Gauge unification and magic fields

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Calibbi, Ferretti, R and Ziegler, Phys. Lett. B **672** (2009) 152, arXiv:0812.0342 [hep-ph]

(thanks to Lorenzo for providing the file)

Unification

Three hints for Grand Unification:

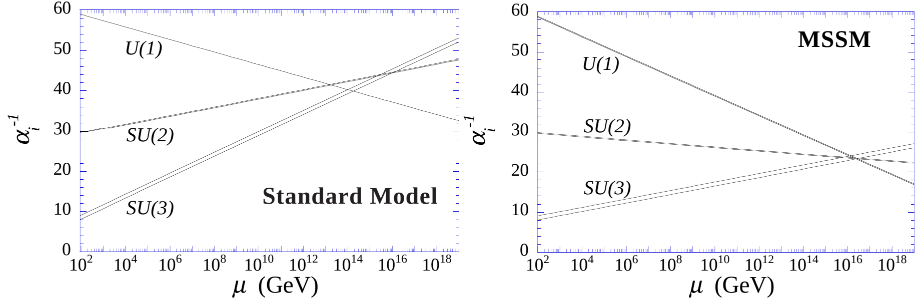
- The structure of the SM quantum numbers
- The successful (within the MSSM) prediction of gauge coupling unification
- The successful prediction of the GUT scale

The structure of the SM quantum numbers

	SU(3)	SU(2)	U(1)		SO(10)
L_i	1	2	-1/2		
e^c_i	1	1	1		
Q_i	3	2	1/6	⇒	16
u^c_i	3^*	1	-2/3		
d^c_i	3^*	1	1/3		
			Y		

Gauge coupling unification in the MSSM

$$\alpha_3^{\exp}(M_Z) = 0.1176 \pm 0.0020$$



$$M_{\rm GUT}^0 \simeq 2 \cdot 10^{16}\,{\rm GeV}, \quad \alpha_U \simeq 1/24$$

The prediction of the GUT scale

- The "experimental" window for M_{GUT} is not too wide
  - $M_{\text{GUT}} \gtrsim 10^{15}\,\text{GeV}$ (p-decay from GUT gauge interactions, unavoidable)
  - $M_{\text{GUT}} \lesssim 10^{19}\,\text{GeV}$ to avoid the transplanckian regime
- The MSSM prediction $M^0_{\text{GUT}} \approx 2 \cdot 10^{16}\,\text{GeV}$ falls in the window
- Note: in the minimal supersymmetric SU(5) $M^0_{\text{GUT}} \approx 2 \cdot 10^{16}\,\text{GeV}$ is not large enough to suppress p-decay from Higgs triplet, model dependent, Yukawa interactions (but that's killing a dead horse[1])
- Some string models predict a GUT scale larger than M^0_{GUT}

[1] Murayama and Pierce, hep-ph/0108104; Bajc, Fileviez Perez, Senjanovic, hep-ph/0204311

A "desert"?

- The MSSM prediction holds if there are no additional fields ("desert") between the weak and the GUT scale
- Many SM extensions predict the existence of fields at intermediate scales: a high energy origin of neutrino masses, supersymmetry breaking, flavour breaking, etc. . .
- The new fields might affect the running of the gauge couplings and thus spoil unification
- Gauge coupling unification can often be *reproduced* by an appropriate choice of the scale at which the new fields live[2], at the price of loosing what in the MSSM can be considered a *prediction*
- Unless the new fields form complete SU(5) representations. Then
  - The MSSM prediction for $\alpha_s(M_Z)$ is preserved at one loop, independently of the scale at which the new fields appear
  - The unification scale is also unchanged at one loop

[2] See for instance Aulakh, Bajc, Melfo, Rasin, Senjanovic, hep-ph/0004031

Gauge coupling unification at one loop

1-loop renormalization group equation of the gauge couplings ($t = \log(\mu/\mu_0)$):

$$\frac{d}{dt}\alpha_i^{-1} = -\frac{1}{2\pi} b_i, \quad \alpha_i \equiv \frac{g_i^2}{4\pi}$$

