On the cosmological backreaction for large distance modifications of gravity

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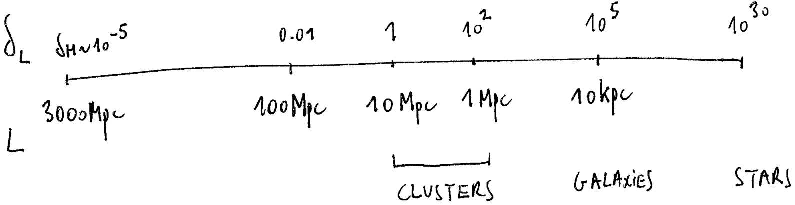
the (inhomogeneous) Universe



$$\delta(x) = \frac{\rho_M(x) - \bar{\rho}_M}{\bar{\rho}_M}$$

$$< \delta(x)\delta(y) >= \int \frac{d^3k}{4\pi k^3} \Delta^2(k) e^{ik.(x-y)} \qquad \delta_L = \Delta(1/L)$$

backreaction for Einstein gravity

the metric of our Universe (Ishibashi and Wald 2006):

$$ds^2 \approx -dt^2(1+2\psi(x,t))+a(t)^2(1-2\psi(x,t))(dx^2+dy^2+dz^2) \qquad (\psi \ll 1)$$

backreaction for Einstein gravity

$$ds^2 = -dt^2(1 + 2\psi(x, t)) + a(t)^2(1 + 2\phi(x, t))(dx^2 + dy^2 + dz^2)$$

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$$\begin{aligned} G_{00} &\approx 3H^2 - 2a^{-2}\nabla^2\phi + a^{-2}\left(3\partial_i\phi\partial_i\phi - 4\psi\nabla^2\phi + 8\phi\nabla^2\phi\right) + \ldots \\ &\approx 8\pi G_N\overline{\rho}_{tot}(t) + 8\pi G_N\overline{\rho}_M(t)\delta(x,t) \\ G^i_i &\approx -(3H^2 + 6\frac{\ddot{a}}{a}) + 2a^{-2}\nabla^2(\phi + \psi) + a^{-2}\left(-3\partial_i\phi\partial_i\phi + \ldots\right) + \ldots \\ &\approx 24\pi G_N\overline{p}_{tot}(t) \end{aligned}$$

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first order: $\phi^{(1)} = -4\pi G_N\overline{\rho}_M\frac{a^2}{\nabla^2}\delta \qquad \psi^{(1)} = -\phi^{(1)}$

local compact system: $\psi \approx -\phi \approx \frac{G_N M}{ra}$

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local compact system: $\psi \approx -\phi \approx \frac{G_N M}{ra}$

higher order: $\psi = -\frac{G_N M}{ra}(1 - \frac{G_N M}{ra} + \ldots)$

nearly all local objects are in the linear Newtonian regime

backreaction for Einstein gravity

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- backreaction linear fluctuations (Kolb et al 2005): $\delta_L \sim \delta_H \frac{a}{(LH_0)^2}$

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$$<\phi\frac{\nabla^2}{a^2}\phi>\approx (4\pi G_N)^2\bar{\rho}_M^2 <\delta\frac{a^2}{\nabla^2}\delta>\sim H^4L^2a^2\delta_L^2 \sim \left(\frac{H}{H_0}\right)^4 \frac{a^4\delta_H^2}{L^2} \sim \delta_H H^2$$

backreaction for Einstein gravity

- backreaction linear fluctuations (Kolb et al 2005): $\delta_L \sim \delta_H \frac{a}{(LH_0)^2}$

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- backreaction from 'nonlinear' compact sources (Gruzinov et al 2006)

$$<\phi \frac{\nabla^2}{a^2} \phi> \sim n_o G_N^2 \frac{M^2}{R} \sim G_N \bar{\rho}_o \frac{R_S}{R}$$

(gravitational energy $\sim M \frac{R_S}{R}$)

backreaction for Einstein gravity

- backreaction linear fluctuations (Kolb et al 2005): $\delta_L \sim \delta_H \frac{a}{(LH_0)^2}$

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(gravitational energy $\sim M\frac{R_S}{R}$)

⋆ backreaction in present universe is small $\sim 10^{-5}$

⋆ if at some era $R \rightarrow R_S$ for the dominant matter sources → order one backreaction

large distance modifications of gravity

alternative to dark energy

$$
\begin{aligned}
G_{\mu\nu}(g) + H_{\mu\nu}(g,\pi) &= 8\pi G_N T_{\mu\nu} \\
E_\pi(g,\pi) &= 0
\end{aligned}
\qquad \left(H_{00}^{(0)} \sim H^2 (\frac{H_0}{H})^n \right)
$$

large distance modifications of gravity

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- homogeneous Universe: dark energy=modified gravity
- gravity is everywhere: Earth,Solar system,galaxies,...,Universe

large distance modifications of gravity

linear perturbations

Brans-Dicke, DGP, $f(R)$, $f(\mathcal{G})$, ...

