Flat scalar spectrum from conformal invariance and possible signatures

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Outline

- Introduction
- Getting flat spectrum of field perturbations
- Reprocessing field perturbations into adiabatic perturbations
- Effects of non-linearity:
  - ✓ Statistical anisotropy
  - ✓ Non-Gaussianity

Introduction

- Primordial scalar perturbations: Gaussian (or nearly Gaussian) random field $\zeta(\vec{x})$
  - $\zeta(\vec{x})$ obeys Wick theorem
  - This suggests the origin: enhanced *vacuum fluctuations of some (almost) free quantum field*
- Flat or nearly flat power spectrum
  - There must be some symmetry behind this property

Candidate theory for origin: inflation

- Symmetry of de Sitter space-time: spatial dilatations supplemented by time translations

$$\vec{x} \to \lambda \vec{x} \,, \quad t \to t - \frac{1}{2H} \log \lambda$$

Another way getting flat spectrum

Conformal plus global symmetry instead of de Sitter symmetry

- Main requirement: long evolution before the hot stage

✓ But otherwise insensitive to regime of cosmological evolution:
works at inflation, contracting (ekpyrotic) phase, "starting the Universe" scenario, etc.

Model:

Conformal complex scalar field ϕ with negative quartic potential.

$$S = \int \sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + \frac{R}{6}|\phi|^2 - (-h^2|\phi|^4)\right]$$

Conformal symmetry in 4 dimensions. Global symmetry $U(1)$.

[Conformal symmetry broken at large fields. To be discussed later.]

Homogeneous and isotropic Universe,

$$ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2]$$

In terms of the field $\chi(\eta, \vec{x}) = a(\eta)\phi(\eta, \vec{x}) = \chi_1 + i\chi_2$,
evolution is Minkowskian,

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\chi - 2h^2|\chi|^2\chi = 0$$