β-function coefficients:

$$b_i = \frac{2}{3}\sum_f S_f + \frac{1}{3}\sum_b S_b - \frac{11}{3} C_2(G) \overset{\text{SUSY}}{\Rightarrow} b_i = \sum_R S_R - 3\, C_2(G)$$

Requiring unification:

$$\frac{1}{\alpha_U} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi} \log\left(\frac{M_{\text{GUT}}}{M_Z}\right), \; i = 1, 2, 3$$

$\alpha_3(M_Z)$, M_{GUT}, α_U can be obtained in terms of $\alpha_1(M_Z)$, $\alpha_2(M_Z)$:

$$\boxed{\frac{1}{\alpha_3} = \frac{1}{\alpha_2} + \frac{b_3 - b_2}{b_2 - b_1}\left(\frac{1}{\alpha_2} - \frac{1}{\alpha_1}\right)},$$

$$\log\left(\frac{M_{\text{GUT}}}{M_Z}\right) = \frac{2\pi}{b_2 - b_1}\left(\frac{1}{\alpha_2} - \frac{1}{\alpha_1}\right), \quad \frac{1}{\alpha_U} = \frac{1}{\alpha_1} - \frac{b_1}{b_2 - b_1}\left(\frac{1}{\alpha_2} - \frac{1}{\alpha_1}\right)$$

Adding to the MSSM

MSSM 1-loop predictions $\leftrightarrow (b_1, b_2, b_3) = (33/5, 1, -3) \equiv (b_1^0, b_2^0, b_3^0)$

$$\frac{1}{\alpha_3^0} = \frac{1}{\alpha_2} + \frac{b_3^0 - b_2^0}{b_2^0 - b_1^0}\left(\frac{1}{\alpha_2} - \frac{1}{\alpha_1}\right), \quad \log\left(\frac{M_{\rm GUT}^0}{M_Z}\right) = \frac{2\pi}{b_2^0 - b_1^0}\left(\frac{1}{\alpha_2} - \frac{1}{\alpha_1}\right)$$

Adding full SU(5) multiplets does not affect $\alpha_3^0(M_Z)$ and $M_{\rm GUT}^0$ (independently of their mass): $(b_1, b_2, b_3) = (b_1^0, b_2^0, b_3^0) + (b_5^N, b_5^N, b_5^N)$

In general, additional (vectorlike) superfields at a scale $Q_0 > M_Z$ give

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i^0}{2\pi}\log\left(\frac{\mu}{M_Z}\right) - \frac{b_i^N}{2\pi}\log\left(\frac{\mu}{Q_0}\right)$$

$$\frac{1}{\alpha_3} - \frac{1}{\alpha_3^0} = \frac{1}{2\pi}\left[(b_3^N - b_2^N) - \frac{(b_3^0 - b_2^0)(b_2^N - b_1^N)}{b_2^0 - b_1^0}\right]\log\left(\frac{M_{\rm GUT}}{Q_0}\right)$$

MSSM unification is not spoiled (independently of the scale Q_0) if[3]:

$$\frac{b_3^N - b_2^N}{b_2^N - b_1^N} = \frac{b_3^0 - b_2^0}{b_2^0 - b_1^0} = \frac{5}{7}$$

[3] Martin and Ramond hep-ph/9501244

Magic fields

“Magic fields” satisfy the condition

$$\boxed{\frac{b_3^N - b_2^N}{b_2^N - b_1^N} = \frac{5}{7}}$$

They can change the unification scale:

$$M_{\text{GUT}} = M_{\text{GUT}}^0 \left(\frac{Q_0}{M_{\text{GUT}}^0} \right)^r, \quad r = \frac{b_3^N - b_2^N}{b_3 - b_2}$$

