On Lorentz violation in superluminal theories

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Outline

- Motivation: modified gravity theories
- A toy model with superluminality
- Instabilities and Lorentz violation
- Pathologies in stress-energy tensor
- Conclusions

Fierz–Pauli linearized massive gravity

$$S = M_{pl}^2 \int d^4x \left[L_{EH}^{(2)}(h_{\mu\nu}) + \frac{\alpha}{4} h_{\mu\nu} h^{\mu\nu} + \frac{\beta}{4} (h_\mu^\mu)^2 \right] \quad ; \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Fierz, Pauli, 1939

Modified gravity theories: examples

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- Ghost free only for $\alpha = -\beta$
- van Dam–Veltman–Zakharov discontinuity — GR is not restored as $m \to 0$
- Non-linear effects allow to avoid discontinuity at small distances

  Vainshtein, 1972
- On non-flat background Boulware–Deser ghost appears

  Boulware, Deser, 1972

Modified gravity theories: examples

New covariant non-linear generalization of Fierz–Pauli theory

$$S = M_{Pl}^2 \int d^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right] ;$$

$$\mathcal{L}_i = \mathcal{L}_i(\delta^\mu_\nu - \sqrt{g^{\mu\rho}\eta_{ab}\partial_\rho\phi^a\partial_\nu\phi^b})$$

de Rham, Gabadadze, 2010
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- Free of Boulware-Deser ghosts

Hassan, Rosen, 2011

- Provides self-accelerated cosmological solutions, $\Lambda = c(\alpha_i) m_g^2$

Gumrukcuoglu, Lin, Mukohyama, 2011
de Felice et al., 2013

Modified gravity theories: examples

Decoupling limit of new massive gravity: $M_{Pl} \to \infty$; $m_g \to 0$; $M_{Pl} m_g^2 = \text{const}$

$$\Downarrow$$

Galileon theory

$$S = \int d^4x \left[\frac{1}{2} (\partial_\mu \pi)^2 - \partial_\mu^2 \pi (\partial_\mu \pi)^2 + \ldots \right] \; ; \quad \partial_\mu \pi \to \partial_\mu \pi + c_\mu \quad \text{invariance}$$

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Features

- Ghost-free
- Late time cosmic acceleration
- Alternative to inflation (Galilean Genesis)

Creminelli, Nicolis, Trincherini, 2010

One and the same feature of all these theories — presence of superluminal modes

- Galileon

  Nicolis, Rattazzi, Trincherini, 2009

- Massive Gravity

  Burrage et al., 2011
  Gruzinov, 2011
  de Rham, Gabadadze, Tolley, 2011

Superluminality itself looks not so bad...

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$\Downarrow$

Possible problems with causality

$\Downarrow$

Chronology protection conjecture — there are no closed timelike curves.
Proven for some cases.

de Rham et al., 2011

More general problem — UV-completion

Adams et al., 2006

Key moment: number of derivatives in self-coupling should be greater than 2. Take for example Galileon second term.

$$\mathcal{L} = \partial_\mu \pi \, \partial^\mu \pi + \frac{c}{\Lambda^4} \partial_\mu^2 \pi (\partial_\nu \pi)^2 + \ldots ; \quad c < 0$$

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What is the particular mechanism of Lorentz breaking?

Lets look for a physical difference between static and boosted classical solutions.

Toy model

Toy-model in two dimensions.

$$S = \int d^2x \left[\frac{1}{2}(\partial_\mu \phi \, \partial^\mu \phi) - \frac{m^2}{2}\phi^2 - \frac{1}{4\Lambda^2}(\partial_\mu \phi \, \partial^\mu \phi)^2 + M\delta(x)\phi \right]$$

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Consider classical perturbations on a background of a static solution. It is enough to consider e.o.m. without non-linear ϕ term.

$$\phi'' - m^2\phi = -M\delta(x) \;\Rightarrow\; \tilde{\phi} = \frac{M}{2m}\,\mathrm{e}^{-m|x|}$$

Consider quadratic action for classical perturbations $\phi = \tilde{\phi} + \xi$.

$$S_\xi = \int d^2x \left[\frac{1}{2} Z^{\mu\nu} \partial_\mu \xi \, \partial_\nu \xi - \frac{m^2}{2} \xi^2 \right] ;$$

$$Z^{\mu\nu} = \begin{pmatrix} 1 + \frac{M^2}{4\Lambda^2} \, \mathrm{e}^{-2m|x|} & 0 \\ 0 & -1 - \frac{3M^2}{4\Lambda^2} \, \mathrm{e}^{-2m|x|} \end{pmatrix}$$

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Dispersion relation

$$\omega_\pm = \pm \sqrt{-\frac{Z^{11}}{Z^{00}}}\, k$$

$$\Downarrow$$

$$\left| \frac{d\omega}{dk} \right| > 1$$

Group velocity of the excitations exceeds the speed of light.

Looking for a classical instability

$\det Z^{\mu\nu} < 0 \quad \Rightarrow$ the solution $\tilde{\phi}$ is classically stable.