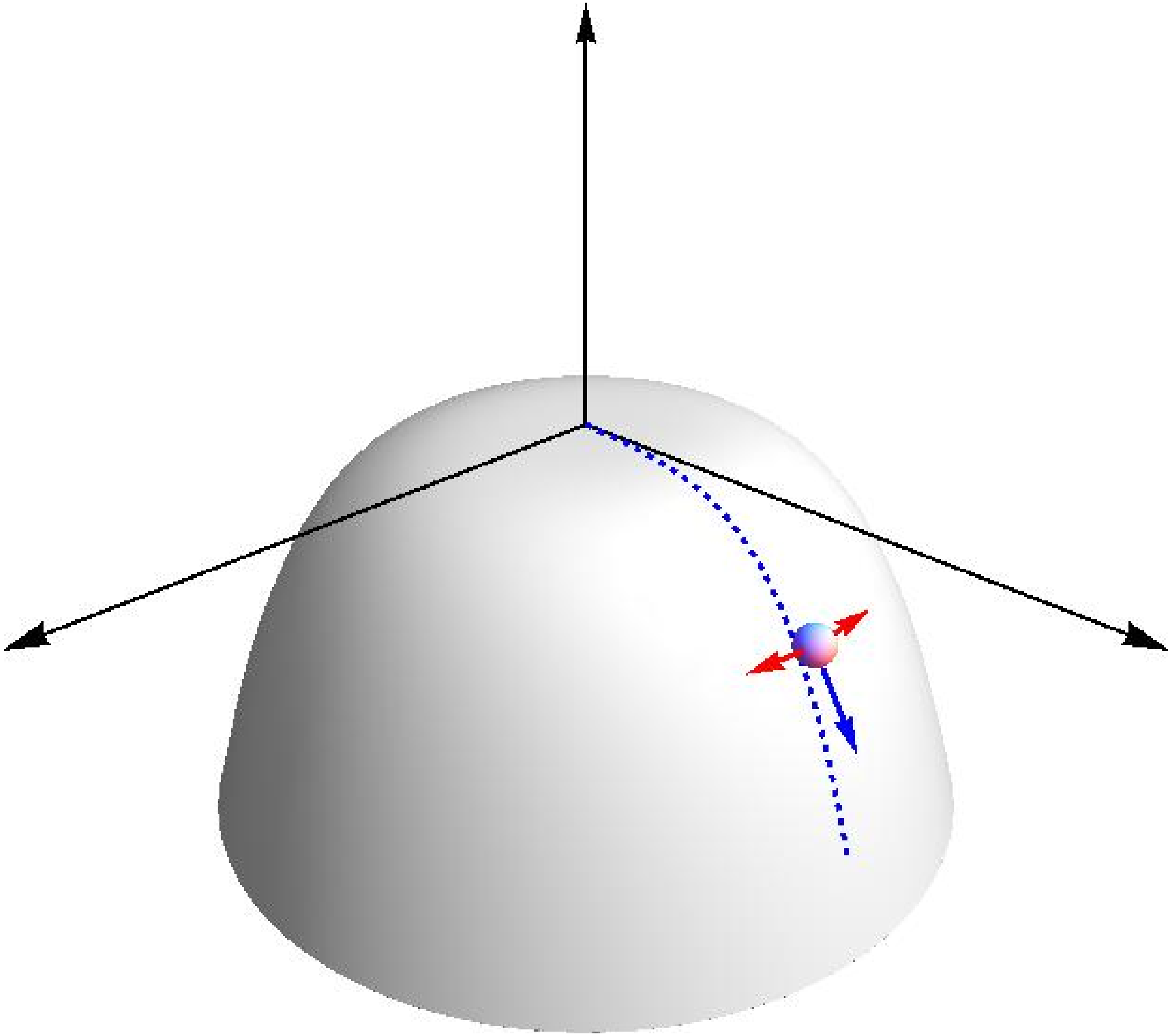
Homogeneous background solution

Attractor (real without loss of generality)

$$\chi_c(\eta) = \frac{1}{h(\eta_* - \eta)}$$

η_* = constant of integration, end time of roll.

$\mathcal{NB}$: Particular behavior $\chi_c \propto (\eta_* - \eta)^{-1}$
dictated by conformal symmetry.



Fluctuations of Im χ automatically have flat spectrum

Linearized equation for fluctuation $\delta\chi_2 \equiv \mathrm{Im}\chi$. Mode of 3-momentum k:

$$\frac{d^2}{d\eta^2}\delta\chi_2 + k^2\delta\chi_2 - 2h^2\chi_c^2\delta\chi_2 = 0$$

[recall $h\chi_c = 1/(\eta_* - \eta)$]

Regimes of evolution:

- Early times, $k \gg 1/(\eta_* - \eta)$, sub-"horizon" regime, χ_c **negligible**, free Minkowskian field

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}}\mathrm{e}^{-ik\eta}A_{\vec{k}} \quad + \quad \mathrm{h.c.}$$

- Late times, $k \ll 1/(\eta_* - \eta)$, super-"horizon" regime, term with χ_c dominates,

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \cdot \frac{1}{k(\eta_* - \eta)} \cdot A_{\vec{k}}$$

✓ Phase of the field ϕ freezes out:

$$\delta\theta = \frac{\delta\chi_2}{\chi_c} = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \cdot \frac{h}{k} \cdot A_{\vec{k}}$$

✓ Power specrum of phase is flat:

$$\langle\delta\theta^2\rangle = \frac{h^2}{2(2\pi)^3}\int \frac{d^3k}{k^3}$$

✓ This is automatic consequence of global $U(1)$ and conformal symmetry

- To see this, consider super-"horizon" regime: $\vec{k}$ negligible,
- equation for $\delta\chi_2$ is equation for spatially homogeneous perturbation.

  - χ_c is solution to full field equation, $e^{i\alpha}\chi_c$ also $\Longrightarrow$
  - $\delta\chi = i\alpha\chi_c$ is solution to perturbation equation $\Longrightarrow$

$$\delta\chi_2 : \quad e^{-ik\eta} \quad \Longrightarrow \quad C(k)\chi_c(\eta) = \frac{1}{k(\eta_* - \eta)}$$

$\mathcal{NB}$: $1/k$ on dimensional grounds.

