The Role of the Dilaton in Composite Higgs models

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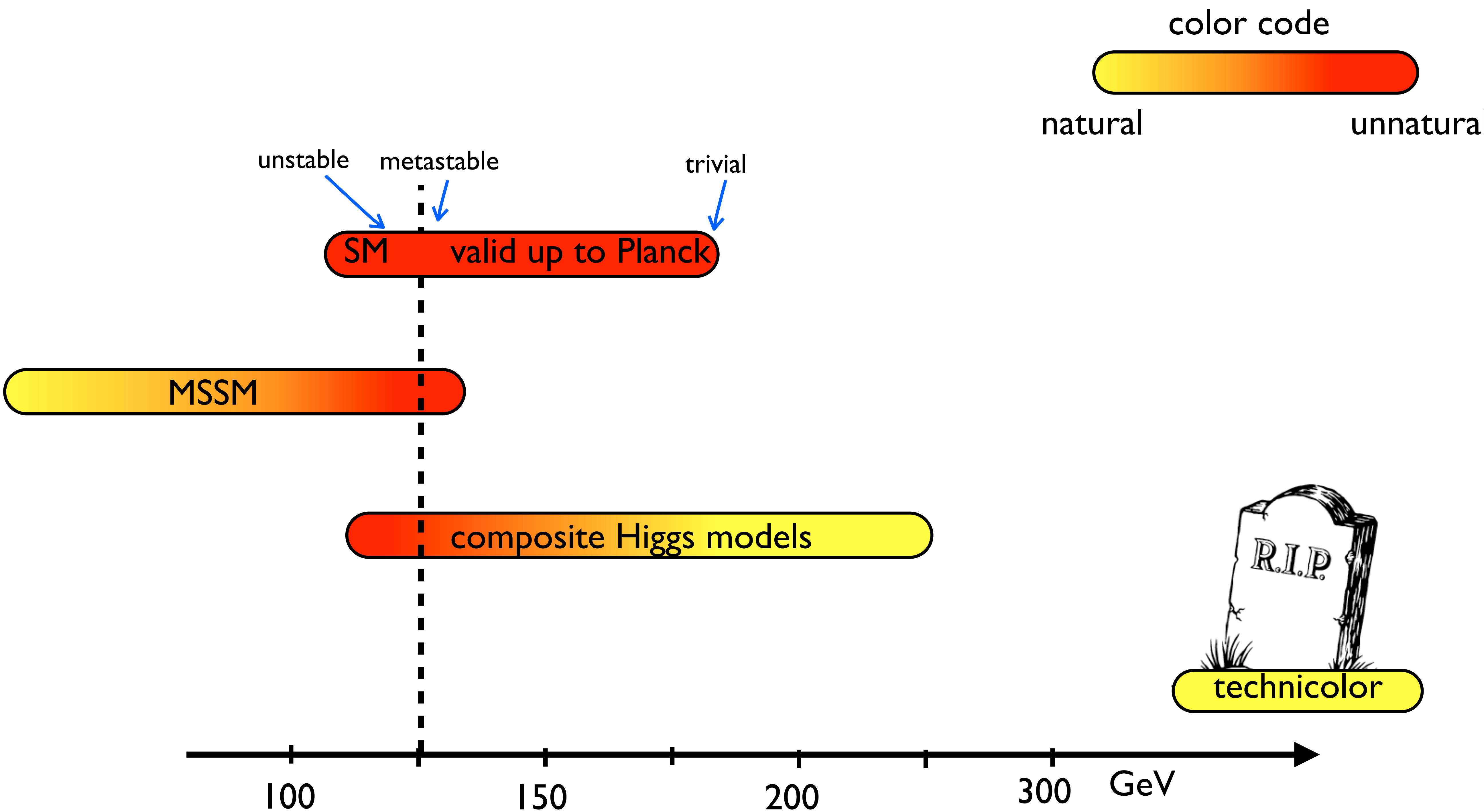
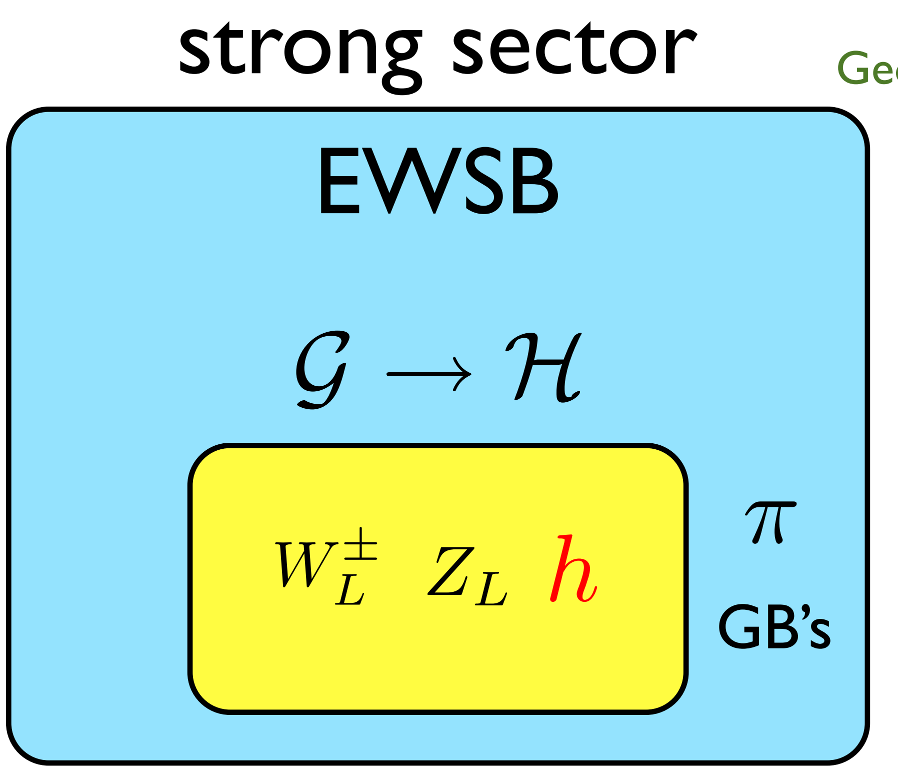
work in progress
with R. Franceschini, L. Martucci and R. Torre

Brussels, June 12th 2013

BSM on 2013


color code
natural
unnatural
unstable
metastable
trivial
SM
valid up to Planck
MSSM
composite Higgs models
R.I.P.
technicolor
100
150
200
300
GeV

Composite Higgs Models



Georgi & Kaplan 1984

Composite Higgs Models

strong sector

Georgi & Kaplan 1984

EWSB

$\mathcal{G} \rightarrow \mathcal{H}$

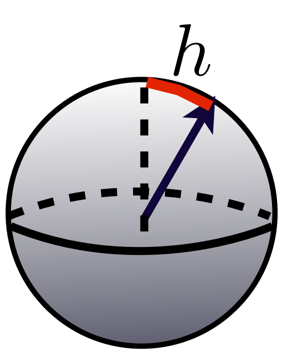
$W_L^{\pm}$ Z_L h

π

GB's

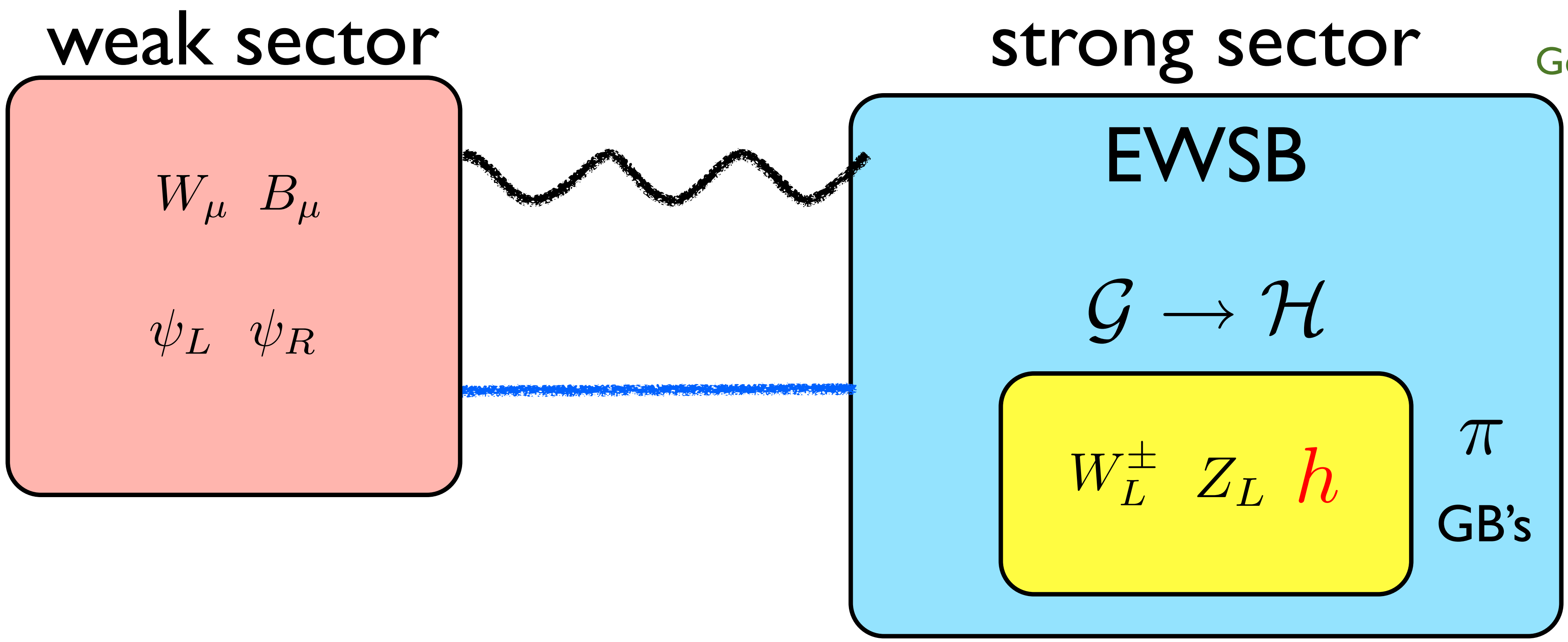
Minimal model: SO(5)/SO(4)= S_4

Agashe, Contino & Pomarol he-ph/0412089



Composite Higgs Models

Georgi & Kaplan 1984



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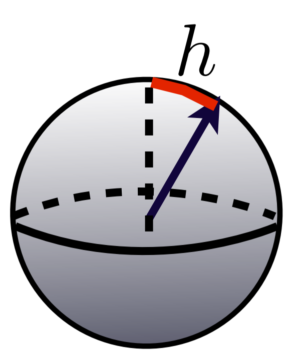
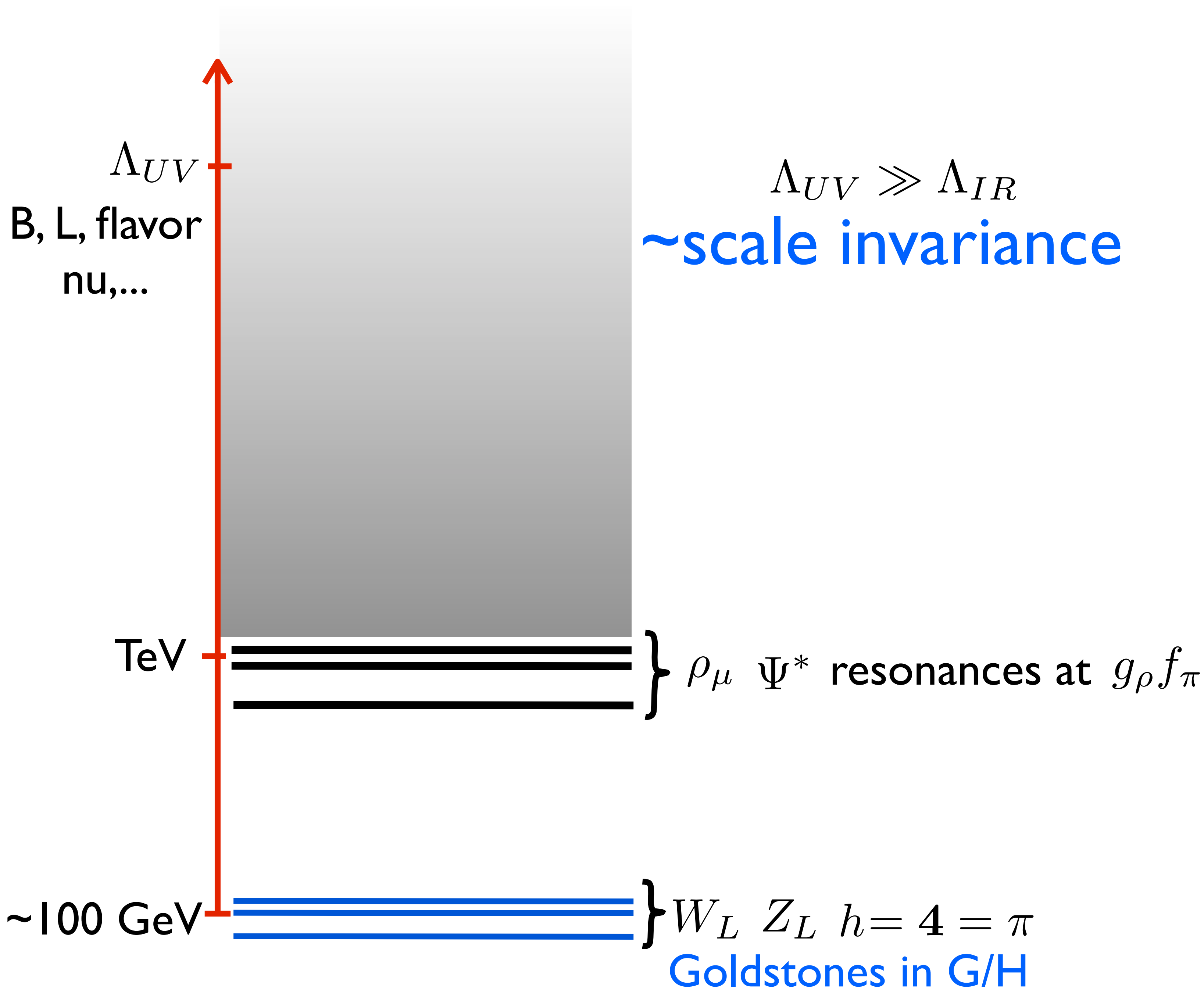