The value of the unified gauge coupling (in the MSSM $\alpha_U^0 \approx 1/24$) becomes:

$$\frac{1}{\alpha_U} = \frac{1}{\alpha_U^0} - \frac{(1-r)b_i^N - r\, b_i^0}{2\pi} \log \left(\frac{M_{\text{GUT}}^0}{Q_0} \right)$$

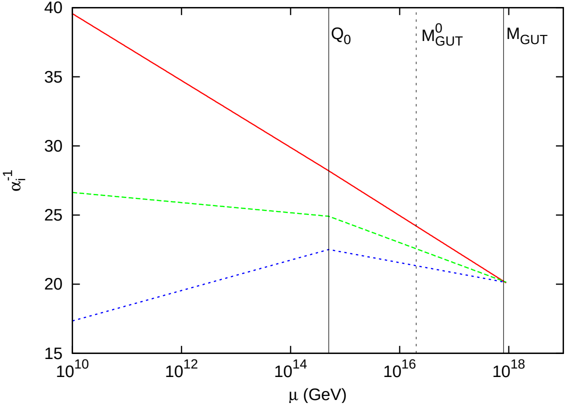
Magic field zoology

$$M_{\mathrm{GUT}} = M^0_{\mathrm{GUT}} \left(\frac{Q_0}{M^0_{\mathrm{GUT}}} \right)^r \qquad r = \frac{b^N_3 - b^N_2}{b_3 - b_2}$$

- $r = 0 \Rightarrow Q_0 < M^0_{\mathrm{GUT}} = M_{\mathrm{GUT}}$: **standard unification**
  $b^N_3 = b^N_2 = b^N_1 \Rightarrow$ GUT scale is unchanged
- $-\infty < r < 0 \Rightarrow Q_0 < M^0_{\mathrm{GUT}} < M_{\mathrm{GUT}}$: **retarded unification**
  Convergence of gauge couplings delayed
- $r = \pm\infty \Rightarrow Q_0 = M^0_{\mathrm{GUT}} < M_{\mathrm{GUT}}$: **fake unification**
  $b_3 = b_2 = b_1$, parallel running $\Rightarrow$ unification only if $Q_0 = M^0_{\mathrm{GUT}}$
- $1 < r < +\infty \Rightarrow M^0_{\mathrm{GUT}} < Q_0 < M_{\mathrm{GUT}}$: **hoax unification**
  Flip of convergence $\Rightarrow$ unification only if $Q_0 > M^0_{\mathrm{GUT}}$
- $0 < r < 1 \Rightarrow Q_0 < M_{\mathrm{GUT}} < M^0_{\mathrm{GUT}}$: **anticipated unification**

Retarded unification

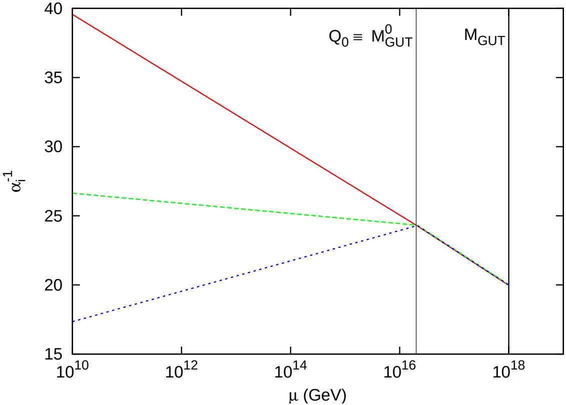
- $-\infty < r < 0 \Rightarrow Q_0 < M^0_{\rm GUT} < M_{\rm GUT}$ $\quad \left[M_{\rm GUT} = M^0_{\rm GUT} \left(\frac{Q_0}{M^0_{\rm GUT}} \right)^r \right]$



Example: $(Q + \bar{Q}) + G$

Fake unification[4]