$\mathcal{NB}$: In fact, equation for $\delta\chi_2$ is precisely the same as equation for minimally coupled massless scalar field in inflating Universe

Comments:

- Mechanism requires long cosmological evolution: need

$$(\eta_* - \eta) \gg 1/k$$

early times, sub-"horizon" regime,
well defined vacuum of the field $\delta\chi_2$.

For $k \sim H_0$ this is precisely the requirement that the horizon problem is solved.

This is probably a pre-requisite for any mechanism that generates density perturbations

- Small explicit breaking of conformal invariance $\Longrightarrow$ tilt of the spectrum

✓ Depends both on the way conformal invariance is broken and on the evolution of scale factor

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Reprocessing field perturbations into adiabatic perturbations

- Assume that conformal evolution ends up at some late time. Scalar potential actually has a minimum at large field.

  Modulus of the field ϕ freezes out at the minimum of the scalar potential. Assume that energy density of ϕ is negligible at that time (important!).

- This talk: assume that modes of interest are superhorizon also in conventional sense at the end of the rolling stage.

  Then the perturbations of phase θ remain frozen.

$\mathcal{NB}$: The opposite case is of interest too, but not yet understood.

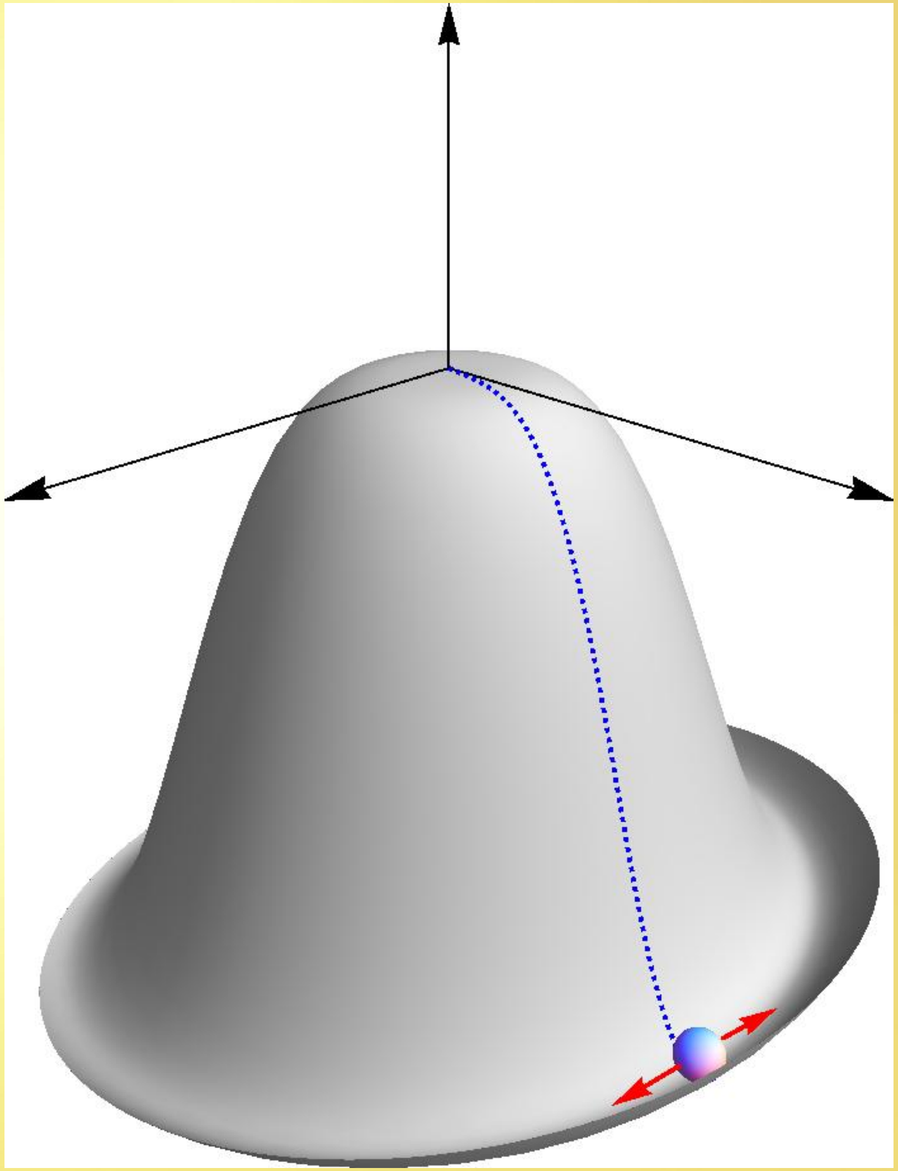
- There are at least two known mechanisms of converting field perturbations into adiabatic perturbations.