$$v = f_\pi \sin h$$

generically $\frac{v^2}{f_\pi^2} \sim \mathcal{O}(1)$

unless F.T.~few%

A PLAUSIBLE SPECTRUM



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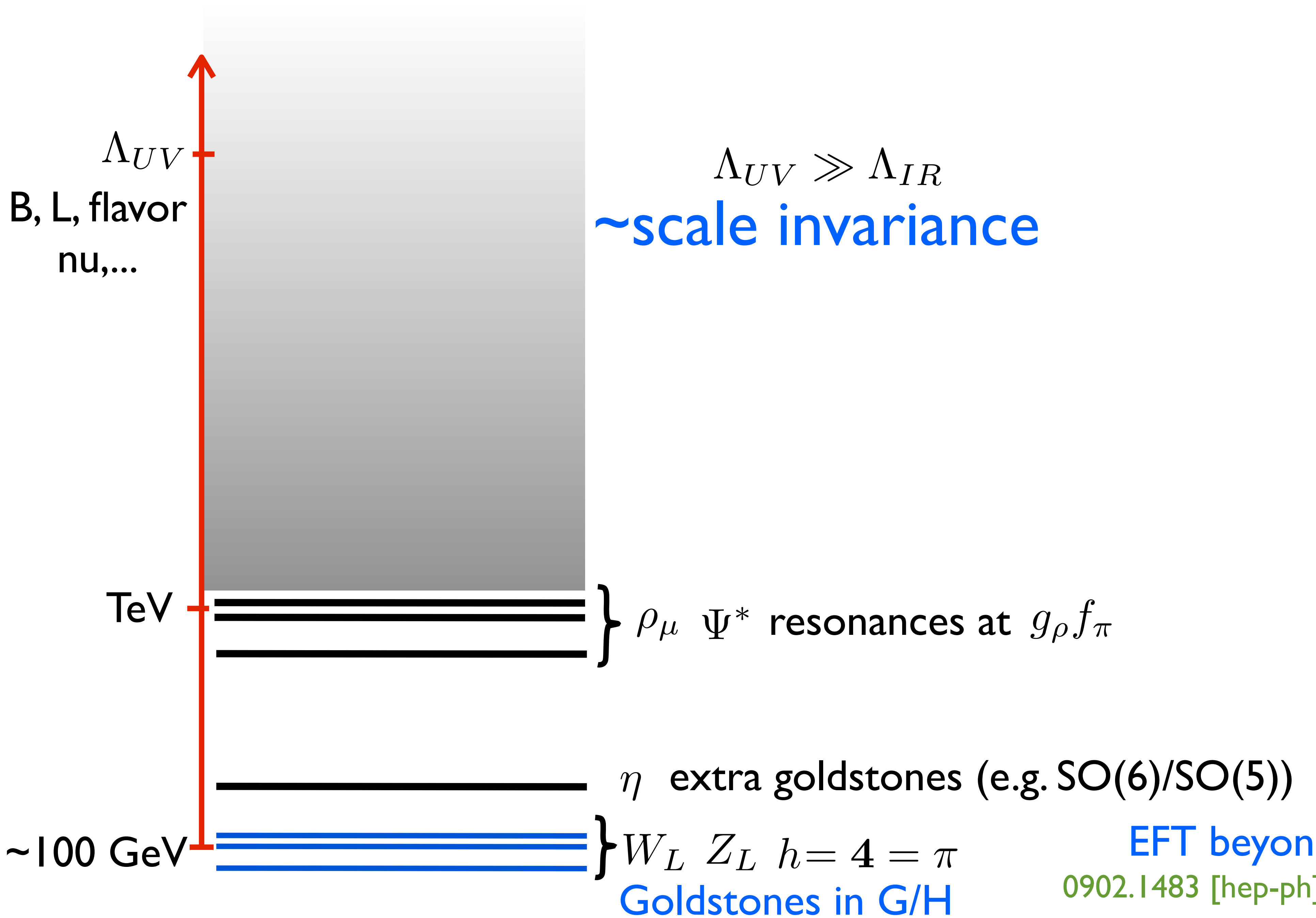
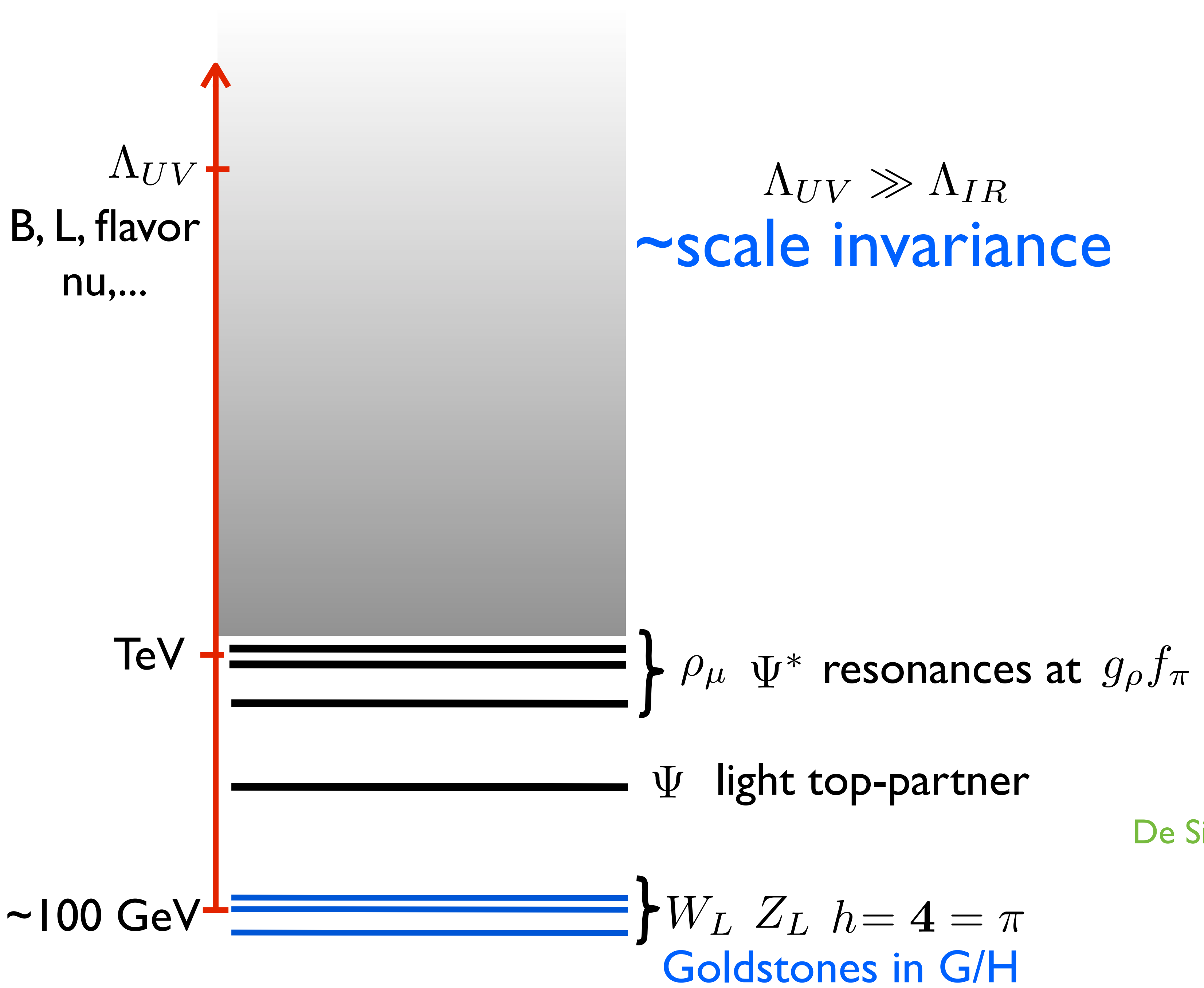


EFT beyond minimal GB sector

0902.1483 [hep-ph] Gripaios, Pomarol, Riva & Serra

A PLAUSIBLE SPECTRUM

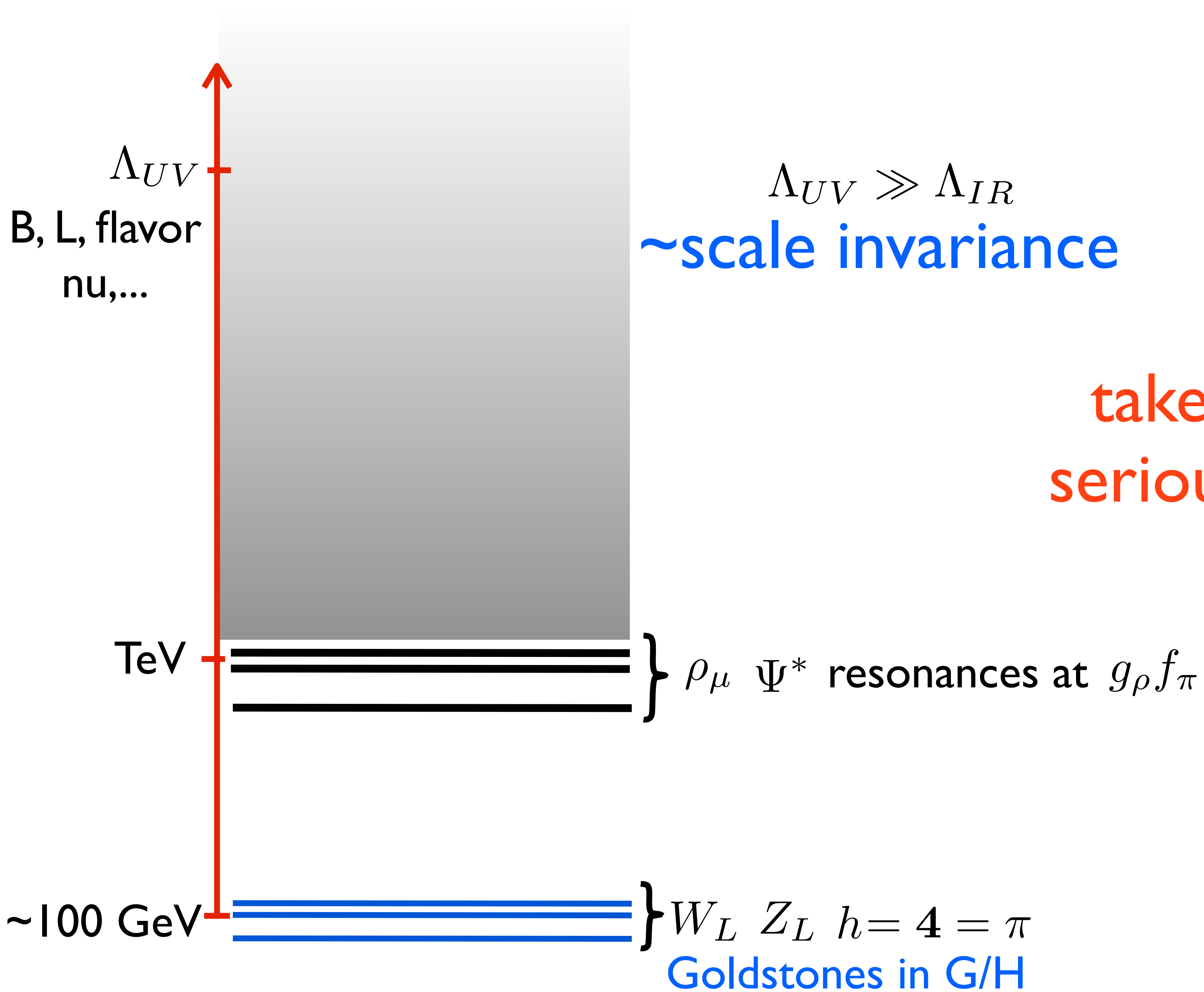


EFT for top-partners

1211.5663 [hep-ph]

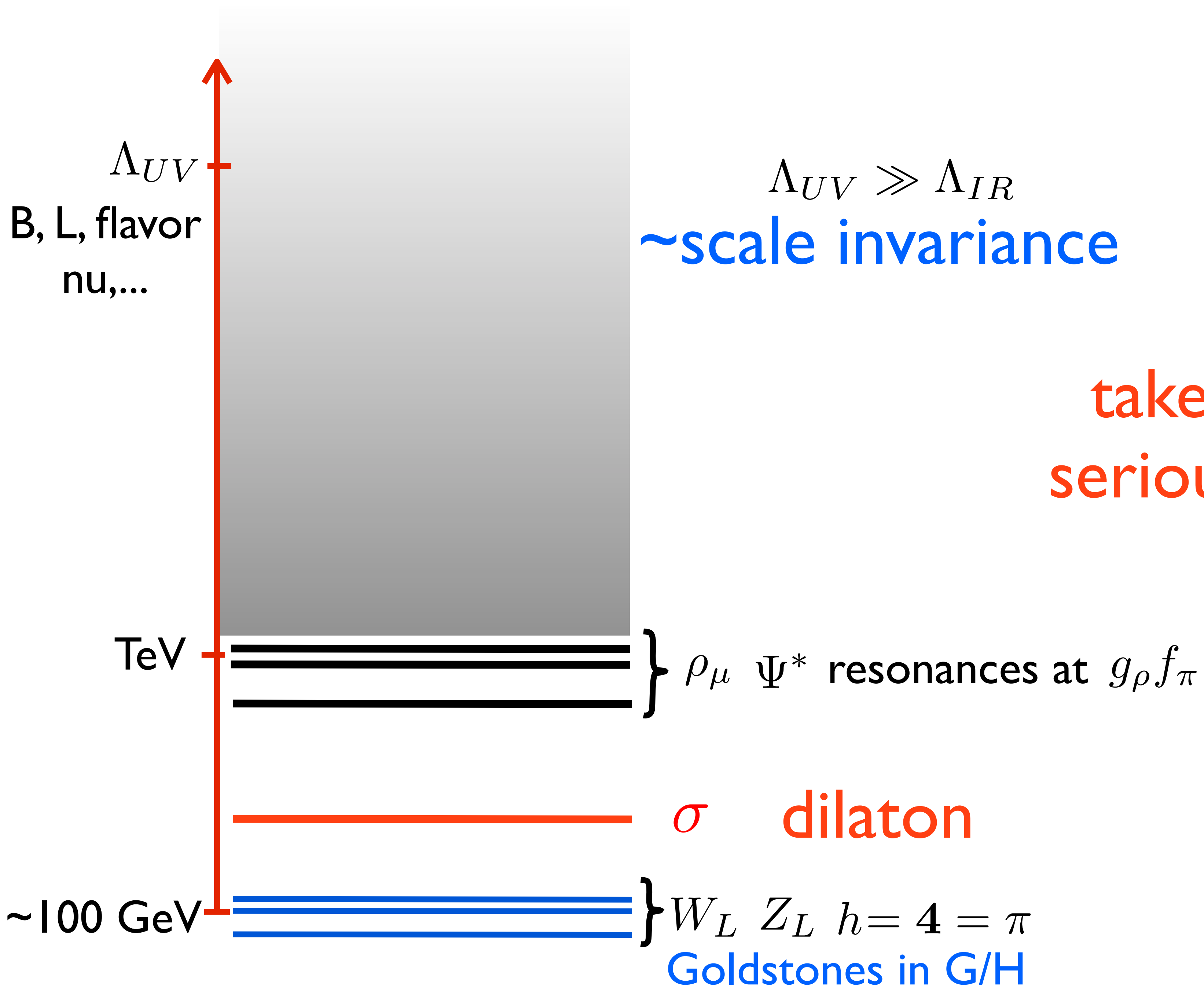
De Simone, Matsedonskyi, Rattazzi & Wulzer