- $r = \pm\infty \Rightarrow Q_0 = M^0_{\mathrm{GUT}} < M_{\mathrm{GUT}}$ $\left[M_{\mathrm{GUT}} = M^0_{\mathrm{GUT}} \left(\frac{Q_0}{M^0_{\mathrm{GUT}}} \right)^r \right]$

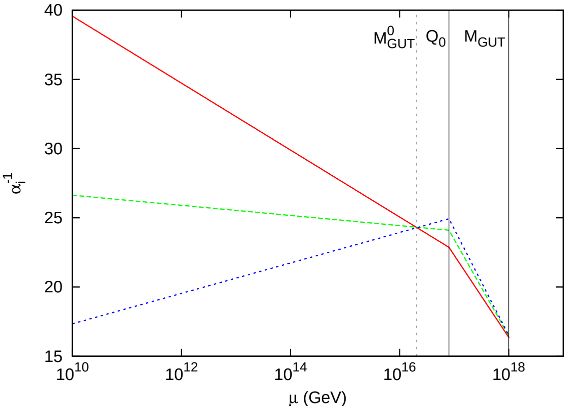


Example: $(6, 2)_{-1/6}$ + c.c. [contained in **210** of $\mathrm{SO}(10)$]

[4] See also B. Dutta, Y. Mimura and R. N. Mohapatra, Phys. Rev. Lett. **100** (2008) 181801

Hoax unification

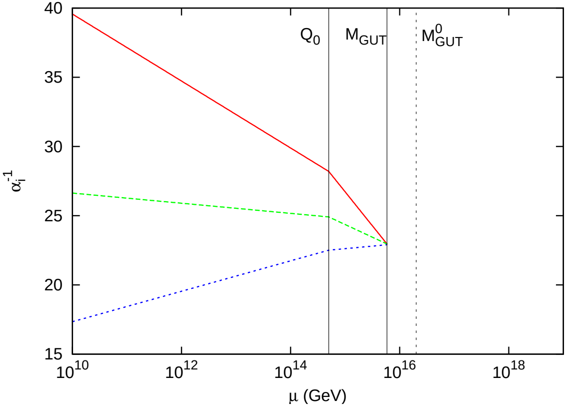
- $1 < r < +\infty \Rightarrow M^0_{\text{GUT}} < Q_0 < M_{\text{GUT}}$ $\qquad \left[M_{\text{GUT}} = M^0_{\text{GUT}} \left(\frac{Q_0}{M^0_{\text{GUT}}} \right)^r \right]$



Example: $W + 2 \times \big((8,2)_{1/2} + \text{c.c.}\big)$ [contained in **120** and **126** of $\mathrm{SO}(10)$]

Anticipated unification

- $0 < r < 1 \Rightarrow Q_0 < M_{\text{GUT}} < M^0_{\text{GUT}}$ $\quad \left[M_{\text{GUT}} = M^0_{\text{GUT}} \left(\frac{Q_0}{M^0_{\text{GUT}}} \right)^r \right]$



Example: $(L + \bar{L}) + (E + \bar{E}) + (V + \bar{V})$

Some properties

- The scale Q_0 is arbitrary, as long as $M_{\mathrm{GUT}} \lesssim M_{\mathrm{Pl}}$, $\alpha_U \lesssim 4\pi$ (perturbative)
- Non-exotic representations: retarded unification $\Leftrightarrow b_3^N - b_2^N = 2$ $(r = -1)$. Therefore:

$$M_{\mathrm{GUT}} = M_{\mathrm{GUT}}^0 \left(\frac{Q_0}{M_{\mathrm{GUT}}^0} \right)^r \Rightarrow \frac{M_{\mathrm{GUT}}}{M_{\mathrm{GUT}}^0} = \frac{M_{\mathrm{GUT}}^0}{Q_0}$$

$$M_{\mathrm{GUT}} \lesssim M_{\mathrm{Pl}} \Leftrightarrow Q_0 \gtrsim 10^{13} - 10^{14}\ \mathrm{GeV}$$