  (We do not suggest anything new here.)

Option # 1: phase θ as curvaton

cf. K. Dimopoulos et.al.' 2003

- Let the phase θ be *pseudo*-Goldstone field interacting with matter
- Generically, phase θ ends up at a slope of its potential
- If mass of θ is small enough, it does not evolve until $H \sim m_\theta$ at radiation domination
- Then θ rolls down its potential, oscillates near the minimum and in the end delivers its energy to matter particles.



- Perturbations in θ become adiabatic density perturbations,

$$\zeta \simeq \Omega_\theta \frac{\delta\theta}{\theta_0} \, , \qquad \text{flat power spectrum}$$

  - Ω_θ: relative energy density of θ at the time its oscillations decay,
  - θ_0: distance to minimum from landing point.

- Without fine tuning

$$\sqrt{\mathcal{P}_\zeta} \sim \Omega_\theta \frac{h}{2\pi^2}$$

✓ $\Omega_\theta \gtrsim 10^{-2}$, otherwise too strong non-Gaussianity $\Longrightarrow$

$$\frac{h}{2\pi} \lesssim 10^{-2} \qquad \text{for correct scalar amplitude.}$$

Option # 2: phase θ as DGZK field

Dvali, Gruzinov, Zaldarriaga' 03

Kofman' 03;

better known in the context of modulated reheating

Simple version

- Let masses and/or widths of some heavy particles X depend on θ,

$$M_X = M_0 + c_M \theta \quad \text{and/or} \quad \Gamma_X = \Gamma_0 + c_\Gamma \theta$$

- Let there be intermediate matter dominated epoch, at which these particles are non-relativistic and dominate the expansion

- This epoch begins when $T \sim M_X$ and ends when $\Gamma_X \sim H \Longrightarrow$ dilution of energy density depends on M_X and $\Gamma_X \Longrightarrow$ energy density becomes inhomogeneous.

- This yields

$$\zeta = a\frac{\delta M_X}{M_X} + b\frac{\delta \Gamma_X}{\Gamma_X} + O\left[\left(\frac{\delta M_X}{M_X}\right)^2, \left(\frac{\delta \Gamma_X}{\Gamma_X}\right)^2\right]$$
$$= a c_M \frac{\delta\theta}{M_X} + b c_\Gamma \frac{\delta\theta}{\Gamma_X}$$

$a, b = O(1)$.

Vernizzi' 03:

$a = -16/3$, $b = -2/3$

- Hence,

$$\sqrt{\mathcal{P}_\zeta} \sim \left(a\frac{c_M}{M_X} + b\frac{c_\Gamma}{\Gamma_X}\right)\frac{h}{2\pi}$$

Small c_M/M_X and $c_\Gamma/\Gamma_X \Longleftrightarrow$ large h (but still $h \ll 1$). Yet $f_{NL} \sim 1$.

No strong constraints from non-Gaussianity.

