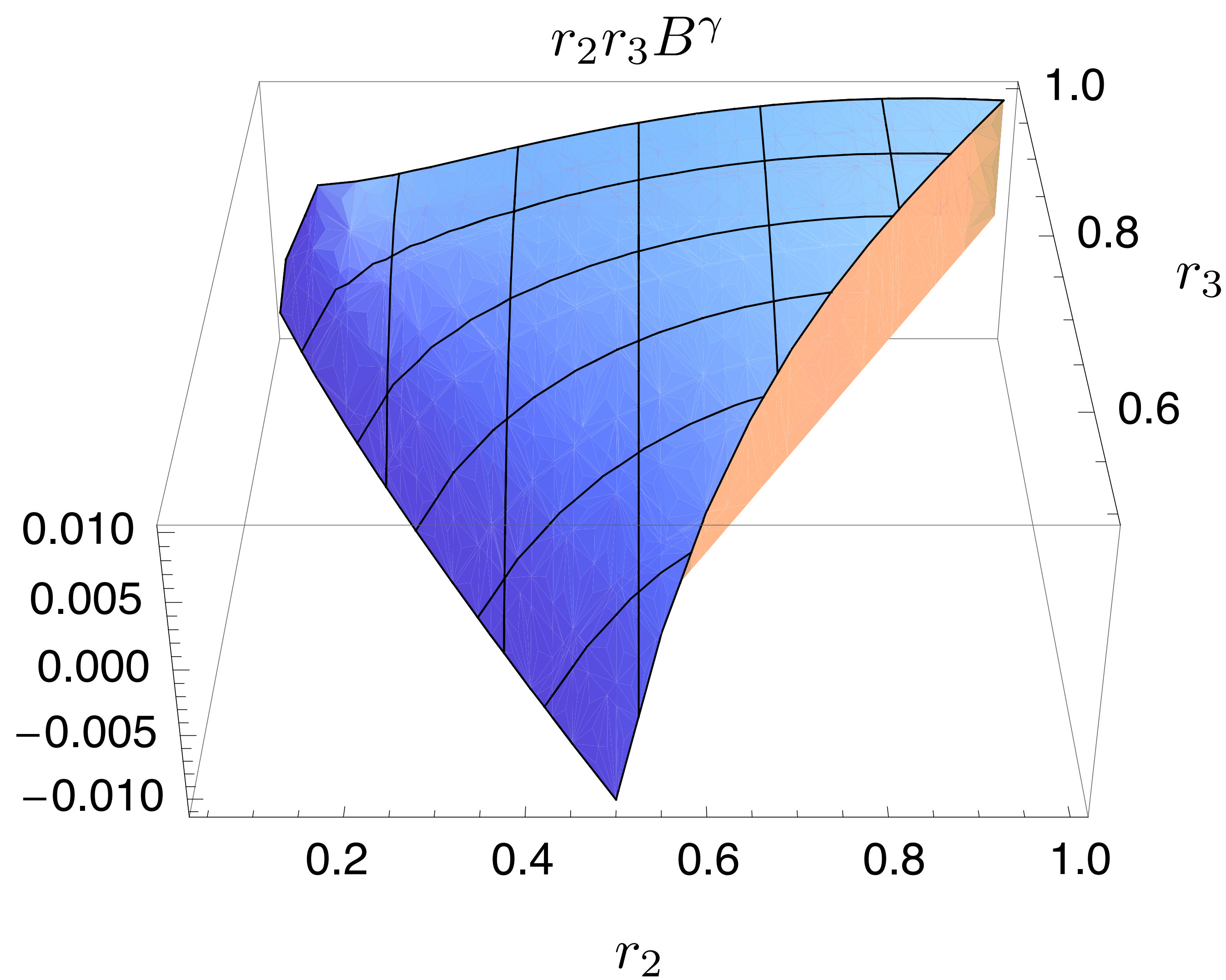
Integrated Tensor Contribution

$$\frac{\delta T}{T}(\hat{n}) = \left[\tfrac{1}{3}\phi + \tfrac{1}{18}\phi^2 + \tfrac{1}{3}\partial^{-2}\Big((\partial_i\phi)^2 - 3\partial^{-2}\partial_i\partial_j(\partial_i\phi\partial_j\phi)\Big)\right]_e$$