Classical stability

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In a boosted frame $(x \to \gamma|x + \beta t|)$

$$Z^{\mu\nu} = \begin{pmatrix} 1 - \frac{M^2}{4\Lambda^2}\gamma^2(3\beta^2 - 1)\,\mathrm{e}^{-2m\gamma|x+\beta t|} & \frac{M^2}{2\Lambda^2}\gamma^2\beta\,\mathrm{e}^{-2m\gamma|x+\beta t|} \\ \frac{M^2}{2\Lambda^2}\gamma^2\beta\,\mathrm{e}^{-2m\gamma|x+\beta t|} & -1 - \frac{M^2}{4\Lambda^2}\gamma^2(3 - \beta^2)\,\mathrm{e}^{-2m\gamma|x+\beta t|} \end{pmatrix}$$

However, det $Z^{\mu\nu} =$ const under Lorentz transformation.

$\Downarrow$

The solution $\tilde{\phi}$ is classically stable in any frame.

Quantum instability

Looking for a quantum instability

Z^{00} change its sign in a boosted frame $\Rightarrow$ Ghosts appear!

$$Z^{00} = 1 - \frac{M^2}{4\Lambda^2}\gamma^2(3\beta^2 - 1)\,\mathrm{e}^{-2m\gamma|x+\beta t|} < 0$$

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Critical boost factor $\gamma > \frac{\Lambda\sqrt{2}}{M}\,\mathrm{e}^{m\gamma|x+\beta t|}$

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The trajectory of $\tilde{\phi}$ should be close to $x = -\beta t$

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Looking for UV dynamics we can take $m = 0$ and regard boosted background as a constant

$$S_\xi = \int d^2x \left[\frac{1}{2} Z^{\mu\nu}\partial_\mu\xi\,\partial_\nu\xi\right];\; Z^{\mu\nu} = \begin{pmatrix} Z^{00} & Z^{01} \\ Z^{01} & Z^{11} \end{pmatrix} = \text{const}$$

Looking for evolution of negative energy modes.

Add interaction with matter scalar field χ.

$$S_\chi = \int d^2x \left[\frac{1}{2}(\partial_\mu \chi \, \partial^\mu \chi) - \frac{m_\chi^2}{2}\chi^2 + \frac{g}{2}\xi\chi\chi \right]$$

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Triples $\xi\chi\chi$ created from vacuum as $Z^{00} < 0$

Required hierarchy of parameters:

$$m_\chi < m\gamma \ll k \ll \Lambda \, ;$$

$$m \ll M \, ; \quad g\frac{M}{m} \ll m_\chi^2$$

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Estimations for background decay rate and energy loss rate

Integral saturated in IR

$$\frac{dE}{dt} \sim \Gamma \sim \frac{g^2}{2\pi m^2 \gamma^2} \lesssim \frac{m_\chi^4}{\Lambda^2}$$

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Small, but non-zero quantity $\Rightarrow$ Lorentz invariance is violated!

Naive estimation for $4D$

$$S_\chi = \int d^2x \left[\frac{1}{2}(\partial_\mu\chi\,\partial^\mu\chi) - \frac{m_\chi^2}{2}\chi^2 + \frac{g'}{2}\xi(\partial_\mu\chi\partial^\mu\chi)\right]$$

This simulates coupling $\frac{g'}{2}\xi^2\chi^2$ in $4D$ as it have the same dimensionality.

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Estimation for background decay rate

Integral saturated in UV

$$\Gamma \sim \frac{g'^2}{8\pi}\Lambda^2$$

Estimation for energy loss rate

$$\frac{dE}{dt} \sim \frac{g'^2}{8\pi}\frac{\Lambda^3}{m\gamma} \, .$$

This can be very high and probably could destroy the classical solution.

Expect some features of $T^{\tilde{\phi}}_{\mu\nu}(\xi)$ as $\gamma \to \gamma_{critical}$

Below $\gamma_{critical}$, $T^{\tilde{\phi}\,boosted}_{\mu\nu}(\xi) = T^{\tilde{\phi}\,static}_{\mu\nu}(\xi)$

Stress-energy tensor pathology

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Looking for accelerated solution $\tilde{\phi}$, $(\beta = t/\tau \simeq 1)$

$$Z^{\mu\nu} = \begin{pmatrix} 1 - t^2/\alpha^2 & -t^2/\alpha^2 \\ -t^2/\alpha^2 & -1 - t^2/\alpha^2 \end{pmatrix} \; ; \quad \alpha^2 = \frac{2\Lambda^2(\tau^2 - t^2)}{M^2} = \text{const}$$

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After quantization of ξ on this “curved” background we get

$$\langle T_{00} \rangle = -\langle T_{11} \rangle = \frac{\alpha^2}{\alpha^2 - t^2} \int_0^\infty \frac{dkk}{\pi}$$

Indication for pathology as $t \to \alpha$, although renormalization of $T_{\mu\nu}$ is needed

Work in progress...

Conclusions & Discussion

- Physical solution in the simple superluminal quantum field theory shows different behavior in static and boosted frames. I.e. Lorentz invariance is broken.
    - Theory is pathological and we don't need to consider it.
    - As the theory is non-Lorentzian we need to complete a Lagrangian with other Lorentz-braking terms.
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    - Theory is pathological and we don't need to consider it.
    - As the theory is non-Lorentzian we need to complete a Lagrangian with other Lorentz-braking terms.
    - It is a spontaneous breaking of Lorentz invariance, so we need to consider a phase with a broken symmetry.
- This analysis seem to be suitable for other theories that admit superluminal propagation on classical backgrounds.

Merci!