$\mathcal{NB}$: In any case, no relationship with tensor perturbations

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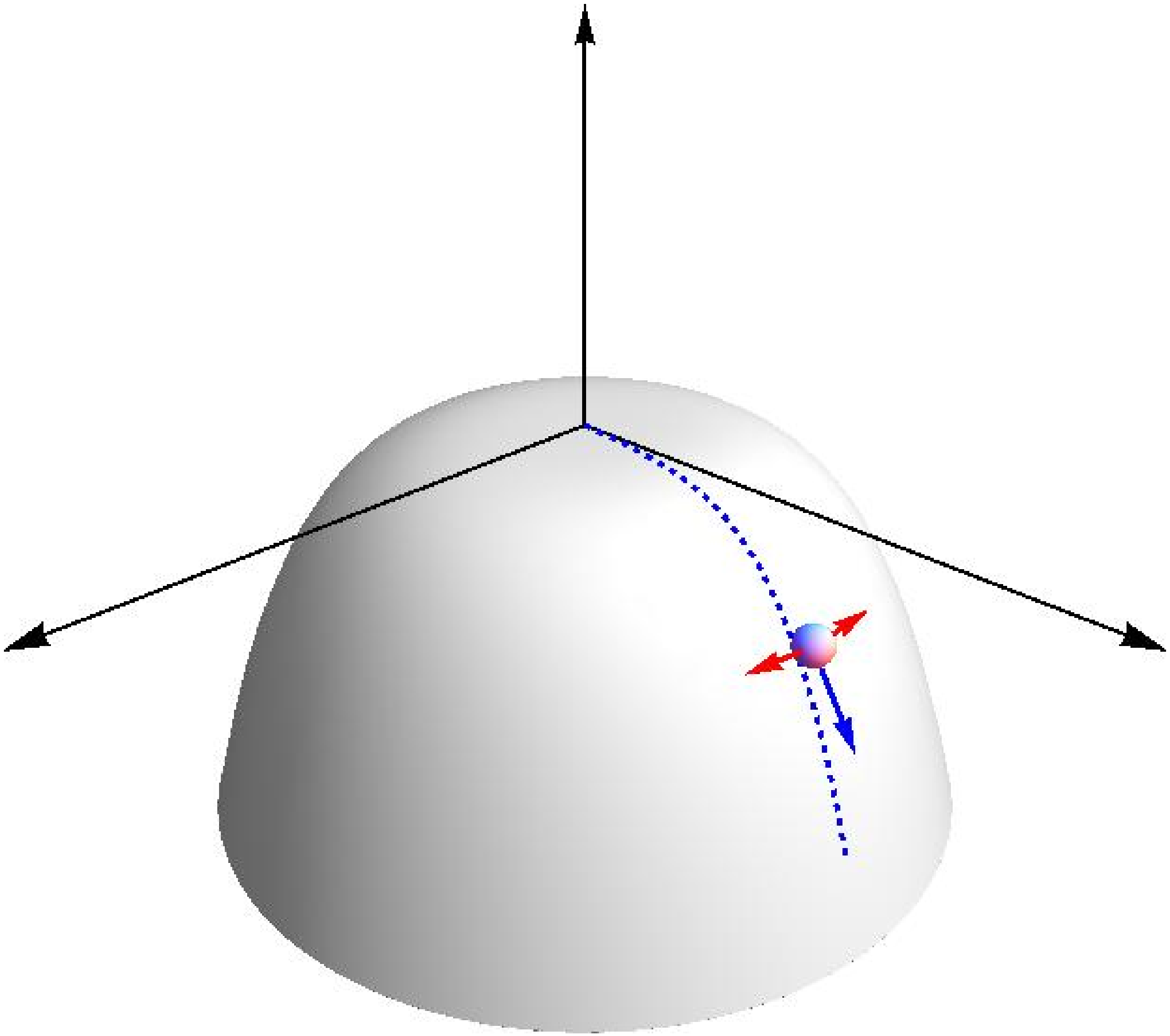
Back to conformal evolution



Peculiarity: perturbations of modulus

- Linear analysis of perturbations of $\chi_1 = \mathrm{Re}\chi$ about the homogeneous real solution χ_c:

$$\frac{d^2}{d\eta^2}\delta\chi_1 + k^2\delta\chi_1 - \mathbf{6}h^2\chi_c^2\delta\chi_1 = 0$$

  - Recall $h\chi_c = 1/(\eta_* - \eta)$.
  - Again initial condition

$$\delta\chi_1 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}}\mathrm{e}^{i\vec{k}\vec{x}-ik\eta}B_{\vec{k}} \quad + \quad \textbf{h.c.}$$