A PLAUSIBLE SPECTRUM



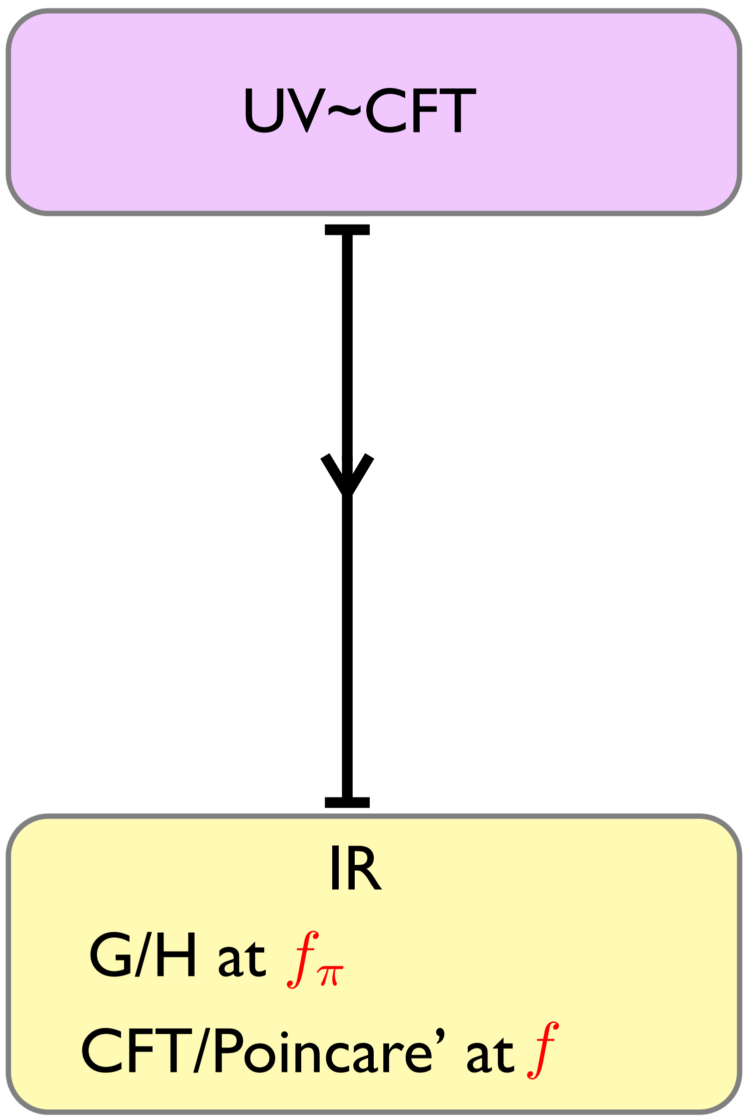
take scale invariance seriously even in the IR

A PLAUSIBLE SPECTRUM



take scale invariance seriously even in the IR

EFT for the Dilaton



σ dilaton $m_\sigma \ll m_\rho$

π GBs $m_\pi \ll m_\rho$

Effective theory for

(G+CFT)/(H+Poincare')?

global sym

spacetime sym

EFT-Dilaton alone: see e.g.
BB, Csaki, Hubisz, Serra & Terning 1209.3299
Chacko & Mishra 1209.3022
Goldberger, Grinstein & Skiba 0708.1463
Hubisz, Csaki & Lee 0705.3844

CFT-DICTIONARY

CONFORMAL GROUP

$$x \longrightarrow x'(x)$$

$$\eta_{\mu\nu} \longrightarrow e^{b(x')}\eta_{\mu\nu} = J(x)^{-\frac{1}{2}}\eta_{\mu\nu}$$

Jacobian

Poincare': $b = 0$

Dilations: $b = const$

Special Conformal: $b(x) = \log\left(1 - 2\epsilon x + \epsilon^2 x^2\right)^2$

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Fields organized w/ Lorentz + scale-dimension

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Lorentz scalar

$$\mathcal{O}(x) \longrightarrow \mathcal{O}'(x') = e^{b(x')\frac{\Delta}{2}} \cdot \mathcal{O}(x(x'))$$

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e.g. Dilations: $x' = e^{\alpha}x$

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Lorentz vector:

$$\mathcal{O}_\mu(x) \longrightarrow \mathcal{O}'_\mu(x') = e^{b(x')\frac{\Delta}{2}} \cdot \Lambda^\nu_\mu \mathcal{O}_\nu(x(x'))$$

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Tensor & Spinor rep. of: $\Lambda^\nu_\mu(x) = J^{-1/4}(x) \cdot \dfrac{\partial x'^{\,\nu}}{\partial x^\nu}$

Unbroken CFT

Lagrangian exactly
marginal

$$S_{CFT} = \sum_{\mathcal{O}} \int d^4x\, \mathcal{O}(x)$$

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$$\left|\frac{\partial x'}{\partial x}\right| = e^{-2b(x')}$$

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Lagrangian exactly marginal

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$\Delta_{\mathcal{O}} = 4$ no scale

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Invariant correlations: $\langle \mathcal{O}'(x_1')\mathcal{O}'(x_2')\ldots\mathcal{O}'(x_n')\rangle = \langle \mathcal{O}(x_1')\mathcal{O}(x_2')\ldots\mathcal{O}(x_n')\rangle$

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Example:

$$e^{-2\alpha\Delta}\langle \mathcal{O}(e^{-\alpha}x')\mathcal{O}(e^{-\alpha}y')\rangle = \langle \mathcal{O}(x')\mathcal{O}(y')\rangle \longrightarrow \langle \mathcal{O}(x)\mathcal{O}(y)\rangle = \frac{const}{|x-y|^{2\Delta}}$$

DILATON BASICS

$$\text{CFT} \xrightarrow{\langle \mathcal{O}(x) \rangle = f^{\Delta}} \text{Poincare'}$$

DILATON BASICS

CFT $\xrightarrow{\langle \mathcal{O}(x) \rangle = f^{\Delta}}$ Poincare'

GOLDSTONE=DILATON

$$\sigma(x) \xrightarrow{\text{dilations}} \sigma'(x') = \sigma(e^{-\alpha} x') - \alpha$$

shift-symmetry

spacetime tr.

$$\sigma(x) \xrightarrow{\text{special conform.}} \sigma'(x') = \sigma(x(x')) - \frac{1}{4} \log J(x(x'))$$

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scale=1

linear notation: $\chi(x) \equiv e^{\sigma(x)} \begin{cases} \to \chi'(x') = e^{-\alpha}\cdot\chi(e^{-\alpha}x') \\ \to \chi'(x') = J(x(x'))^{-\frac{1}{4}}\cdot\chi(x(x')) \end{cases}$

DILATON BASICS

DILATON BASICS

dilaton restores conformality

$$\mathcal{L}_{IR} \supset \mathcal{O}(x) \longrightarrow \mathcal{O}(x) \times \chi^{4-\Delta_{\mathcal{O}}}$$

dilaton couples to non-marginality

$$\mathcal{L}_{IR} \supset \mathcal{O}(x)\left[1 + (4-\Delta)\frac{\sigma}{f} + \ldots\right] = \mathcal{O}(x) + \frac{\sigma}{f}\partial_\mu D^\mu + \ldots$$

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like the SM-scalar: it couples to the mass

$$\frac{1}{f}\sigma T^\mu_\mu = \frac{v}{f}\sigma\left[2m_W^2 W^2 + m_\psi \psi\psi \dots\right]$$

overall rescaling

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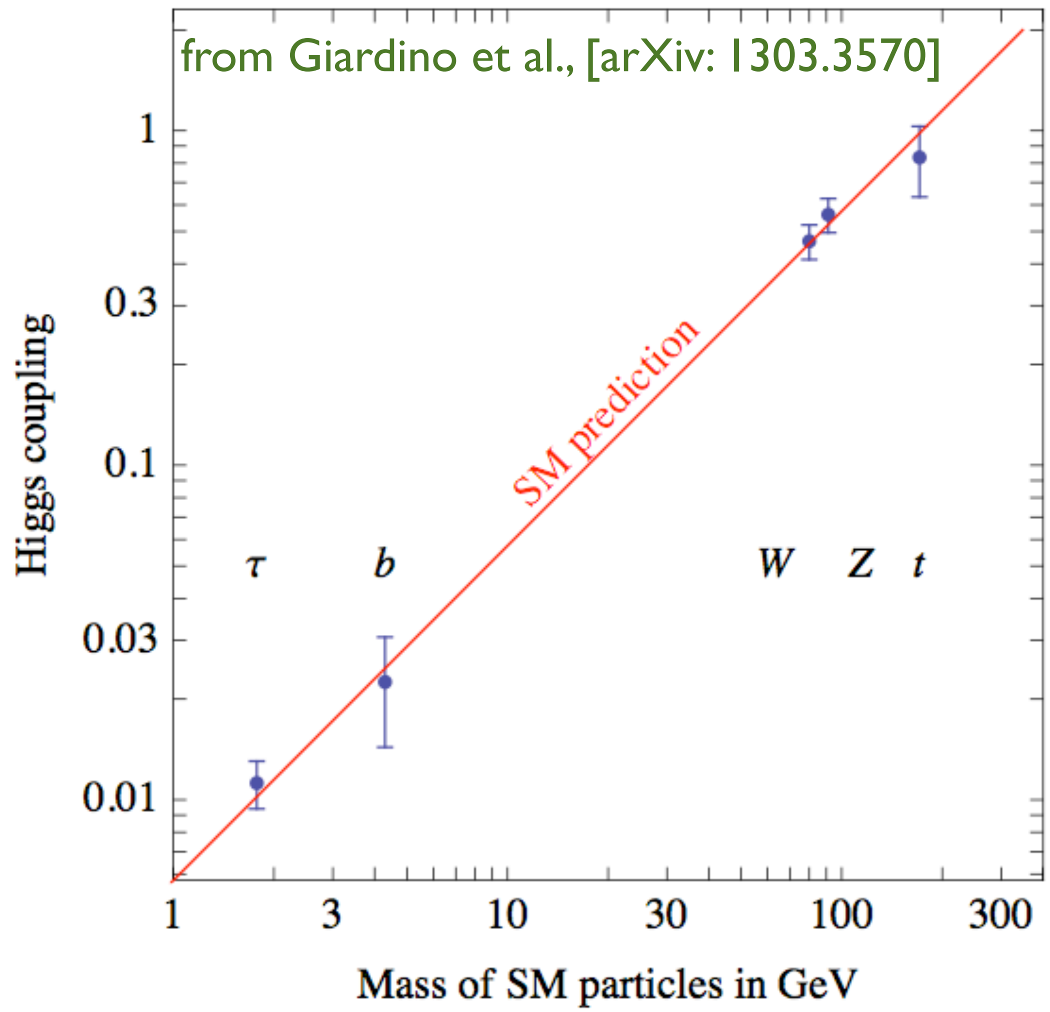
Fit to Higgs couplings



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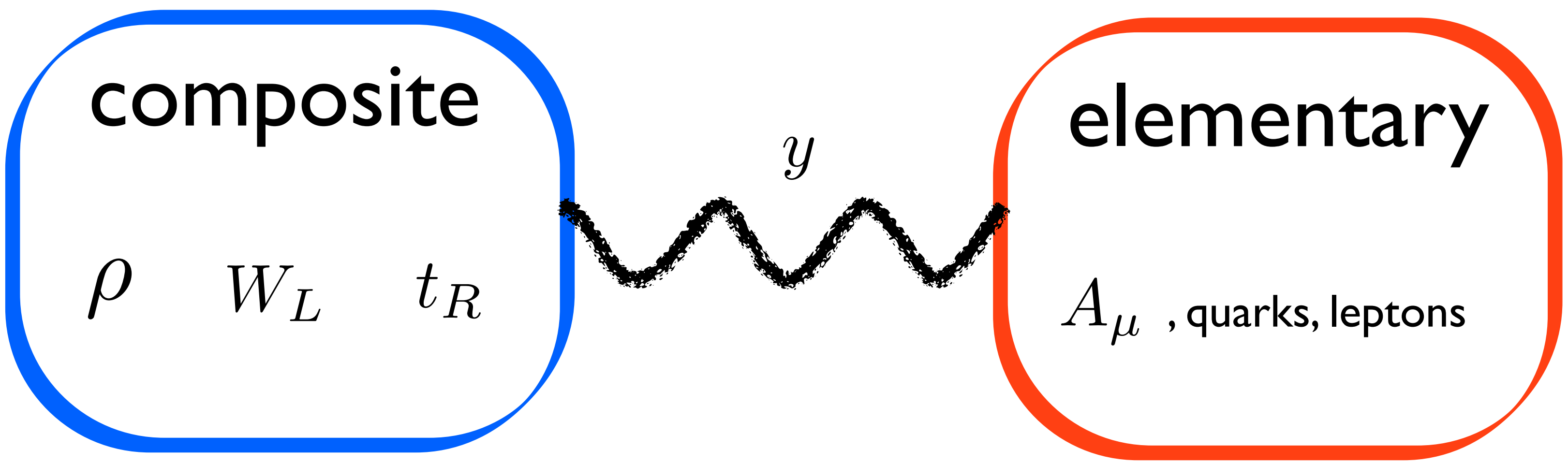
overall rescaling

Higgs-like Dilaton?

SO(4)/SO(3)+ dilaton with $v \sim f$ within 10%

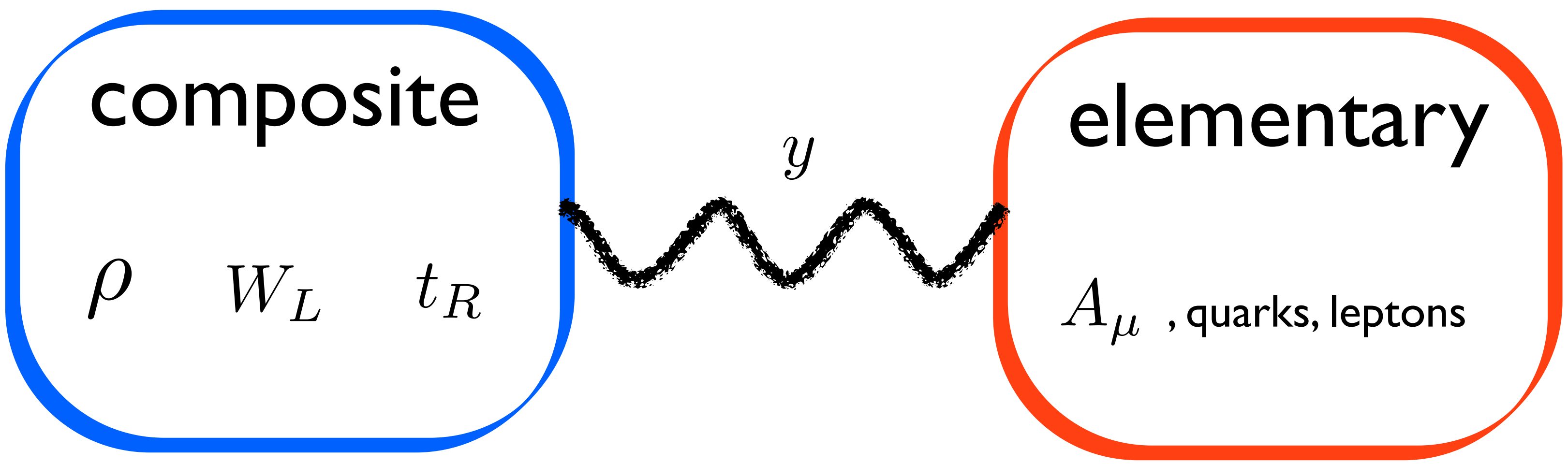
BB, Csaki, Hubisz, Serra, Terning 1209.3299;
Chacko, Franceschini, Mishra 1209.3259