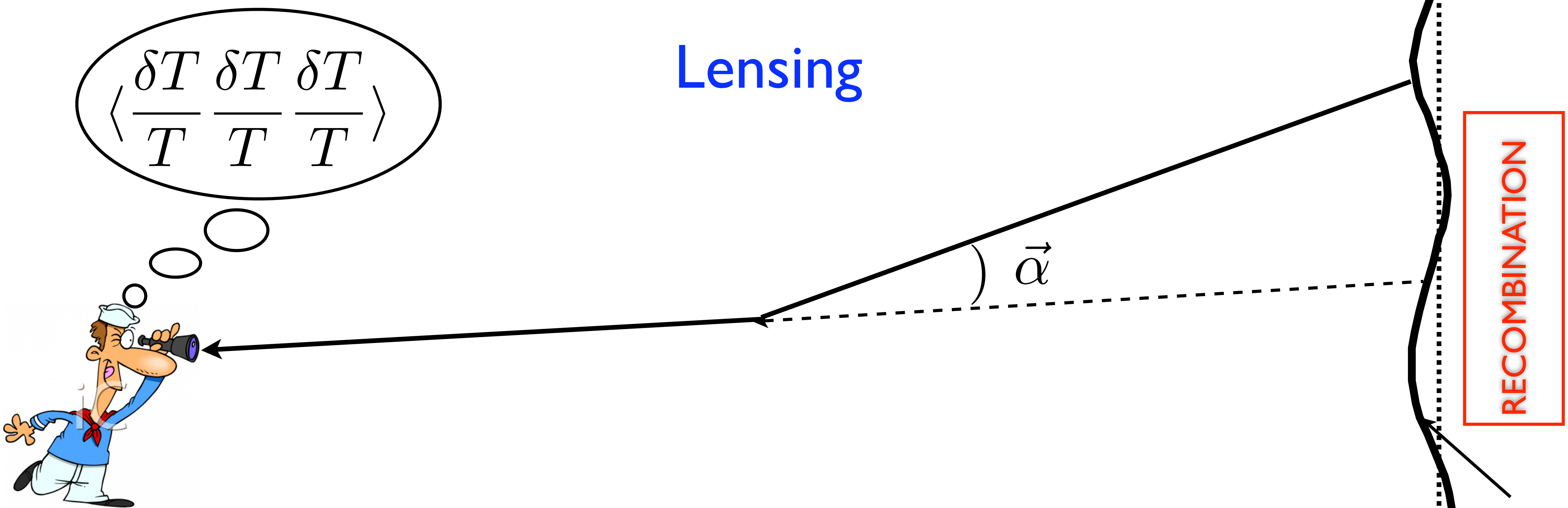
$$+ \int_{\tau_e}^{\tau_o} \mathrm{d}\tau \left(\Phi' + \Psi' + \omega_i'\hat{n}^i - \boxed{\tfrac{1}{2}\gamma_{ij}'\hat{n}^i\hat{n}^j}\right) + \tfrac{1}{3}\vec{\alpha}\cdot\vec{\nabla}_{\hat{n}}\phi_e\,,$$

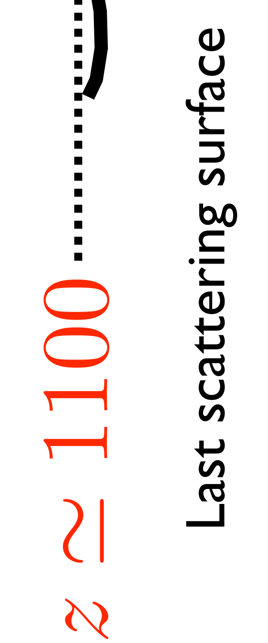
Tensor bispectrum is of the equilateral type.

$$f_{\mathrm{NL}}^{\mathrm{equil}} = -0.61$$



Lensing





$$\frac{\delta T}{T}(\hat{n})\Big|_{\rm lens} = \frac{\delta T}{T}(\hat{n}+\vec{\alpha}) \simeq \frac{\delta T}{T}(\hat{n}) + \vec{\alpha}\cdot\vec{\nabla}_{\hat{n}}\frac{\delta T}{T}(\hat{n})$$

$$\boxed{\vec{\alpha} = -2\int_{\tau_e}^{\tau_o} \mathrm{d}\tau \frac{\tau-\tau_e}{\tau_o-\tau_e}\vec{\nabla}_{\parallel}\phi\,.}$$