  - But now the solution is

$$\delta\chi_1 = \frac{1}{4\pi}\sqrt{\frac{\eta_* - \eta}{2}}H_{\mathbf{5/2}}{}^{(1)}\left[k(\eta_* - \eta)\right]\cdot B_{\vec{k}} \quad + \quad \textbf{h.c.}$$

- In super-"horizon" regime, $k \ll 1/(\eta_* - \eta)$,

$$\delta\chi_1 = \frac{3}{4\pi^{3/2}} \frac{1}{k^2\sqrt{k}(\eta_* - \eta)^2}$$

✓ Red spectrum:

$$\langle \delta\chi_1^2 \rangle \propto \int \frac{d^3k}{k^5}$$

✓ Large $\delta\chi_1$ at small $(\eta_* - \eta)$

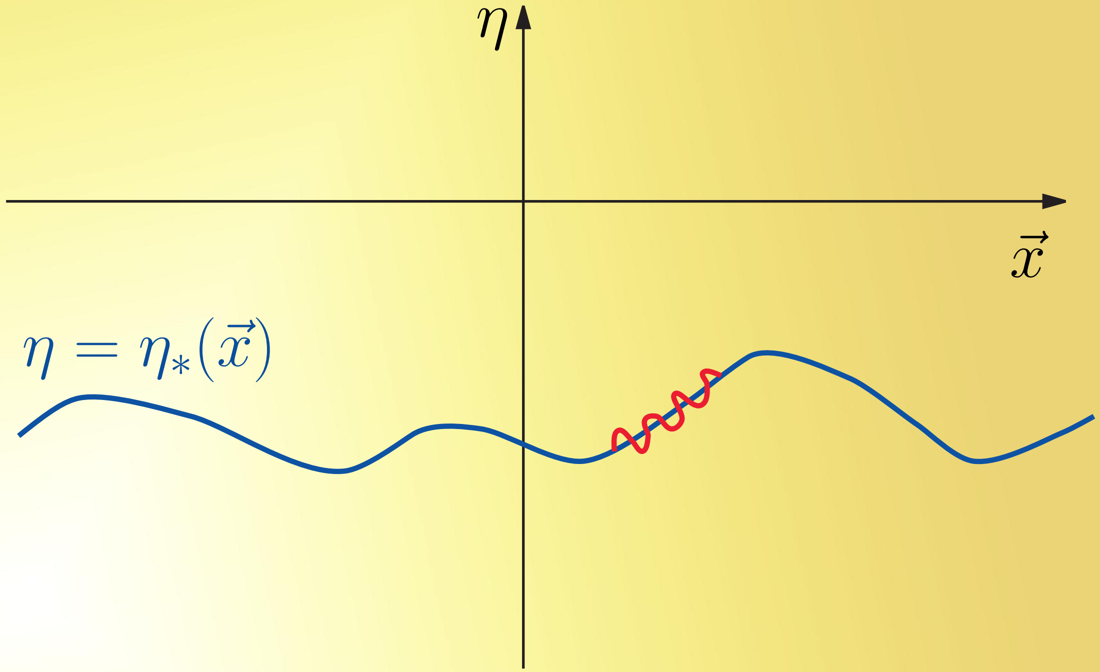
[Recall $\chi_c = 1/[h(\eta_* - \eta)]$]

- Again by symmetry: now translations of conformal time:

$\chi_c \propto 1/(\eta_* - \eta) \Longrightarrow$ spatially homogeneous solution to perturbation equation $\delta\chi = \partial_\eta \chi_c$.

- Interpretation: shift

$$\eta_* \longrightarrow \eta_* + \delta\eta_*(\vec{x})$$



✓ Background for perturbations $\delta\chi_2 = \mathrm{Im}\chi$ (in other words, for phase θ) is no longer spatially homogeneous.