$$G_{00}^{(0)} + H_{00}^{(0)} + \frac{1}{a^2}\left(-2\nabla^2\phi + \nabla^2(c_1\phi + c_2\psi + c_3\pi)\right) \approx 8\pi G_N\bar{\rho}_M(t)(1+\delta(x,t))$$

$$G_i^{i\,(0)} + H_i^{i\,(0)} + \frac{1}{a^2}\left(2\nabla^2(\phi+\psi) + \nabla^2(c_4\phi + c_5\psi + c_6\pi)\right) \approx 0$$

$$E_\pi^{(0)} + \frac{1}{a^2}\nabla^2(c_7\phi + c_8\psi + c_9\pi) \approx 0$$

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for a compact source: $\phi^{(1)} \approx \frac{\alpha_1 G_N M}{ra}$ $\psi^{(1)} \approx \frac{\alpha_2 G_N M}{ra}$ $\pi^{(1)} \approx \frac{\alpha_3 G_N M}{ra}$

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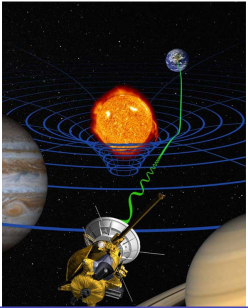

Cassini: $\frac{\alpha_1}{\alpha_2} = (-1 \pm 10^{-5})$
(Bertotti et al, 2003)

ruled out, **if** the linearization is valid inside Solar system

large distance modifications of gravity

nonlinear terms (Vainshtein 1972; Deffayet et al 2001; Navarro, KVA 2006)

$$H_{00}, H^i_i, E_\pi \sim \frac{(\nabla^2)^{m+1}\phi^{n+1}}{a^{2(m+1)}H_0^{2m}}$$

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$$H_{00}, H^i_i, E_\pi \sim \frac{(\nabla^2)^{m+1}\phi^{n+1}}{a^{2(m+1)}H_0^{2m}}$$

$$\begin{aligned} \phi &\approx \alpha_1 \frac{G_N M}{ra}\left(1 + \frac{(G_N M)^n}{(ra)^{2m+n}H_0^{2m}} + \ldots\right) \\ &\approx \alpha_1 \frac{G_N M}{ra}\left(1 + (\frac{R_V}{ra})^{2m+n} + (\frac{R_V}{ra})^{4m+2n} + \ldots\right) \qquad R_V \equiv (\frac{(G_N M)^n}{H_0^{2m}})^{\frac{1}{2m+n}} \end{aligned}$$

large distance modifications of gravity

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$$\approx \alpha_1 \frac{G_N M}{ra}\left(1 + (\frac{R_V}{ra})^{2m+n} + (\frac{R_V}{ra})^{4m+2n} + \dots\right) \qquad R_V \equiv (\frac{(G_N M)^n}{H_0^{2m}})^{\frac{1}{2m+n}}$$

- $f(R)$, Brans-Dicke: $m = 0$, $R_V = R_S$ $\rightarrow$ ruled out
  (Erickcek et al 2006; Navarro and KVA 2006; Faulkner et al 2006)
- massive gravity: $m = 2, n = 1$ $R_V = \left(G_N M/H_0^4\right)^{1/5}$
- DGP, $f(\mathcal{G})$: $m = 1, n = 1$ $R_V = (G_N M/H_0^2)^{1/3}$

large distance modifications of gravity

estimating the backreaction

$$< \frac{(\nabla^2)^{m+1}\phi^{n+1}}{a^{2(m+1)}H_0^{2m}} > \sim n_o \frac{(G_N M)^{n+1}}{R^{2m+n}H_0^{2m}} \sim G_N \bar{\rho}_o \left(\frac{R_V}{R}\right)^{2m+n}$$

if $R \to R_V$ for the dominant structures $\to$ order one backreaction.

large distance modifications of gravity

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if $R \rightarrow R_V$ for the dominant structures $\rightarrow$ order one backreaction.

loophole: if nonlinear term is a total derivative, average gravitational energy is zero

$f(\mathcal{G})$ gravity

FLRW solutions

- full equations:

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,(R+f(\mathcal{G})) \qquad (\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau})$$

$$G_{\mu\nu} + 4T_{\alpha\mu\beta\nu}\nabla^\alpha\nabla^\beta f_{\mathcal{G}} + \frac{1}{2}g_{\mu\nu}(\mathcal{G}f_{\mathcal{G}} - f) = 8\pi G_N T_{\mu\nu}$$

$$T_{\alpha\mu\beta\nu} = R_{\alpha\mu\beta\nu} - (R_{\alpha\beta}g_{\nu\mu} - \ldots) + \frac{R}{2}(g_{\alpha\beta}g_{\mu\nu} - g_{\alpha\nu}g_{\mu\beta})$$