- Combinations of magic sets at different scales still preserve unification. Merging two or more sets at the same scale gives again a magic set

Examples

Field content	b_1^N	b_2^N	b_3^N	r	type
$(6,2)_{-1/6} + \text{c.c.}$	2/5	6	10	∞	fake
$(Q+\bar{Q}) + G$	1/5	3	5	-1	retarded
$(U^c+\bar{U}^c) + (D^c+\bar{D}^c) + W$	2	2	2	0	usual
$(D^c+\bar{D}^c) + G + ((1,3)_1 + \text{c.c.})$	4	4	4	0	usual
$(L+\bar{L}) + ((6,1)_{1/3} + (1,3)_1 + \text{c.c.})$	5	5	5	0	usual
$(Q+\bar{Q}) + (D^c+\bar{D}^c) + ((8,2)_{1/2} + \text{c.c.})$	27/5	11	15	∞	fake
$W + 2\,((8,2)_{1/2} + \text{c.c.})$	48/5	18	24	3	hoax
$W + ((6,2)_{-1/6} + \text{c.c.}) + ((1,1)_2 + \text{c.c.})$	26/5	8	10	-1	retarded
$((3,3)_{2/3} + (6,2)_{-1/6} + (6,1)_{4/3} + \text{c.c.})$	18	18	18	0	usual
$2W + ((6,2)_{5/6} + \text{c.c.})$	10	10	10	0	usual
$((3,3)_{2/3} + (6,2)_{5/6} + (6,1)_{-2/3} + \text{c.c.})$	18	18	18	0	usual
$((8,1)_1 + (\bar{3},1)_{4/3} + \text{c.c.}) + (8,3)_0$	16	16	16	0	usual
$((8,1)_1 + (6,1)_{1/3} + \text{c.c.}) + (8,3)_0$	52/5	16	20	∞	fake

Table: Simplest irreducible magic sets that can be built from SM representations belonging to $\mathrm{SO}(10)$ representations up to **210** and do not correspond to full SU(5) multiplets or anticipated unification

Examples

Field content	b_1^N	b_2^N	b_3^N	r
$(Q+\bar{Q})+G$	1/5	3	5	-1
$(E^c+\bar{E}^c)+2W+2G$	6/5	4	6	-1
$2\,(L+\bar{L})+W+2G$	6/5	4	6	-1
$(Q+\bar{Q})+(U^c+\bar{U}^c)+(D^c+\bar{D}^c)+W+G$	11/5	5	7	-1
$3\,(D^c+\bar{D}^c)+2W+G$	6/5	4	6	-1
$(U^c+\bar{U}^c)+(L+\bar{L})+2W+2G$	11/5	5	7	-1
$(Q+\bar{Q})+2\,(D^c+\bar{D}^c)+(E^c+\bar{E}^c)+W+G$	11/5	5	7	-1
$2\,(Q+\bar{Q})+(D^c+\bar{D}^c)+2\,(E^c+\bar{E}^c)+G$	16/5	6	8	-1
$2\,(Q+\bar{Q})+(U^c+\bar{U}^c)+3\,(D^c+\bar{D}^c)$	16/5	6	8	-1
$2\,(Q+\bar{Q})+2\,(U^c+\bar{U}^c)+(L+\bar{L})+G$	21/5	7	9	-1
$2\,(Q+\bar{Q})+2\,(D^c+\bar{D}^c)+G+(V+\bar{V})$	31/5	9	11	-1

Table: Simplest irreducible magic sets which provide retarded unification. We show only fields belonging to representations of $\mathrm{SO}(10)$ up to **45**.