✓ Red spectrum of $\delta\eta_*(\vec{x})$:

$$\sqrt{\mathcal{P}_{\delta\eta_*}} = \frac{3h}{2\pi k}$$

- Have to study perturbations of $\mathrm{Im}\chi$ in *spatially inhomogeneous background*, slowly varying in space,

$$\chi_c = \frac{1}{h(\eta_*(\vec{x}) - \eta)}$$

- Back to equation for perturbations of $\delta\chi_2 = \mathrm{Im}\chi$

$$\frac{d^2}{d\eta^2}\delta\chi_2 - \frac{\partial^2}{\partial\vec{x}^2}\delta\chi_2 - \frac{2}{(\eta_*(\mathbf{x}) - \eta)}\delta\chi_2 = 0$$

  - Initial condition as $\eta \to -\infty$:

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}}\mathrm{e}^{i\vec{k}\vec{x} - ik\eta}A_{\vec{k}} \quad + \quad \mathrm{h.c.}$$

  - $\eta_*(\vec{x})$: long ranged field, derivative expansion appropriate
    - Zeroth order in $\partial_i\eta_*$: local shift of conformal time, $\delta\eta_*(\vec{x})$
    - First order in $\partial_i\eta_*$: local Lorentz boost with $v_i = -\partial_i\eta_*$; background is locally homogeneous and isotropic in a reference frame other than cosmic frame.

- Solution at the first order: time shift and boost of the original solution

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \mathrm{e}^{i\vec{k}(\vec{x}+\vec{v}\eta)-i(k+\vec{k}\vec{v})\eta_*(\vec{x})} f(q(\vec{x}), \eta - \eta_*(\vec{x})) \cdot A_{\vec{k}}$$

$$f(k,\eta) \propto H_{3/2}[q(\eta_* - \eta)]$$

$\vec{q} = \vec{k} + k\vec{v} =$ boosted momentum; $v_i(\vec{x}) = -\partial_i \eta_*(\vec{x})$.

- Super-"horizon" regime:

$$\delta\theta = \frac{\delta\chi_2}{\chi_c} = \frac{h}{(2\pi)^{3/2}\sqrt{2kq}} \mathrm{e}^{ik\vec{x}-ik\eta_*(\vec{x})} \cdot A_{\vec{k}}$$

  - Modes of $\delta\eta_*$ longer than present cosmological horizon: η_*, $\vec{v}$ are constants throughout our Universe $\Longrightarrow$

    anisotropy, constant in space vector $v_i = -\partial_i \eta_*$

  ✓ Effect of order $\vec{v}$ cancels out nevertheless (!)

- Derivative expansion to the second order: perturbative solution.
  - Super-"horizon" regime:

$$\delta\theta = \frac{\delta\chi_2}{\chi_c} = \frac{h}{(2\pi)^{3/2}\sqrt{2kq}} \mathrm{e}^{ik\vec{x} - ik\eta_*(\vec{x})} \left(1 - \frac{\pi}{2k}\frac{k_i k_j}{k^2}\partial_i\partial_j\eta_*\right) \cdot A_{\vec{k}}$$

  - Scalar power spectrum

$$\mathcal{P}(\vec{k}) = A_s(k)\left(1 - \frac{\pi}{k}\frac{k_i k_j}{k^2}\partial_i\partial_j\eta_* - \frac{3}{2}\frac{(\vec{k}\vec{v})^2}{k^2}\right)$$

  - Statistical anisotropy due to constant in space tensor $\partial_i\partial_j\eta_*|_{\text{long wavelengths}}$
    $\Longrightarrow$ CMB correlators $\langle a_{l,m} a^*_{l\pm 2,m}\rangle$, etc.

✓ $v_i = -\partial_i\eta_* \propto h$

✓ Quadrupole of general form [unlike usual factorized $(\vec{k}\vec{v}/k)^2$]

✓ Momentum dependence $1/k$

$\mathcal{NB}$: Power spectrum of $\partial^2\eta_*$ is blue $\Longrightarrow$

$$\langle(\pi\partial_i\partial_j\eta_*)^2\rangle_{\text{long wavelengths}} \simeq \frac{9h^2}{4}\int_0^{H_0} kdk \simeq h^2 H_0^2$$

Statistical anisotropy effect on perturbations of wave vector k is

$$h\frac{H_0}{k}$$

$\Longrightarrow$ effect on CMB suppressed by $1/l$

✓ Stronger limitations from cosmic variance

$\mathcal{NB}$: Power spectrum of $\vec{v}$ is flat$\Longrightarrow$

$$\langle(\vec{v})^2\rangle_{\text{long wavelengths}} \simeq h^2\int_{H_0}\frac{dk}{k} \simeq h^2 \log H_0$$