- Friedmann equation:

$$G_{00}^{(0)} + H_{00}^{(0)} = 3H^2 + 12H^3\dot{f}_{\mathcal{G}}^{(0)} - \frac{1}{2}(\mathcal{G}^{(0)}f_{\mathcal{G}}^{(0)} - f^{(0)}) = 8\pi G_N\bar{\rho}_M$$

$$\mathcal{G}^{(0)} = 24H^2\frac{\ddot{a}}{a} \sim H^4$$

- cosmic self acceleration (De Felice, Tsujikawa 2009):

$$\mathcal{G}^{(0)}f_{\mathcal{G}}^{(0)} - f^{(0)} \sim H^2(\frac{H_0}{H})^n\,,\quad f_{\mathcal{G}\mathcal{G}}^{(0)} \sim \frac{1}{H^6}(\frac{H_0}{H})^n\,,\ldots$$

$f(\mathcal{G})$ gravity

linear perturbations

$$
\begin{aligned}
G^{(1)}_{00} + H^{(1)}_{00} &\approx \frac{1}{a^2}\nabla^2\left(-2\phi - 8H\dot{f}^{(0)}_{\mathcal{G}}\phi - 4H^2 f^{(0)}_{\mathcal{G}\mathcal{G}}\widetilde{\mathcal{G}}\right) \approx 8\pi G_N \bar{\rho}_M \delta \\
G^{i\,(1)}_{i} + H^{i\,(1)}_{i} &\approx \frac{1}{a^2}\nabla^2\left(2(\phi+\psi) + 8\ddot{f}^{(0)}_{\mathcal{G}}\phi + 8\dot{f}^{(0)}_{\mathcal{G}}H\psi + 8\frac{\ddot{a}}{a}f^{(0)}_{\mathcal{G}\mathcal{G}}\widetilde{\mathcal{G}}\right) \approx 0 \\
\widetilde{\mathcal{G}} &\approx -\frac{1}{a^2}\nabla^2\left(16\frac{\ddot{a}}{a}\phi + 8H^2\psi\right)
\end{aligned}
$$

$f(\mathcal{G})$ gravity

linear perturbations

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G^{(1)}_{00} + H^{(1)}_{00} &\approx \frac{1}{a^2}\nabla^2\left(-2\phi - 8H\dot{f}^{(0)}_{\mathcal{G}}\phi - 4H^2 f^{(0)}_{\mathcal{G}\mathcal{G}}\widetilde{\mathcal{G}}\right) \approx 8\pi G_N \bar{\rho}_M \delta \\
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$$
\pi \equiv \frac{\widetilde{\mathcal{G}}}{\mathcal{G}^{(0)}}
$$

$f(\mathcal{G})$ gravity

linear perturbations

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\begin{aligned}
G_{00}^{(1)} + H_{00}^{(1)} &\approx \frac{1}{a^2}\nabla^2\left(-2\phi + c_1\phi + c_2\pi\right) \approx 8\pi G_N \bar{\rho}_M \delta \\
G_i^{i\,(1)} + H_i^{i\,(1)} &\approx \frac{1}{a^2}\nabla^2\left(2(\phi+\psi) + c_3\phi + c_4\psi + c_5\pi\right) \approx 0 \\
E_\pi^{(1)} &\approx \frac{1}{a^2}\nabla^2\left(d_1\phi + d_2\psi\right) + d_3 H^2\pi \approx 0
\end{aligned}
$$

$$
c_i \sim (\frac{H_0}{H})^n\,,\ d_i \sim 1
$$

$f(\mathcal{G})$ gravity

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$$
c_i \sim (\frac{H_0}{H})^n\,,\ d_i \sim 1
$$

$$
\begin{aligned}
\pi^{(1)} &= (\frac{H}{H_0})^n \frac{4\pi G_N \bar{\rho}_M \delta}{\nabla^2/a^2 - H^2(\frac{H}{H_0})^n} \\
\phi^{(1)} &= -\frac{4\pi G_N \bar{\rho}_M a^2 \delta}{\nabla^2(1+(\frac{H_0}{H})^n)} + (\frac{H_0}{H})^n \pi^{(1)} \\
\psi^{(1)} &= \frac{4\pi G_N \bar{\rho}_M a^2 \delta}{\nabla^2(1+(\frac{H_0}{H})^n)} + (\frac{H_0}{H})^n \pi^{(1)}
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$$
\begin{aligned}
\pi^{(1)} &= \left(\frac{H}{H_0}\right)^n \frac{4\pi G_N\bar{\rho}_M\delta}{\nabla^2/a^2 - H^2\left(\frac{H}{H_0}\right)^n} \\
\phi^{(1)} &= -\frac{4\pi G_N\bar{\rho}_M a^2\delta}{\nabla^2\left(1+\left(\frac{H_0}{H}\right)^n\right)} + \left(\frac{H_0}{H}\right)^n\pi^{(1)} \\
\psi^{(1)} &= \frac{4\pi G_N\bar{\rho}_M a^2\delta}{\nabla^2\left(1+\left(\frac{H_0}{H}\right)^n\right)} + \left(\frac{H_0}{H}\right)^n\pi^{(1)}
\end{aligned}
$$