PARTIAL COMPOSITENESS



$$\mathcal{L}_{CFT}^{UV} + \mathcal{L}_{elem} + \sum_i y_i \mathcal{O}_{elem,i} \mathcal{O}_{CFT,i}^{UV}$$

PARTIAL COMPOSITENESS



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$\Delta_{elem,i}^{UV}$

$\Delta_{CFT,i}^{UV}$

spurion dimensions

$$[y_i] = 4 - \Delta_{CFT,i}^{UV} - \Delta_{elem,i}^{UV}$$

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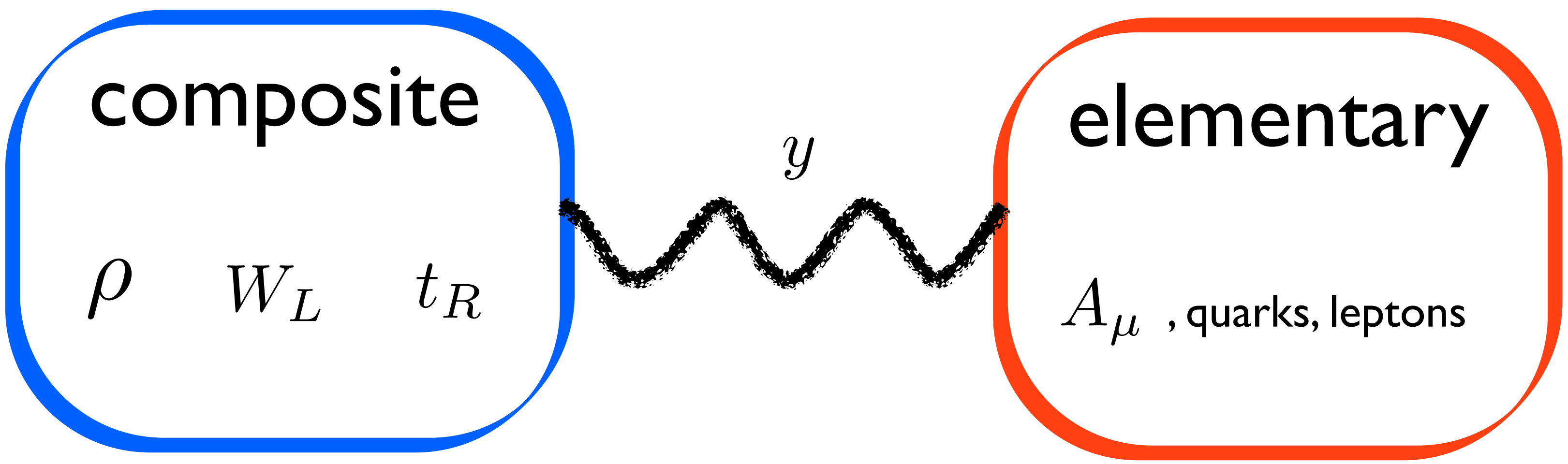

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$\Delta_{elem,i}^{UV}$ $\Delta_{CFT,i}^{UV}$

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condense $\langle \mathcal{O}(x) \rangle = f^{\Delta}$

$$\mathcal{L}_{CFT}^{IR} + \mathcal{L}_{elem} + \sum_i y_i \mathcal{O}_{elem,i} \mathcal{O}_{CFT,i}^{IR}$$

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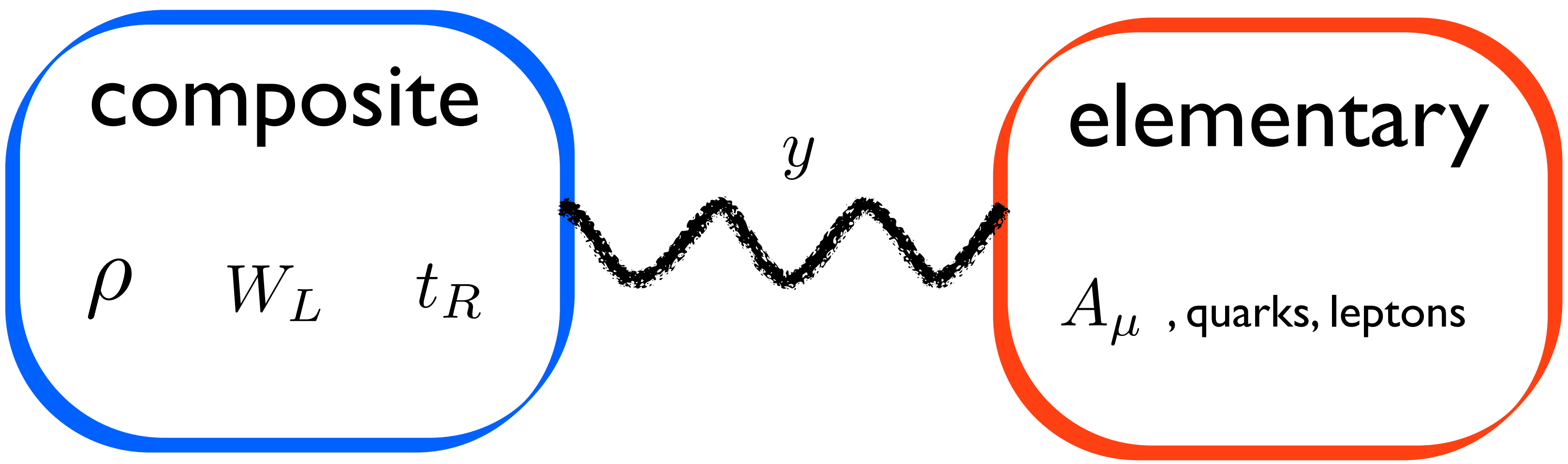

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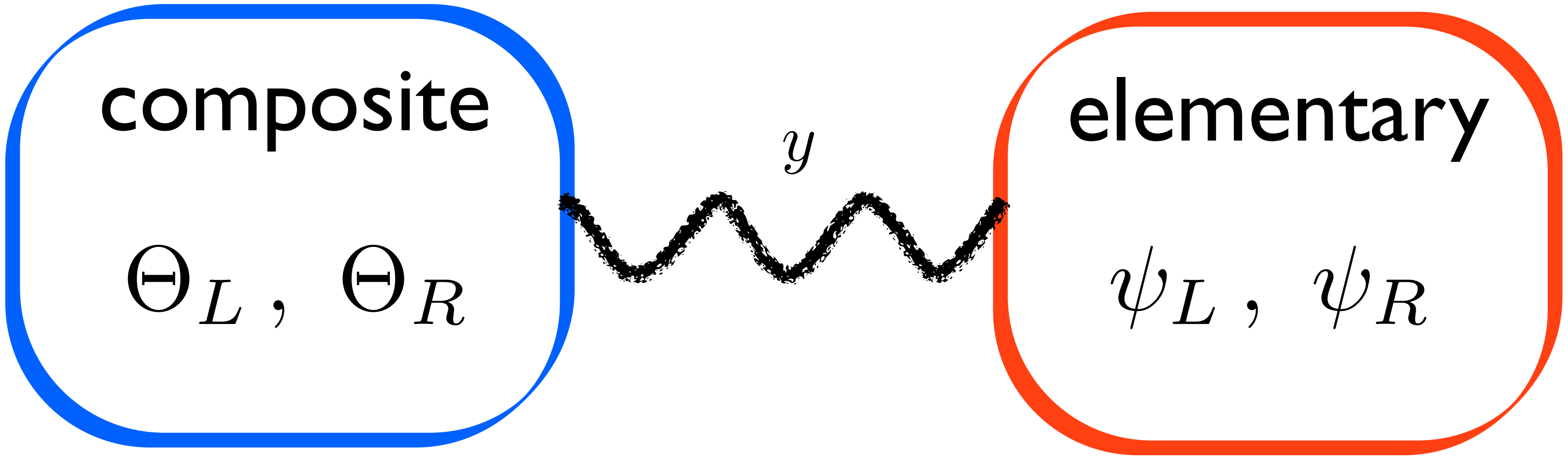
$$\mathcal{L}_{CFT}^{IR} + \mathcal{L}_{elem} + \sum_i y_i \mathcal{O}_{elem,i} \mathcal{O}_{CFT,i}^{IR} \times \overbrace{\chi^{\left(\Delta_{CFT,i}^{UV} - \Delta_{CFT,i}^{IR} + \Delta_{elem,i}^{UV} - \Delta_{elem,i}^{IR}\right)}}^{\text{compensate}}$$

FERMION COUPLINGS



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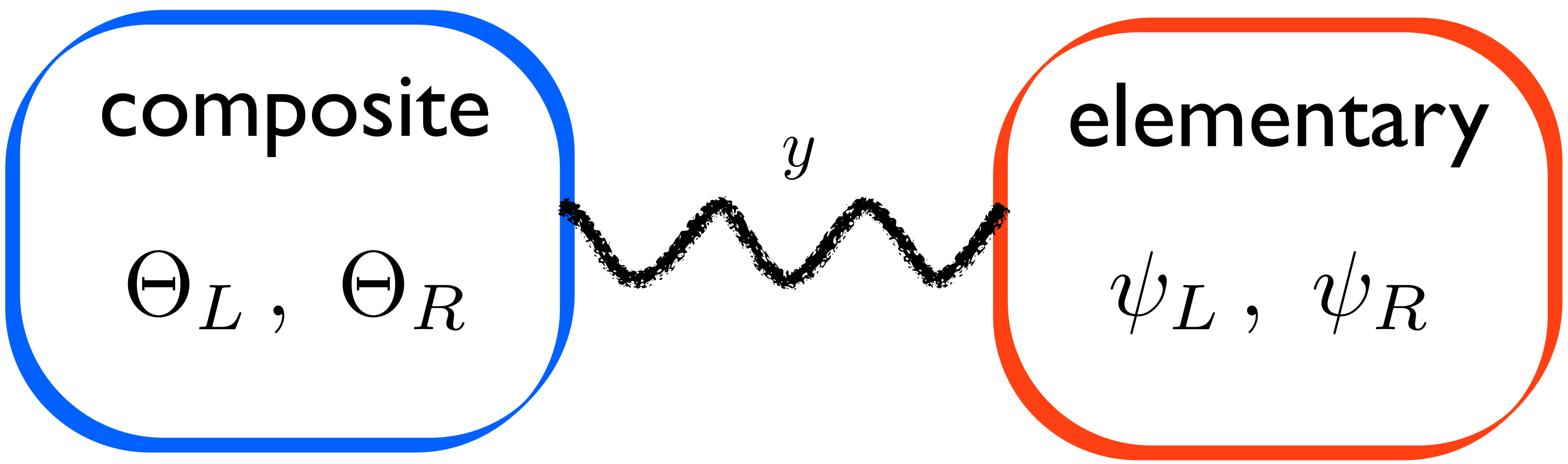


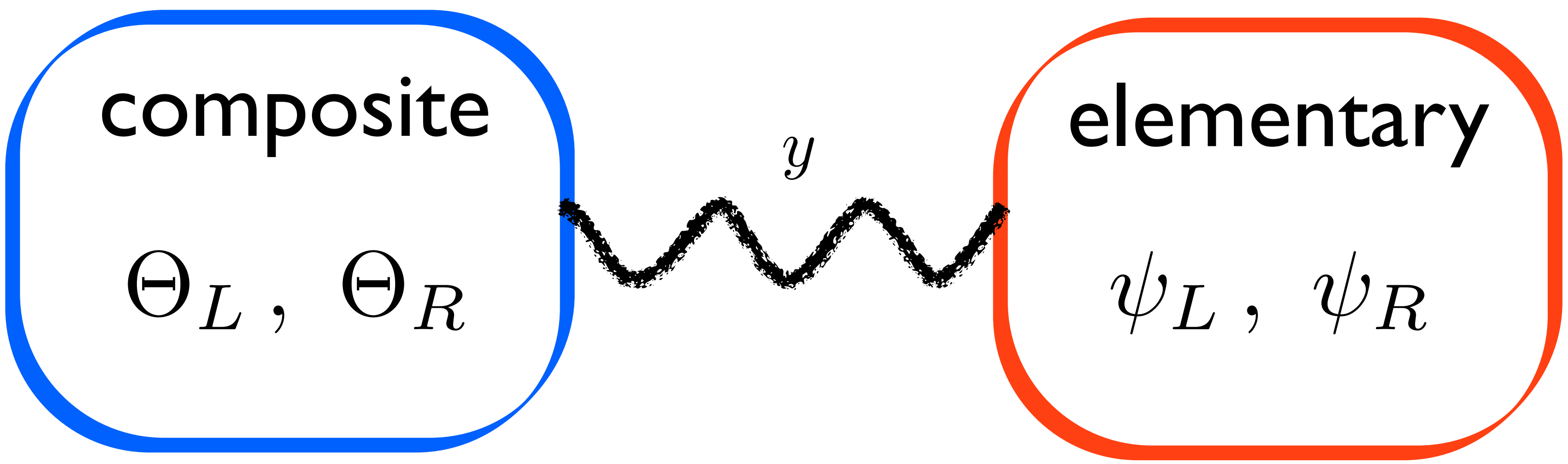
$$\mathcal{L}_{mix} = y_L\psi_L\Theta_R + y_R\psi_R\Theta_L$$

$$[y_{R,L}] = -\gamma_{L,R}$$

$3/2$

$5/2 + \gamma_R$

FERMION COUPLINGS



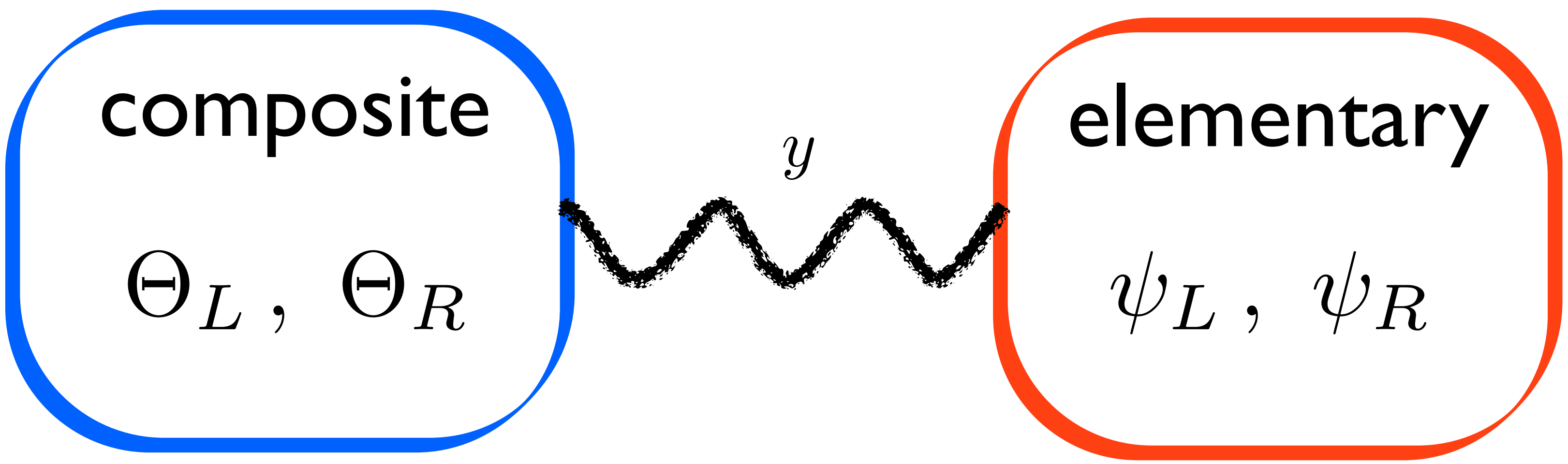
$$\mathcal{L}_{mix} = y_L \psi_L \Theta_R + y_R \underbrace{\psi_R}_{3/2} \underbrace{\Theta_L}_{5/2+\gamma_R}$$

$$[y_{R,L}] = -\gamma_{L,R}$$

integrate out the CFT: $\sim y_L y_R v\, \psi_L \psi_R$ compensate: $\sim y_L y_R v\, \psi_L \psi_R \times \chi^{1+\gamma_L+\gamma_R}$

$$\boxed{\mathcal{L} \supset m_\psi\, \psi_L \psi_R \left[1 + \frac{\sigma}{f}(1+\gamma_L+\gamma_R)\right]}$$