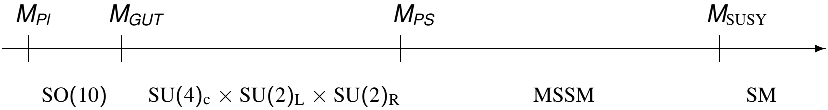
Origin of magic fields

Magic field sets at $Q_0 < M_{\text{GUT}}$ can indeed arise from the spontaneous breaking of a SUSY $\mathrm{SO}(10)$ GUT at the scale M_{GUT}. For example:

$$W = 16\ 45_H\overline{16} + 16_H\ 16\ 10 + \overline{16}_H\ \overline{16}\ 10 + 45_H\ 45\ 54 \\ + 16_H\ 45\ \overline{16}' + \overline{16}_H\ 45\ 16' + M\ 10\ 10 + M\ 54\ 54 + M\ \overline{16}'\ 16'$$

- $\mathcal{O}(1)$ dimensionless couplings
- Mass scale $M \approx M_{\text{GUT}}$
- $\langle 45_H \rangle \approx M_{\text{GUT}}$ in the the T_{3R} direction.
- All fields get a mass of order M_{GUT} except $Q + \bar{Q}$, G

Magic fields in two-step $SO(10)$ breaking



Matching at the PS breaking scale:

$$\frac{1}{\alpha_4} = \frac{1}{\alpha_3} \qquad \frac{1}{\alpha_L} = \frac{1}{\alpha_2} \qquad \frac{1}{\alpha_R} = \frac{5}{3}\frac{1}{\alpha_1} - \frac{2}{3}\frac{1}{\alpha_3}$$

Magic condition:

$$\frac{b_4 - b_L}{b_L - b_R} = \frac{1}{3} \qquad (b_4^0, b_L^0, b_R^0) = (-6, 1, 1)$$

New unification scale:

$$\ln \frac{M_{\text{GUT}}}{M^0_{\text{GUT}}} = \left(\frac{b_3 - b_2}{b_4 - b_L} - 1 \right) \ln \frac{M^0_{\text{GUT}}}{M_{\text{PS}}}$$

Applications: magic Kaluza-Klein tower

Consider "unification" in GUT-scale extra-dimensions compactified on orbifold

- Boundary conditions for bulk fields allow new symmetry breaking mechanisms (and chirality)
- GUT breaking by boundary conditions $\Rightarrow$ the single floors in the KK towers do not correspond to complete GUT multiplets $\Rightarrow$ effect on $\alpha_3(M_Z)$ (sometimes welcome)
- Fields in the "bulk" correspond to KK towers of fields in 4D
- The magic condition on the zero modes ensures that the MSSM $\alpha_3(M_Z)$ prediction is not affected by the KK towers at the tree level

Example: 5D SUSY $\mathrm{SO}(10)$ on $S^1/(Z_2 \times Z_2')$

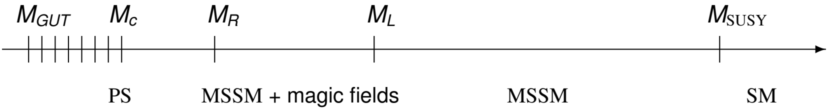
- Vector (V, Σ) and chiral (Φ_1, Φ_2) hypermultiplets in the bulk
- SM matter, Higgs fields etc. on the branes

(V, Σ)	(Φ_1, Φ_2)	
V_{++}, Σ_{--}	Φ_{1++}, Φ_{2--}	PS adjoints
V_{+-}, Σ_{-+}	Φ_{1+-}, Φ_{2-+}	$\mathrm{SO}(10)$/PS adjoints

.