$f(\mathcal{G})$ gravity

nonlinear terms

$$
\begin{aligned}
S &= \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}f(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau}) \\
&= \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left(f_{\mathcal{G}}(\mathcal{G})(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau} - \mathcal{G}) + f(\mathcal{G})\right)
\end{aligned}
$$

$$
\mathcal{L}^{(3)} \approx f^{(0)}_{\mathcal{G}\mathcal{G}}\widetilde{\mathcal{G}}\frac{8}{a^4}\left(\nabla^2\phi\nabla^2\psi - \partial_i\partial_j\phi\partial_i\partial_j\psi\right) \sim (\frac{H_0}{H})^n\frac{\pi(\nabla^2\phi\nabla^2\psi - \partial_i\partial_j\phi\partial_i\partial_j\psi)}{H^2}
$$

$f(\mathcal{G})$ gravity

nonlinear terms

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}f(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau})$$
$$= \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\left(f_{\mathcal{G}}(\mathcal{G})(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau} - \mathcal{G}) + f(\mathcal{G})\right)$$

$$\mathcal{L}^{(3)} \approx f^{(0)}_{\mathcal{G}\mathcal{G}}\widetilde{\mathcal{G}}\frac{8}{a^4}\left(\nabla^2\phi\nabla^2\psi - \partial_i\partial_j\phi\partial_i\partial_j\psi\right) \sim \left(\frac{H_0}{H}\right)^n\frac{\pi(\nabla^2\phi\nabla^2\psi - \partial_i\partial_j\phi\partial_i\partial_j\psi)}{H^2}$$

$$\pi(\nabla^2\phi\nabla^2\psi - \partial_i\partial_j\phi\partial_i\partial_j\psi) \overset{PI}{=} \psi(\nabla^2\phi\nabla^2\pi - \partial_i\partial_j\phi\partial_i\partial_j\pi) \overset{PI}{=} \phi(\nabla^2\pi\nabla^2\psi - \partial_i\partial_j\pi\partial_i\partial_j\psi)$$

'Galileian' symmetry (Nicolis et al 2008):

$\pi \to \pi + a_\pi + b^i_\pi x_i \quad \phi \to \phi + a_\phi + b^i_\phi x_i \quad \psi \to \psi + a_\psi + b^i_\psi x_i$

- shift symmetry $\to$ total derivatives in eqs.
- extra 'Galileian' symmetry $\to$ no extra degrees of freedom

$f(\mathcal{G})$ gravity

nonlinear terms

- local effect

$$\phi = \alpha_1 \frac{G_N M}{ra}(1 + (\frac{R_V}{ra})^3 + \ldots) \qquad (R_V \equiv (G_N M/H^2)^{1/3})$$

all localized structure with $(R_V/R)^3 \sim \delta \geq 1$ is in the (gravitational) nonlinear regime

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all localized structure with $(R_V/R)^3 \sim \delta \geq 1$ is in the (gravitational) nonlinear regime

- average effect

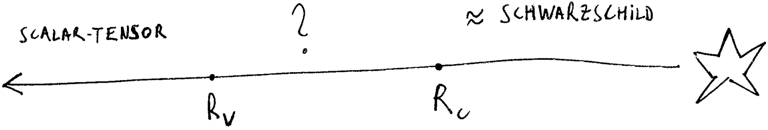
$$< \nabla^2 \phi \nabla^2 \pi - \partial_i \partial_j \phi \partial_i \partial_j \pi >= 0$$

no backreaction...

$f(\mathcal{G})$ gravity

matching the linear with the nonlinear regime

- isolated body on a cosmic background (Navarro,KVA 2005)



- N-bodies on a cosmic background? (one can not just add forces!)

summary/conclusions/outlook...

- cosmology takes place in our world

- it is not impossible to modify GR in the infrared
  - consistent with the SS constraints (nonlinear Vainshtein mechanism)
  - without abandoning the FLRW framework

- 'gravitational energy' averages out to zero, because of the shift symmetry in the nonlinear term, $f(\mathcal{G})$, DGP (Lue 2005)

- a full matching of the linear and nonlinear regime, N-body simulations: still quite a challenge