FERMION COUPLINGS



$$\mathcal{L}_{mix} = y_L \psi_L \Theta_R + y_R \psi_R \Theta_L$$

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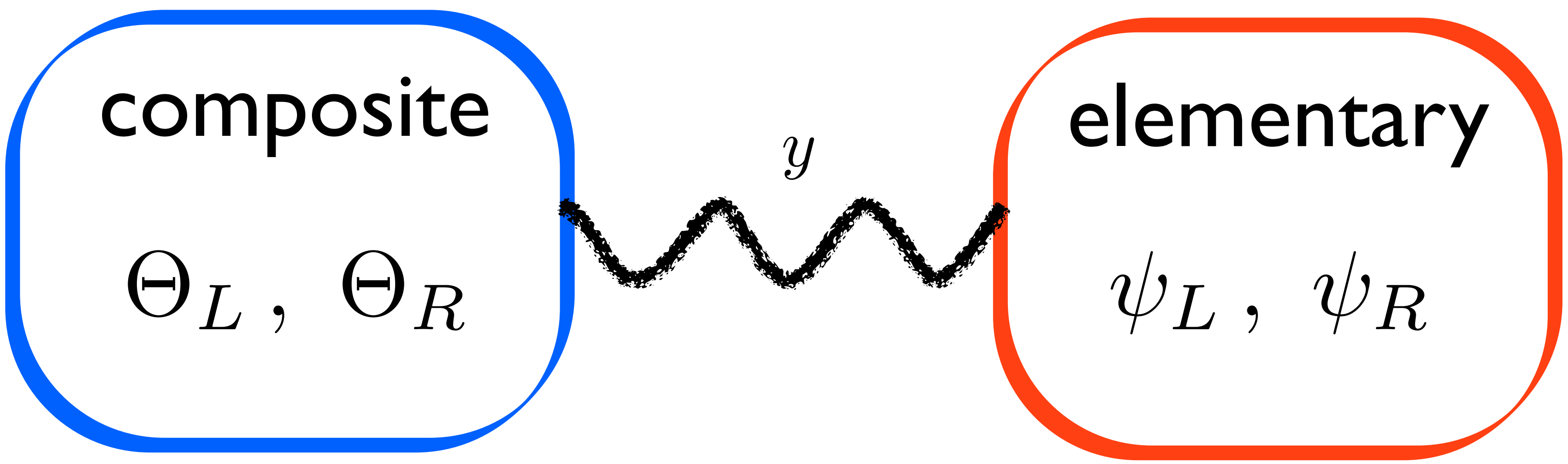

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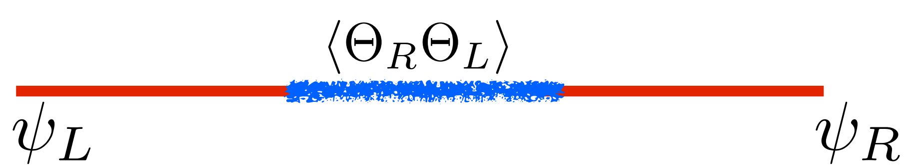
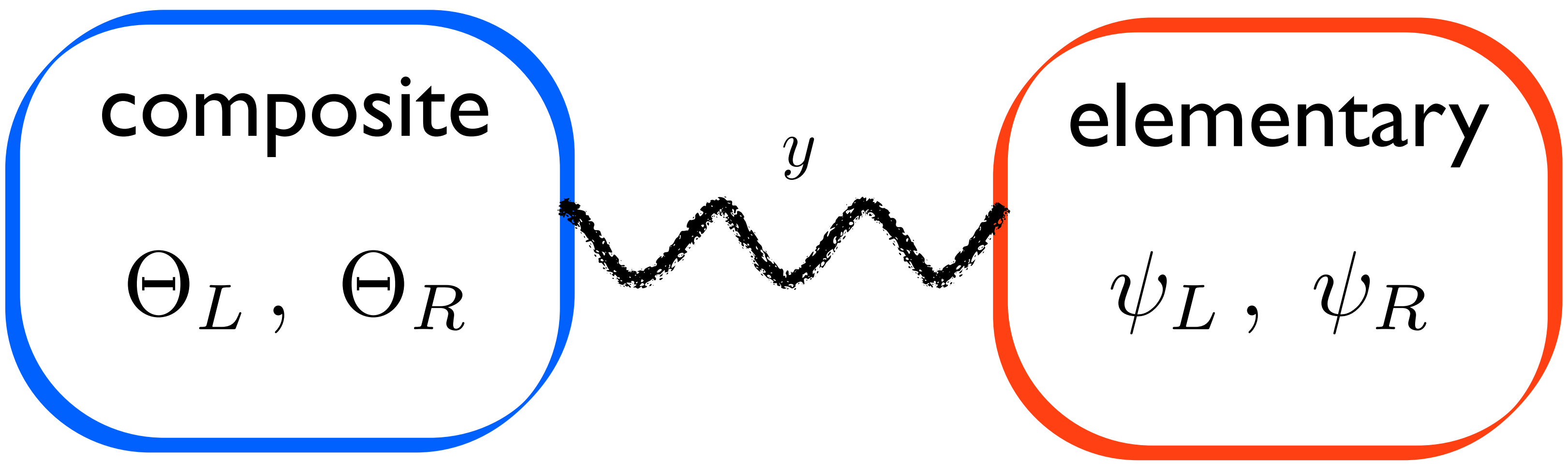


$$m_\psi \sim y_L(f)\, y_R(f)\, f$$

$$y(f) = y(\Lambda) \left(\frac{f}{\Lambda} \right)^\gamma$$

FERMION COUPLINGS



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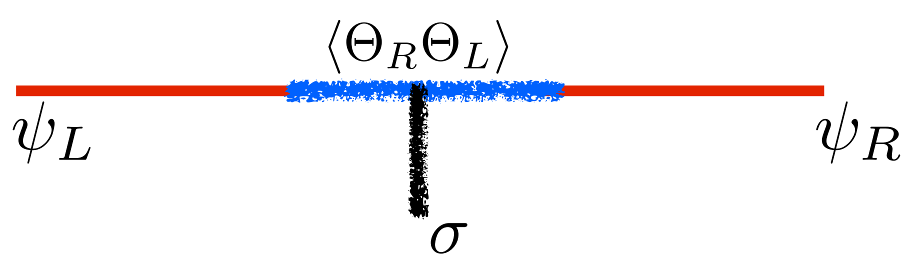

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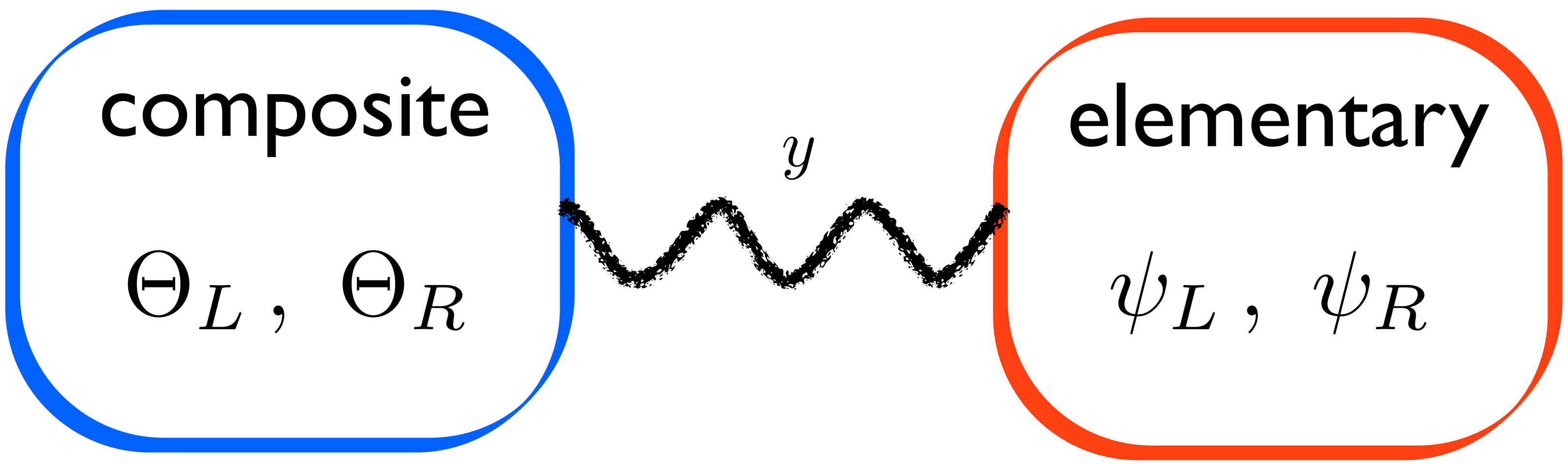
compensate: $f \longrightarrow f\chi = f e^{\sigma/f}$

$$\mathcal{L} \supset m_\psi\, \psi_L \psi_R \left[1 + \frac{\sigma}{f}(1 + \gamma_L + \gamma_R) \right]$$

FERMION COUPLINGS



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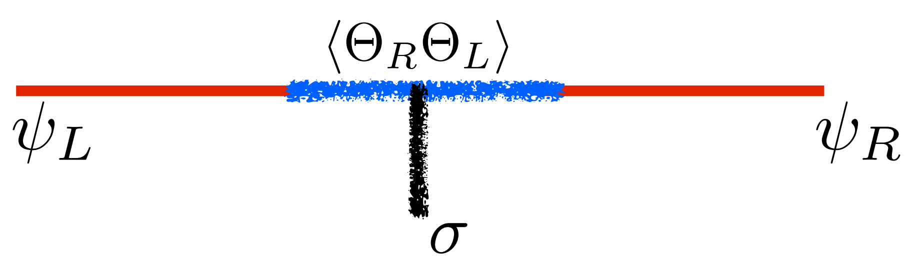

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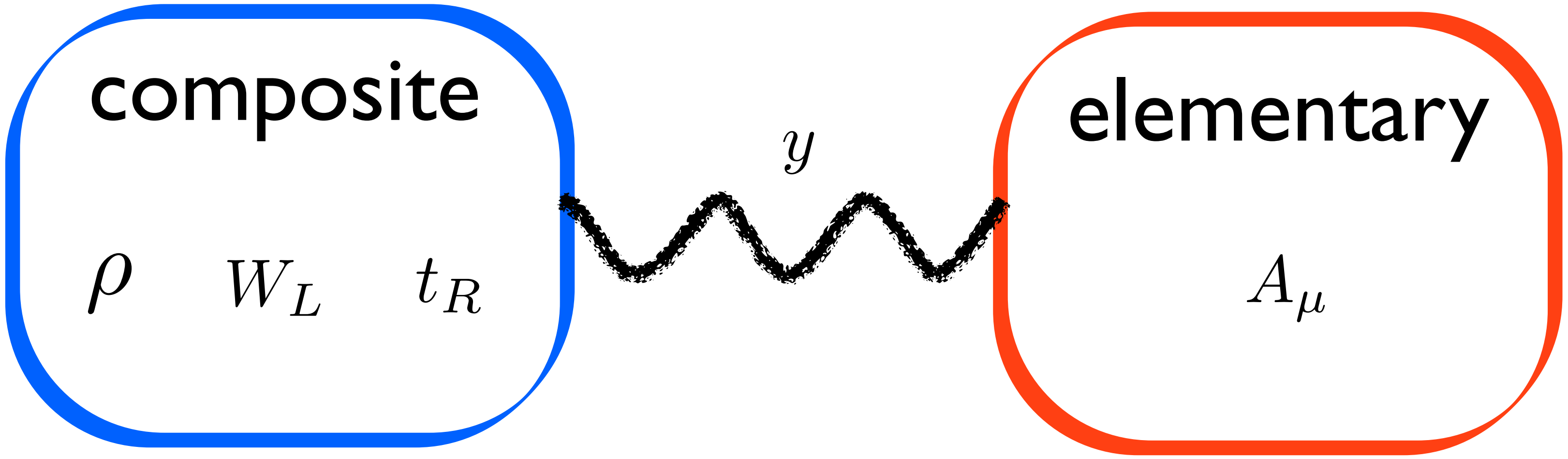
gamma positive for light fermions

(gamma~0.1 for taus)

PHOTON AND GLUON COUPLINGS



$$\mathcal{L}_{mix} \supset -\frac{1}{4g^2}F_{\mu\nu}^2 + A_\mu \mathcal{J}^\mu$$

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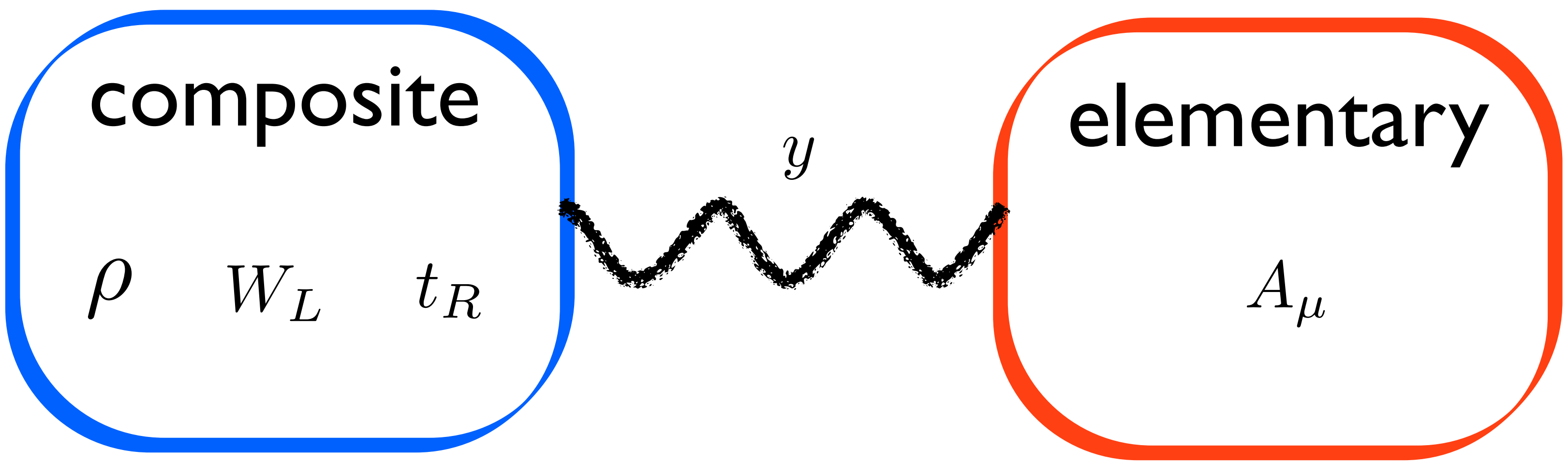