Lensing

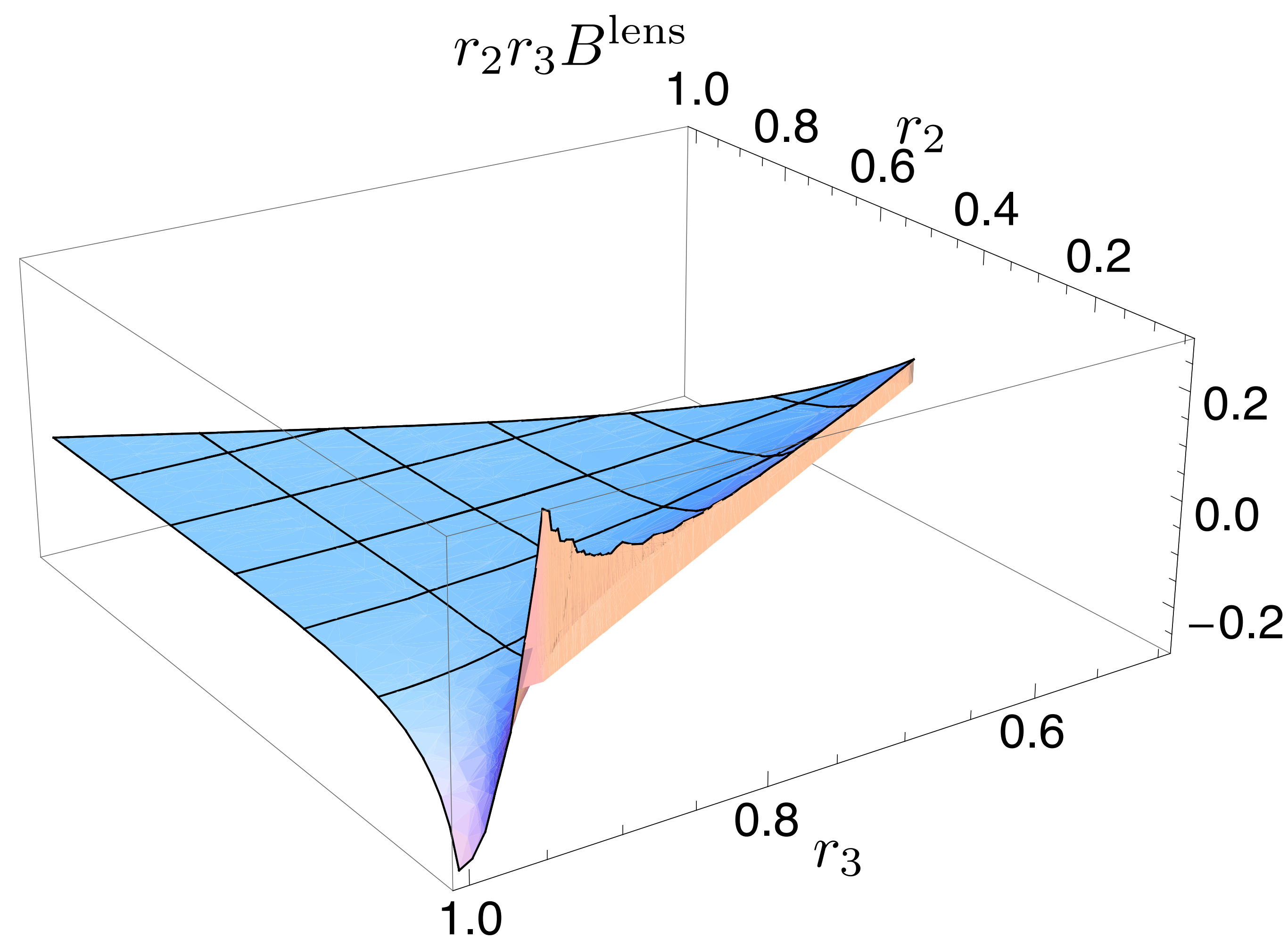
$$\frac{\delta T}{T}(\hat{n}) = \left[\tfrac{1}{3}\phi + \tfrac{1}{18}\phi^2 + \tfrac{1}{3}\partial^{-2}\Big((\partial_i\phi)^2 - 3\partial^{-2}\partial_i\partial_j(\partial_i\phi\partial_j\phi)\Big)\right]_e$$
$$+ \int_{\tau_e}^{\tau_o} \mathrm{d}\tau \left(\Phi' + \Psi' + \omega_i'\hat{n}^i - \tfrac{1}{2}\gamma_{ij}'\hat{n}^i\hat{n}^j\right) + \boxed{\tfrac{1}{3}\vec{\alpha}\cdot\vec{\nabla}_{\hat{n}}\phi_e},$$