- Each step, three chiral multiplets $\Sigma,\ \Phi_1,\ \Phi_2$ cancel the contribution of the gauge fields V. Both even and odd levels do not spoil unification.
- Fields on the PS branes are necessary to form a magic set with the zero-mode Φ_{1++}, e.g. (4,1,2)+(6,1,1)+(1,1,3)

Applications: multi-scale models[5]



	f_i	f_i^c	h	ϕ	F	$\bar{F}$	F^c	$\bar{F}^c$	F'_c	$\bar{F}'_c$	X_c	Φ	H	ϕ_L	ϕ_R
$SU(2)_L$	2	1	2	1	2	2	1	1	1	1	1	1	2	3	1
$SU(2)_R$	1	2	2	1	1	1	2	2	2	2	3	1	2	1	3
$SU(4)_c$	4	$\bar{4}$	1	15	4	$\bar{4}$	$\bar{4}$	4	$\bar{4}$	4	1	15	1	1	1

- $\mu < M_L$: MSSM field content
- $M_L < \mu < M_R$: MSSM fields, $F + \overline{F}$, ϕ, G_Φ (magic set adding H, ϕ_L, ϕ_R)
- $\mu > M_R$: all the fields in the table
- $\mu > M_c$: “magic” KK state

$\Rightarrow M_{\rm GUT} = (M^0_{\rm GUT})^2/M_L$ (while fields above M_R and KK do not modify $M_{\rm GUT}$)

[5] Ferretti, King, R hep-ph/0609047; Calibbi, Ferretti, R, Ziegler 0812.0087

Applications: GMSB with magic messengers

Gauge mediation with incomplete GUT multiplets has been already studied[6]. Magic fields as messengers give additional constraints to the spectrum:

$$W = S\bar{\Psi}_i\Psi_i + M\bar{\Psi}_i\Psi_i$$

$\Psi_i, \bar{\Psi}_i$ form a magic set of fields and S is the spurion with $\langle F_S \rangle \neq 0$
Gaugino masses at the scale μ:

$$M_a(\mu) = \frac{\alpha_a(\mu)}{4\pi} b_a^N \frac{F_S}{M}$$

Scalar masses:

$$\tilde{m}_i^2(\mu) = \sum_a 2 \left(\frac{\alpha_a(\mu)}{4\pi} \right)^2 C_a^i b_a^N \left[\frac{\alpha_a^2(Q_0)}{\alpha_a^2(\mu)} - \frac{b_a^N}{b_a^0} \left(1 - \frac{\alpha_a^2(Q_0)}{\alpha_a^2(\mu)} \right) \right] \left| \frac{F_S}{M} \right|^2$$

Gaugino masses sum rule:

$$7\frac{M_3}{\alpha_3} - 12\frac{M_2}{\alpha_2} + 5\frac{M_1}{\alpha_1} = 0$$

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[6] Martin, hep-ph/9608224

Applications: GMSB with magic messengers

Two examples:

$$Q\bar{Q} + G \;\Rightarrow\; M_1 : M_2 : M_3 = 1 : 30 : 200,\;\; m_{\tilde{e}^c}/m_{\tilde{q}} \sim 1/20$$

$$Q\bar{Q} + G + U^c\bar{U}^c + D^c\bar{D}^c + W \quad\Rightarrow\quad \begin{array}{l} M_1 : M_2 : M_3 = 1 : 5 : 20 \\ m_{\tilde{e}^c}/m_{\tilde{q}} \sim 1/15 \end{array}$$

Typical spectrum (assuming the selectron mass close to the present experimental limit):

	M_1	M_2	M_3	$m_{\tilde{e}^c}$	$m_{\tilde{q}}$
$Q\bar{Q}\ G$	25 GeV	750 GeV	5 TeV	100 GeV	2 TeV
$Q\bar{Q}\ G\ U^c\bar{U}^c\ D^c\bar{D}^c\ W$	75 GeV	400 GeV	1.5 TeV	100 GeV	1.5 TeV

Summary

- Magic fields may represent a useful model building tool in the context of grand unified theories
- They preserve the successful MSSM *prediction* for $\alpha_3(M_Z)$
- They can easily raise the GUT scale up to the Planck scale, thus softening the constraints from p-decay