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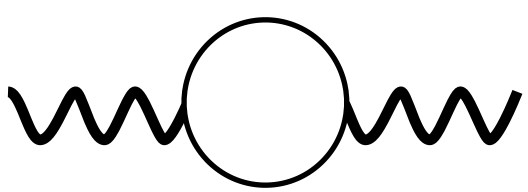
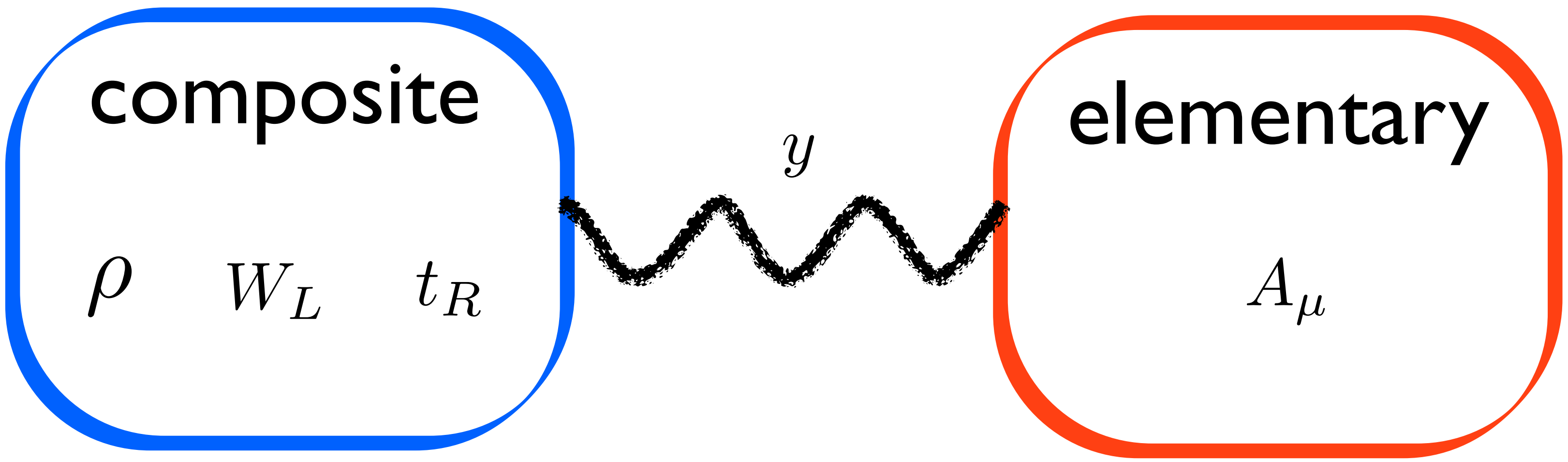
integrate out the CFT: $\quad -\frac{1}{4g^2(\mu)} F_{\mu\nu}^2$



$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} - \frac{b_{UV}^{CFT}}{8\pi^2} \log \frac{\Lambda}{f} - \frac{b_{IR}^{comp}}{8\pi^2} \log \frac{f}{\mu} - \frac{b_{elem}}{8\pi^2} \log \frac{\Lambda}{\mu}$$

PHOTON AND GLUON COUPLINGS



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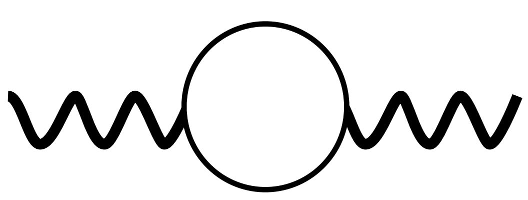
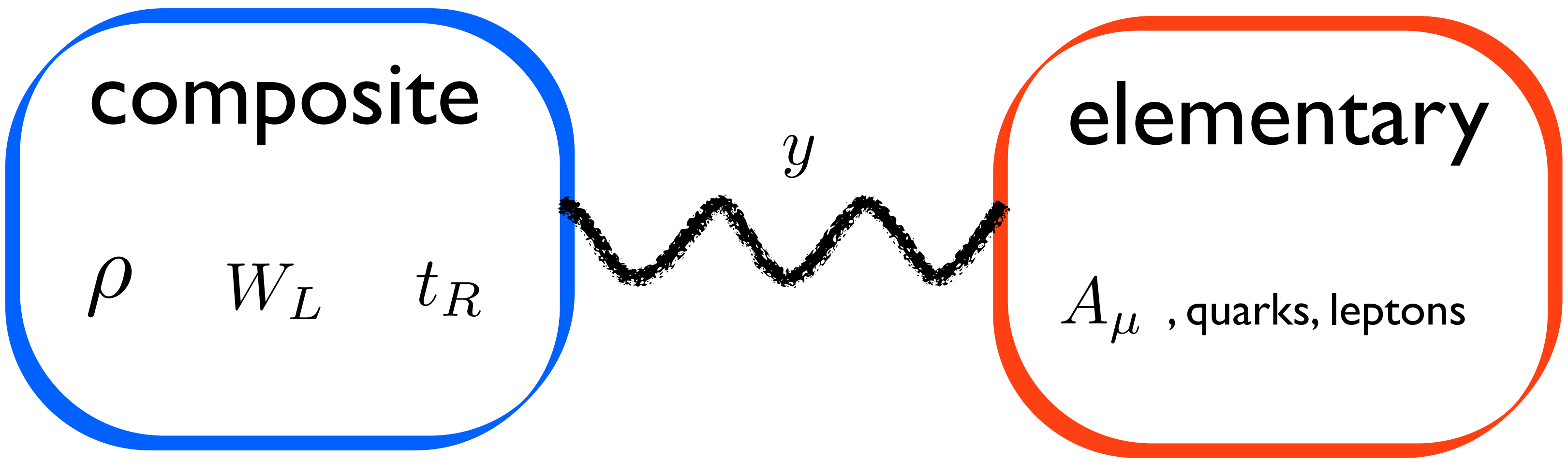


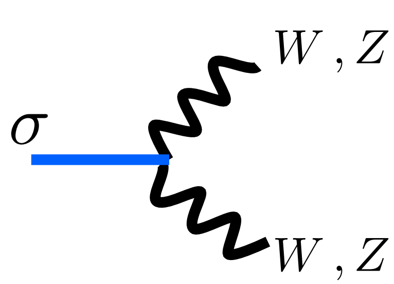
$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} - \frac{b_{UV}^{CFT}}{8\pi^2} \log\frac{\Lambda}{f} - \frac{b_{IR}^{comp}}{8\pi^2} \log\frac{f}{\mu} - \frac{b_{elem}}{8\pi^2} \log\frac{\Lambda}{\mu}$$

compensate: $f \longrightarrow f\chi = f e^{\sigma/f}$ $\longrightarrow$ $\mathcal{L} = -\frac{1}{2}\left(\frac{\beta_{IR}^{comp}}{g} - \frac{\beta_{UV}^{CFT}}{g}\right)\frac{\sigma}{v} F_{\mu\nu}^2$

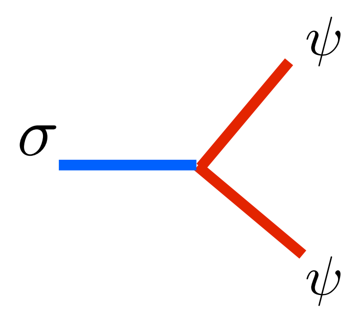
DILATON COUPLINGS: SUMMARY



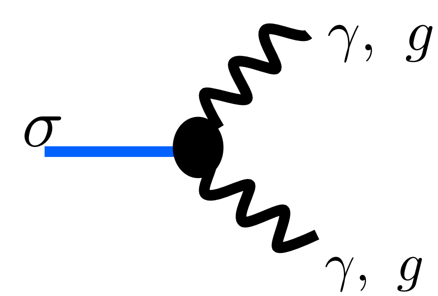
$$\mathcal{L} = \overbrace{\frac{v}{f}}^{\text{overall rescaling}}\sigma\left\{\left[2m_W^2 W_\mu^2 + m_Z^2 Z^2 + m_\psi \psi \overbrace{(1+\gamma)}^{\text{anomalous dim.}}\psi \dots\right] + 2\overbrace{(\beta_{UV}^{CFT} - \beta_{IR}^{comp})}^{\text{beta-functions}}/g\, F_{\mu\nu}^2\right\}$$



$SM \times \frac{v}{f}$

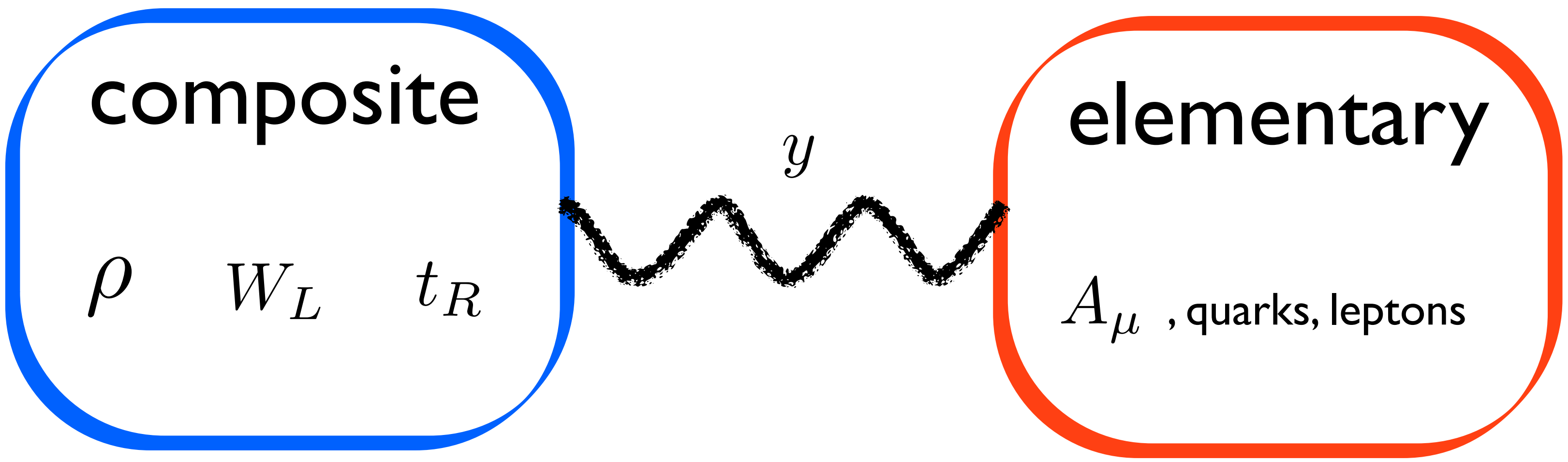


$SM \times \frac{v}{f}(1+\gamma)$



$\frac{v}{f}(\beta_{UV}^{CFT} - \beta_{IR}^{comp})$

DILATON COUPLINGS: SUMMARY



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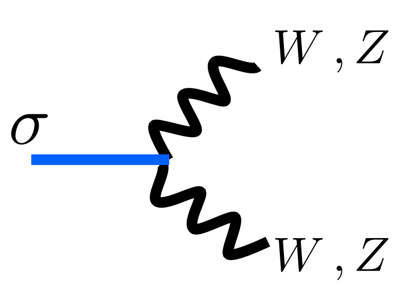

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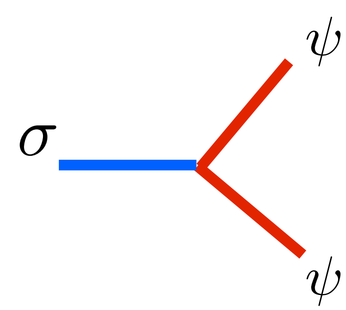

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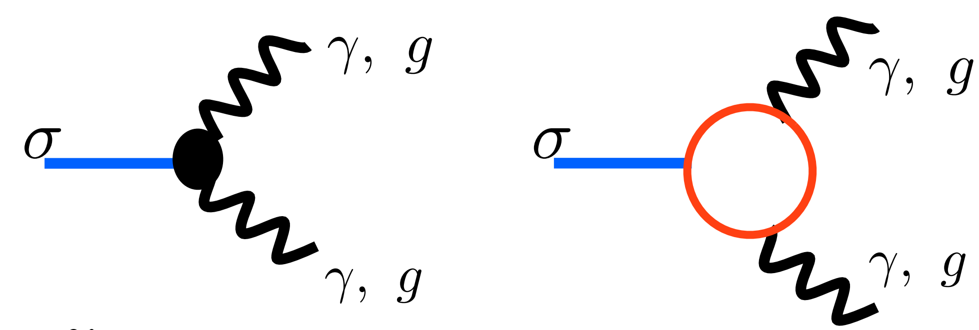


$$\frac{v}{f}(\beta_{UV}^{CFT} - \beta_{IR}^{comp} + loops)$$

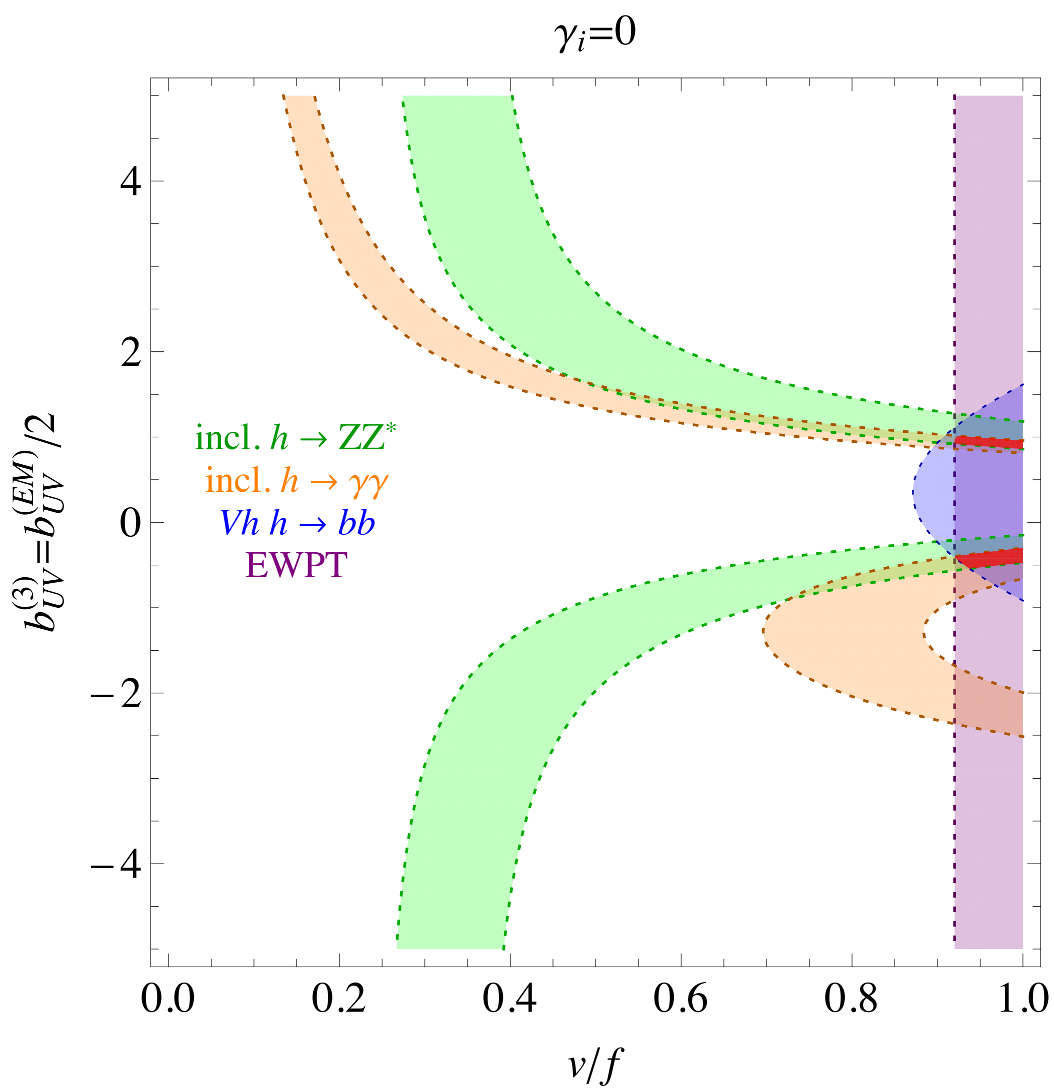
example w/ composite top-right for Higgs-like Dilaton: $\frac{v}{f}(\beta_{UV}^{CFT} + \beta_{SM}^{\gamma} - \beta_{t_R,W_L}^{\gamma})$