- local

$$f_{\rm NL}^{\rm local} = -\cos(2\theta)$$

- equilateral

$$f_{\rm NL}^{\rm equil} = 2.87$$

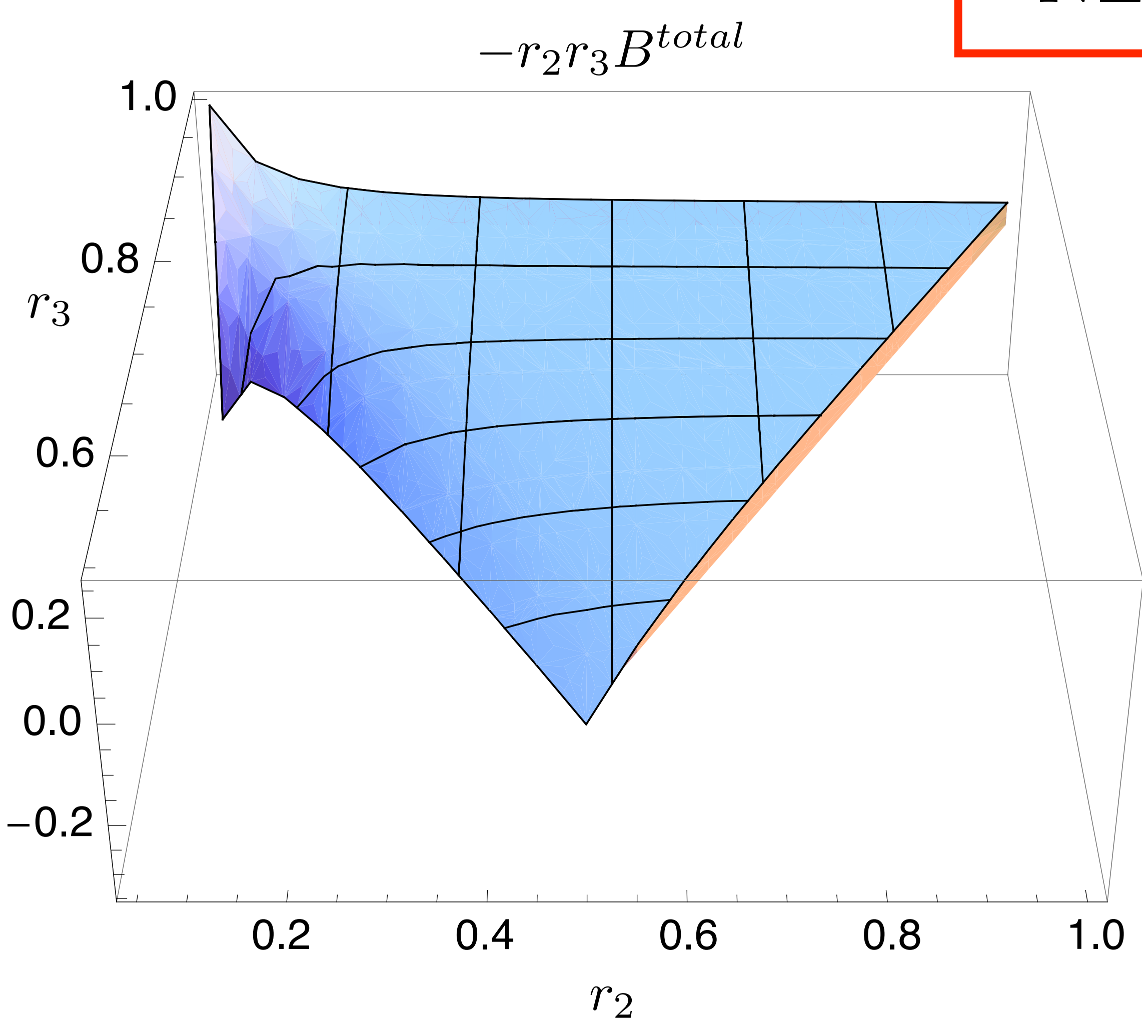


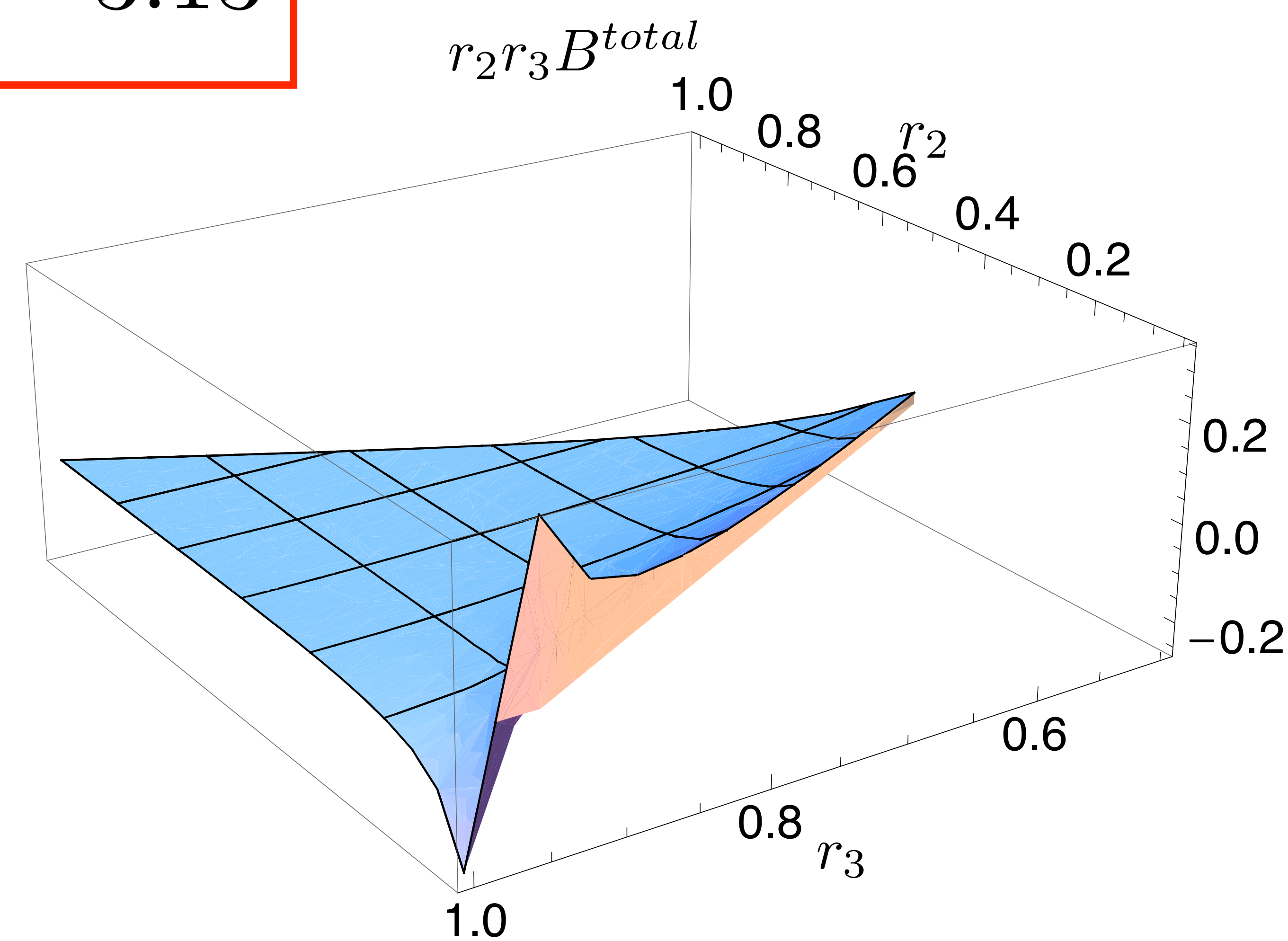
Total Bispectrum

$$\frac{\delta T}{T}(\hat{n}) = \left[\tfrac{1}{3}\phi + \tfrac{1}{18}\phi^2 + \tfrac{1}{3}\partial^{-2}\Big((\partial_i\phi)^2 - 3\partial^{-2}\partial_i\partial_j(\partial_i\phi\partial_j\phi)\Big)\right]_e$$
$$+ \int_{\tau_e}^{\tau_o} \mathrm{d}\tau \left(\Phi' + \Psi' + \omega_i'\hat{n}^i - \tfrac{1}{2}\gamma_{ij}'\hat{n}^i\hat{n}^j\right) + \tfrac{1}{3}\vec{\alpha}\cdot\vec{\nabla}_{\hat{n}}\phi_e\,,$$

- Total bispectrum dominated by lensing ==>mostly equilateral.

$$\boxed{f_{\mathrm{NL}}^{\mathrm{equil}} = 3.13}$$





Conclusions and outlook

- Total bispectrum dominated by lensing ==>mostly equilateral. $f_{\rm NL}^{\rm equil} = 3.13$
- Squeezed limit $f_{\rm NL}^{\rm local} = -1/6 - \cos(2\theta)$
- Out of reach of PLANCK satellite.
- Refinements of computation
  - Small scale plasma effects.
    - Non-linear DM at short scales. (Bartolo, Matarrese, Riotto 06, Pitrou, Bernardeau, Uzan 08)
    - Perturbed recombination (Khatri, Wandelt 08, Senatore, Tassev, Zaldarriaga 08)
    - Full control of various effects (Nitta, Komatsu, Bartolo, Matarrese, Riotto 08)
  - Inclusion of tilt.
  - Full-Sky computation.

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Non-Gaussianity is a powerful tool to test theories of the early universe.