Scalar power spectrum

$$\mathcal{P}(\vec{k}) = A_s(k)\left(1 + c_1\cdot h\cdot\frac{H_0}{k}\cdot\frac{k_i}{k}\frac{k_j}{k}w_{ij} - c_2\cdot h^2(\log H_0)^2\cdot\left(\frac{\vec{k}\vec{u}}{k}\right)^2\right)$$

Non-Gaussianity

- Perturbations of phase $\delta\theta$ frozen out at

$$\delta\theta(\vec{x}) = \frac{h}{(2\pi)^{3/2}\sqrt{2k}q(\vec{x})}\mathrm{e}^{i\vec{k}\vec{x} - ik\eta_*(\vec{x})}A_{\vec{k}} \quad + \quad \text{h.c.}$$

$\eta_*(\vec{x})$ and $q(\vec{x}) = k - k_i\partial_i\eta_*$ are random fields $\Longrightarrow$ Non-Gaussianity of very special kind $\Longrightarrow$ non-Gaussianity in adiabatic perturbations: a new random field $\eta_*(\vec{x})$ in exponent.

✓ $\eta_*(\vec{x})$ is long ranged Gaussian field whose correlation function increases with distance,

$$\langle\eta_*(\vec{x})\eta_*(0)\rangle - \langle\eta_*^2(0)\rangle \equiv -\Delta(\vec{x})$$

$$\Delta(\vec{x}) = \frac{9}{16\pi^3}h^2\int_{p\lesssim H_0}\frac{d^3p}{p^5}\left(1 - \mathrm{e}^{i\vec{p}\vec{x}}\right) = \frac{3}{8\pi^2}h^2\vec{x}^2\log\frac{1}{H_0|\vec{x}|} > 0$$

- Higher correlation functions:
  - Odd correlation functions vanish, just like for Gaussian field
  - Non-trivial effect on four-point correlation function $\Longrightarrow$ backreaction of $\delta\theta$ on η_* need to be taken into account $\Longrightarrow$
  - In-In formalizm

$$\langle\mathcal{O}\rangle = \left\langle \left[\overline{T}\exp\left(i\int\limits_{-\infty}^{0} dx_0 d^3x \mathcal{H}_I\right)\right]\mathcal{O}_{(I)}\left[T\exp\left(-i\int\limits_{-\infty}^{0} dx_0 d^3x \mathcal{H}_I\right)\right]\right\rangle$$

- For instance, consider configuration of points:

$$\left\langle \delta\theta\left(\frac{\vec{q}}{2}+\vec{Q}\right)\delta\theta\left(\frac{\vec{q}}{2}-\vec{Q}\right)\delta\theta\left(-\frac{\vec{q}}{2}+\vec{P}\right)\delta\theta\left(-\frac{\vec{q}}{2}-\vec{P}\right)\right\rangle =$$

$$\frac{h^2\pi^2}{16}\frac{1}{qQ^4P^4}\left[1-3\left(\frac{\vec{q}\vec{Q}}{qQ}\right)^2\right]\left[1-3\left(\frac{\vec{q}\vec{P}}{qP}\right)^2\right]+O(q/Q)$$

✓ Non-trivial, direction-dependent correlation between distant pairs of points due to long-range correlations in $\eta_*(\vec{x})$

✓ Momentum dependence $1/q$ instead of $1/q^5$

To summarize:

- Flat (or nearly flat) spectrum of scalar perturbations may be consequence of conformal + global symmetry, rather than de Sitter symmetry
  - A simple model of this sort: conformally coupled complex scalar field with negative quartic potential
  - Evolution of scale factor arbitrary, except that it must be long

- Peculiar property which hopefully has potentially observable consequences: strong fluctuations along roll down direction
  - Perturbations of wavelengths exceeding the present horizon: quadrupole statistical anisotropy of a general form
  - Perturbations of wavelengths smaller than the present horizon: non-Gaussianity of a special kind.

What if the world started out conformal indeed?