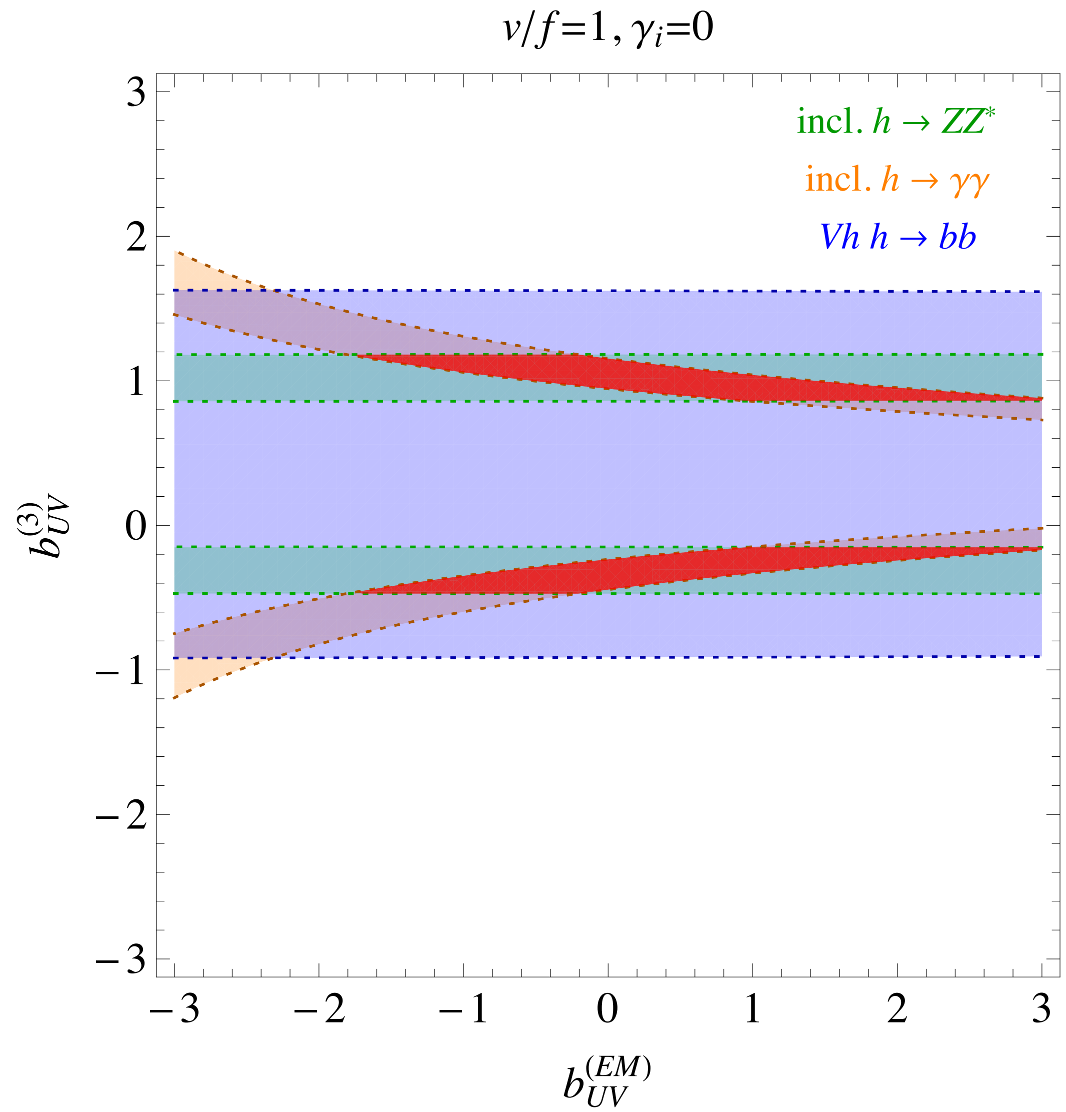
Higgs-like dilaton: fitting data

rates scale with: v^2/f^2

$$R = \frac{\sigma}{\sigma_{SM}} \times \frac{BR}{BR_{SM}}$$

easily fit data if v~f
&
m=125 GeV





EFT: Goldstones+Dilaton

dilaton restores the CFT

covariant conformal derivatives

$$\mathcal{L}_{IR}(\phi, \partial_\mu \phi) \longrightarrow \mathcal{L}_{CFT} = \chi^4 \mathcal{L}_{IR}\left(\frac{\phi}{\chi^\Delta}, \frac{\nabla_\mu \phi}{\chi^{\Delta+1}}\right) + \frac{f^2}{2}(\partial_\mu \chi)^2 + \underbrace{\ldots}_{O(p^\wedge 4)}$$

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IR contains the Goldstones of G/H including the scalar H

$$\pi^{\hat{a}} = (\underbrace{\pi^{i=1,2,3}}_{(Z_L, W_L^\pm)}, h, \ldots) \qquad \mathcal{L}_{IR}^{(2)} = \frac{1}{2} f_\pi^2 \partial_\mu \pi^{\hat{a}} \partial_\mu \pi^{\hat{b}} g_{\hat{a}\hat{b}}(\pi)$$

Pi's restore G

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Pi's restore G

scale dim. can be suitably chosen: $\Delta_\pi = 0$ "angles"

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Pi's restore G

scale dim. can be suitably chosen: $\Delta_\pi = 0$ "angles"

$$\boxed{\mathcal{L}_{CFT+G}^{(2)} = \frac{1}{2}\chi^2 f_\pi^2 \left[\partial_\mu \pi^{\hat{a}} \partial_\mu \pi^{\hat{b}} g_{\hat{a}\hat{b}}(\pi)\right] + \frac{1}{2} f^2 (\partial\chi)^2}$$

all GB's restore G+CFT

GEOMETRY: THE CONE MANIFOLD

$$\mathcal{L}^{(2)}_{CFT+G} = \frac{1}{2}\left[\left(\frac{f_\pi^2}{f^2}\right) \chi^2 \left(\partial_\mu \pi^{\hat{a}} \partial_\mu \pi^{\hat{b}} g_{\hat{a}\hat{b}}(\pi)\right) + \frac{1}{2}(\partial\chi)^2\right]$$

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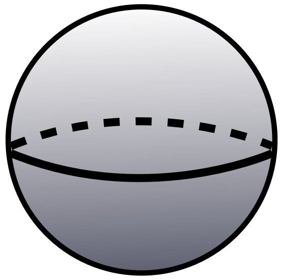

$$f_\pi^2 d\Pi^2 = f_\pi^2 g_{\hat{a}\hat{b}}(\pi)d\pi^{\hat{a}}d\pi^{\hat{b}}$$

GEOMETRY: THE CONE MANIFOLD

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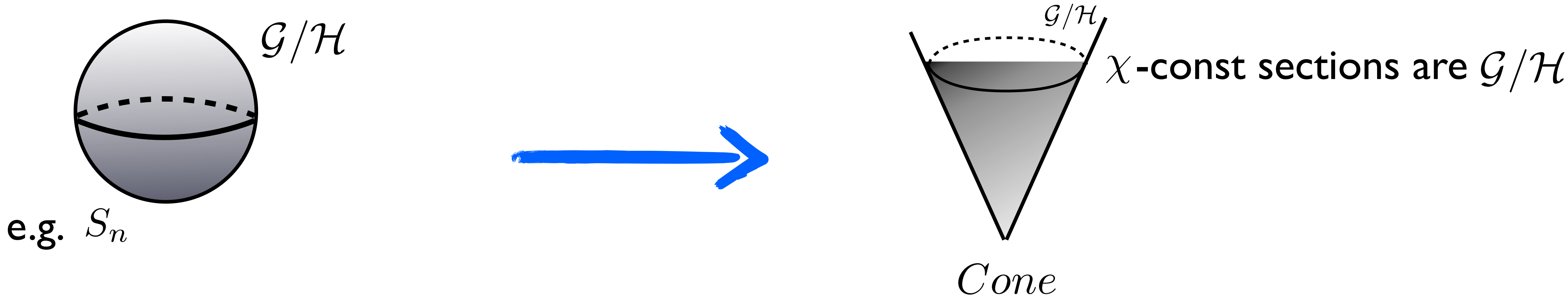

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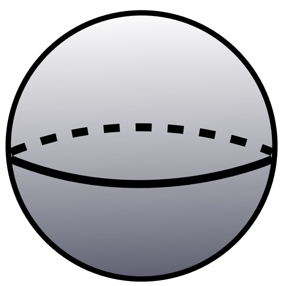

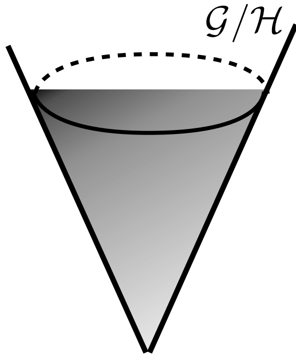

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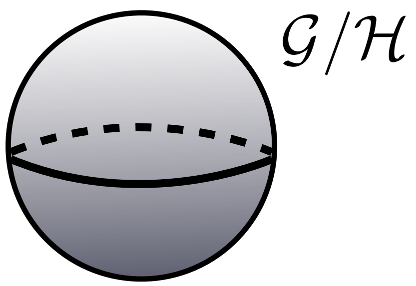
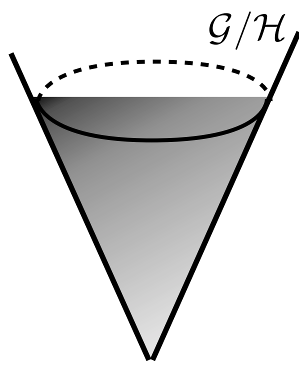


e.g. S_n



χ-const sections are $\mathcal{G}/\mathcal{H}$

$Cone$

$$f_\pi^2 d\Pi^2 = f_\pi^2 g_{\hat{a}\hat{b}}(\pi)d\pi^{\hat{a}}d\pi^{\hat{b}}$$

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e.g. $|\Phi|^2 = f_\pi^2$ “radius”

“speed of light”

$$|\Phi|^2 = \frac{f_\pi^2}{(f^2 - f_\pi^2)}\chi^2$$

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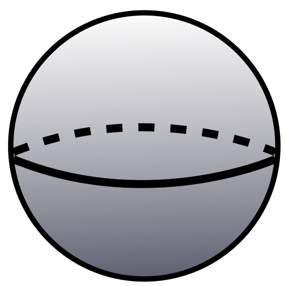

$\mathcal{G}/\mathcal{H}$

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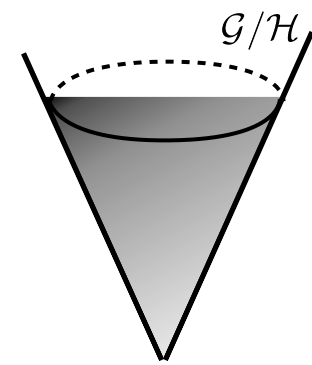

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e.g. $|\Phi|^2 = f_\pi^2$ "radius"

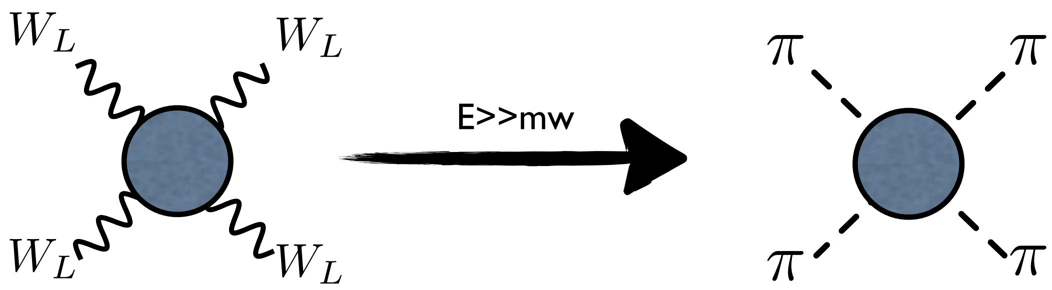
"speed of light"

$$|\Phi|^2 = \frac{f_\pi^2}{(f^2 - f_\pi^2)}\chi^2$$

1) non-compact (dilations): no scale

2) singular at the apex: cut-off=0

WW-scattering



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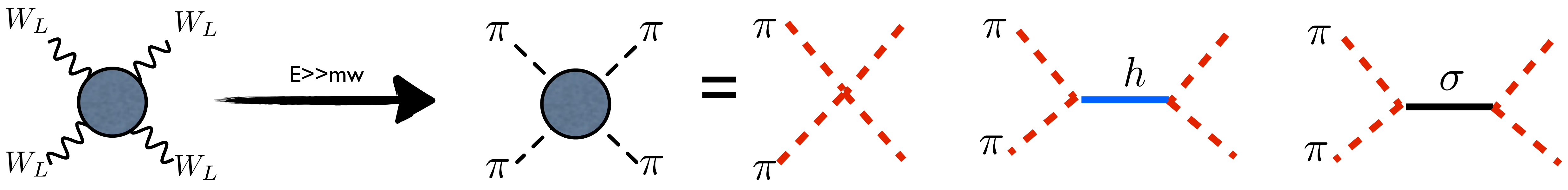

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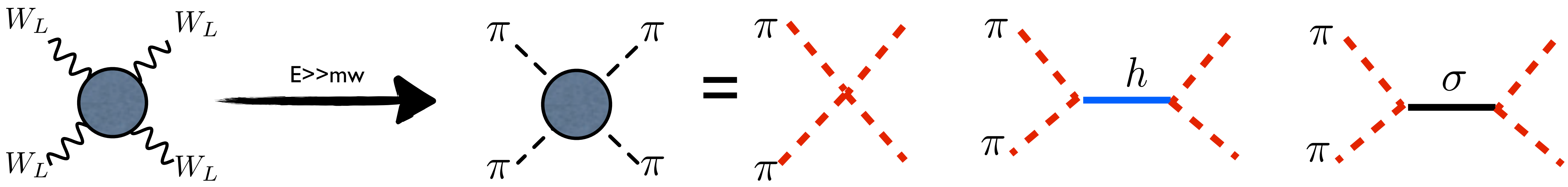
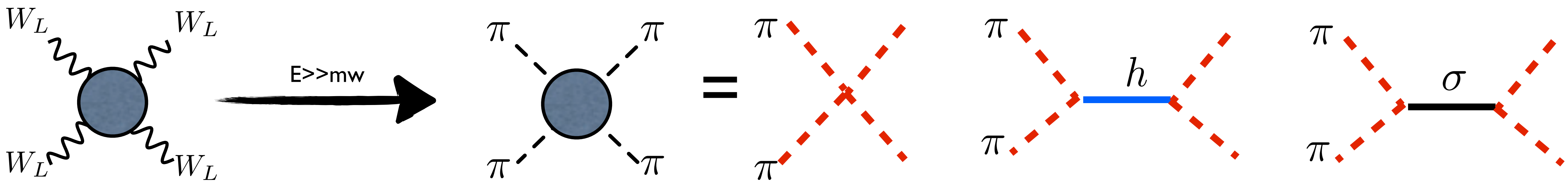


$$SO(n+1)/SO(n) = S_n \longrightarrow A(\pi\pi \to \pi\pi) \sim \frac{s}{f_\pi^2} - \frac{s}{f^2}$$

WW-SCATTERING



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Limit: $f = f_\pi$

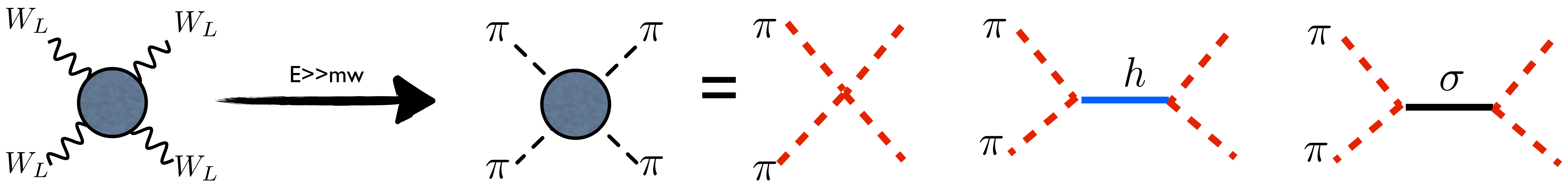
$$A(\pi\pi \to \pi\pi) \sim \left(\frac{s}{f_\pi^2} - \frac{s}{f^2}\right) \to 0$$

$$A(\pi\pi \to hh) \sim \left(\frac{s}{f_\pi^2} - \frac{s}{f^2}\right) \to 0$$

$$A(\pi\pi \to \sigma\sigma) \to 0$$

... all amplitudes vanish!!

WW-SCATTERING



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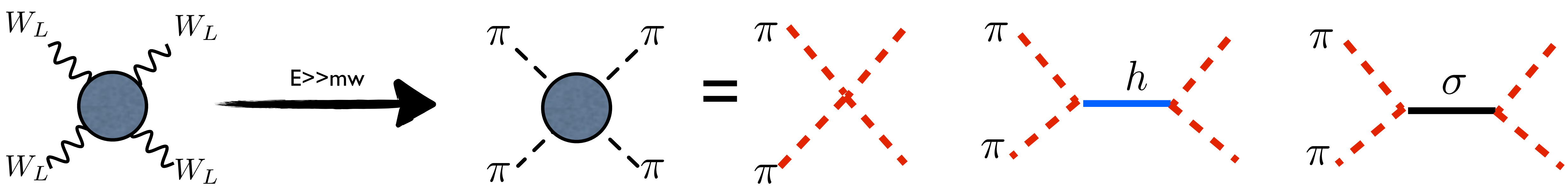
E.g.: the Higgs-like dilaton SO(4)/SO(3)?

$$(f_\pi \equiv)\, v = f\ ?$$

WW-SCATTERING



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... all amplitudes vanish!!

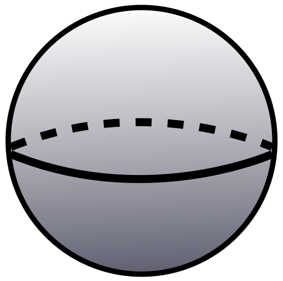
E.g.: the Higgs-like dilaton SO(4)/SO(3)?

$$(f_\pi \equiv)\, v = f\ ?$$

- symmetry or tuning (or dynamics)?
- is it actually weakly coupled?

$\Lambda \sim 4\pi f\ ?$ $\quad \Lambda \gg 4\pi f\ ?$

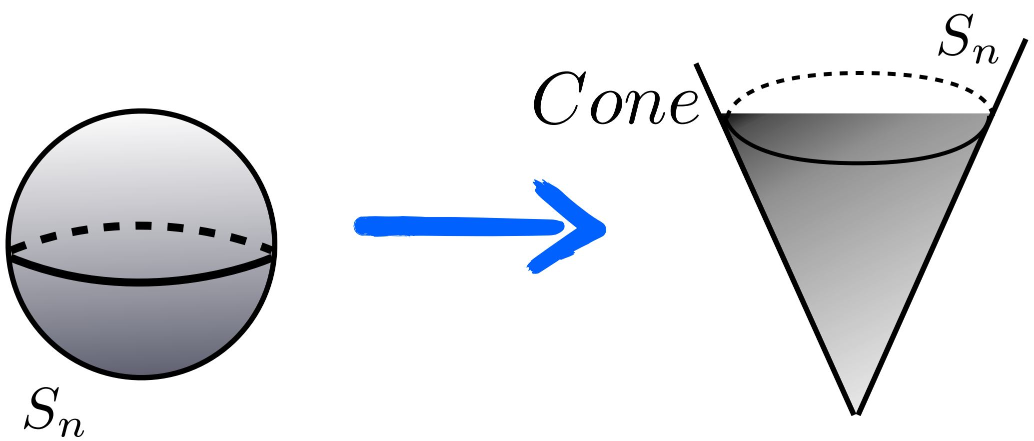
Cone vs Plane



S_n

$$|\Phi|^2 = f_\pi^2$$

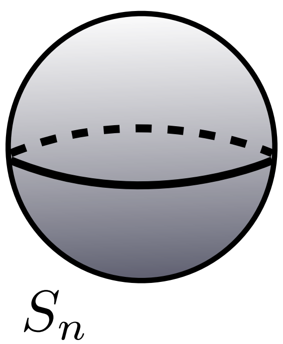
Cone vs Plane



$$|\Phi|^2 = f_\pi^2$$

$$|\Phi|^2 = \frac{f_\pi^2}{(f^2 - f_\pi^2)}\chi^2$$

Cone vs Plane



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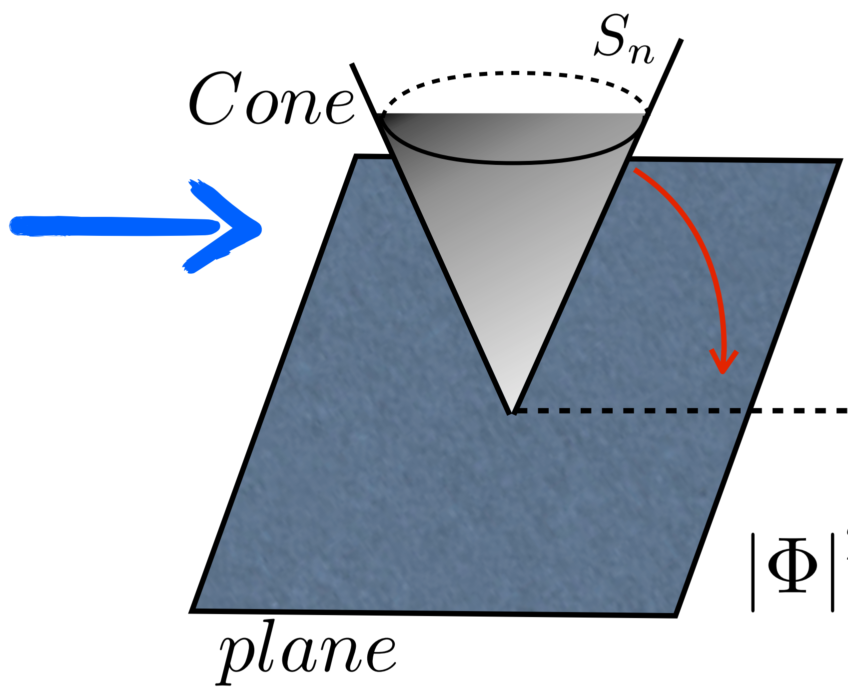


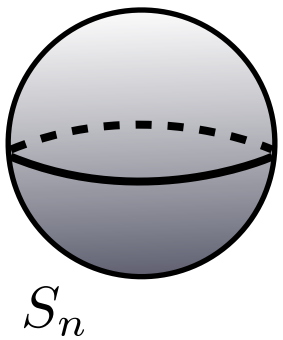
the cone unfolds into a plane

"speed of light"= ∞

$$f_\pi = f$$

$$|\Phi|^2 = \frac{f_\pi^2}{(f^2 - f_\pi^2)}\chi^2$$

Cone vs Plane



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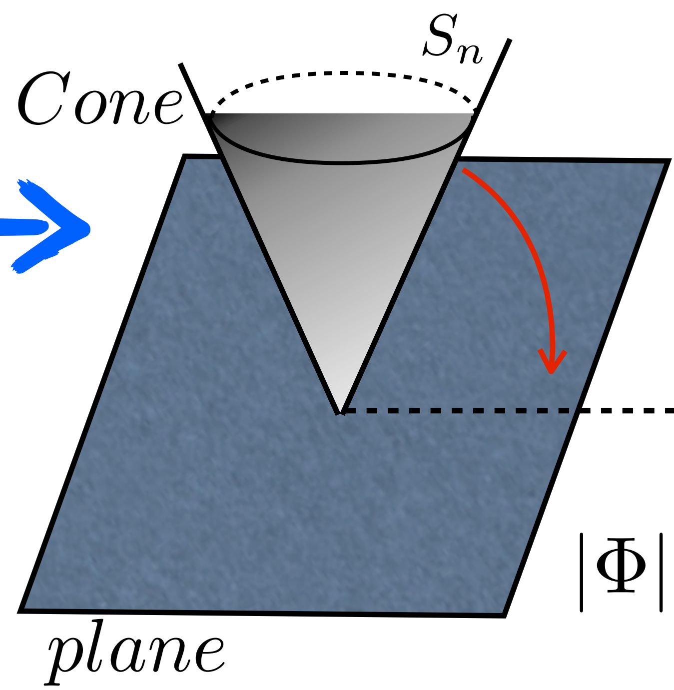

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$$f_\pi = f$$

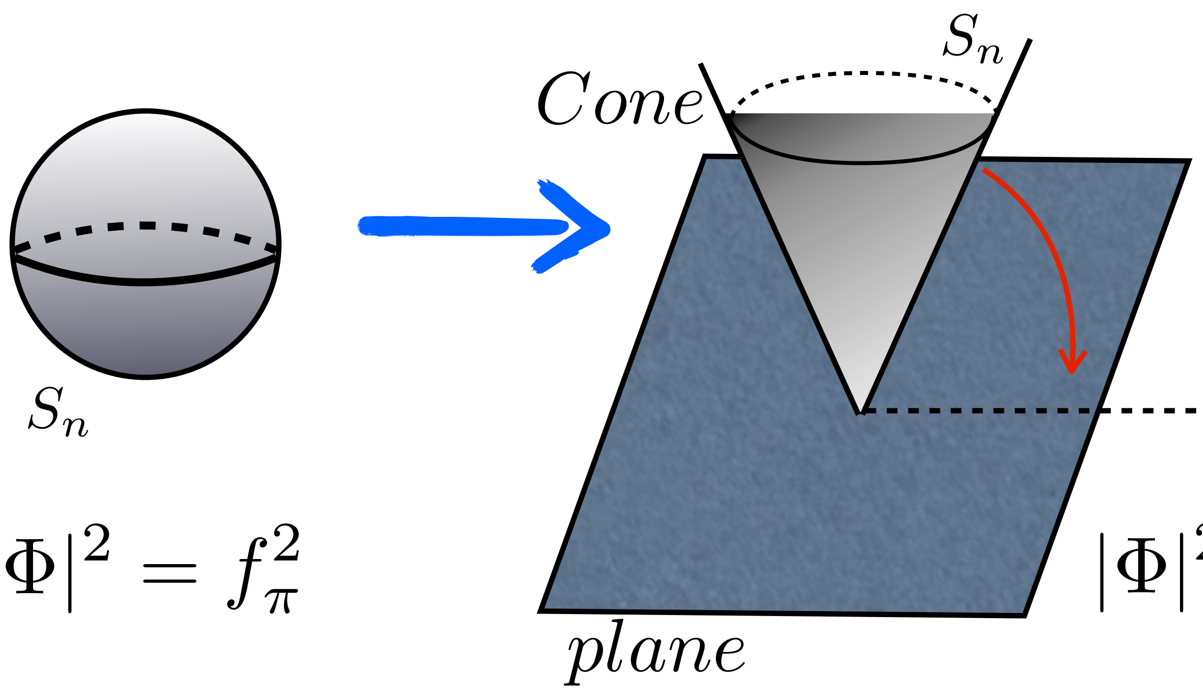
$$|\Phi|^2 = \frac{f_\pi^2}{(f^2 - f_\pi^2)} \chi^2$$

$$d\Omega^2 = \left(\frac{f_\pi}{f}\right)^2 \chi^2 dS_n^2 + d\chi^2 \longrightarrow \chi^2 dS_n^2 + d\chi^2 = d\varphi^2$$

radius goes to 1

radial coordinates of a plane

Cone vs Plane



$$|\Phi|^2 = f_\pi^2$$

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radius goes to 1 *radial coordinates of a plane*

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu\varphi^a\partial_\mu\varphi^a$$ *free theory at O(p^2) in euclidean coordinates*

all amplitudes are trivially vanishing (at this order)

Higher orders?

CCWZ notation: $e^{-i\pi}\partial_\mu e^{i\pi} = i d_\mu^{\hat{a}} T^{\hat{a}} + i E_\mu^a T^a$

Higher orders?

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only pions:

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mixed term:

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a-anomaly Komargodski-Schwimmer

by e.o.m enters in pi-pi scattering

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Higher orders?

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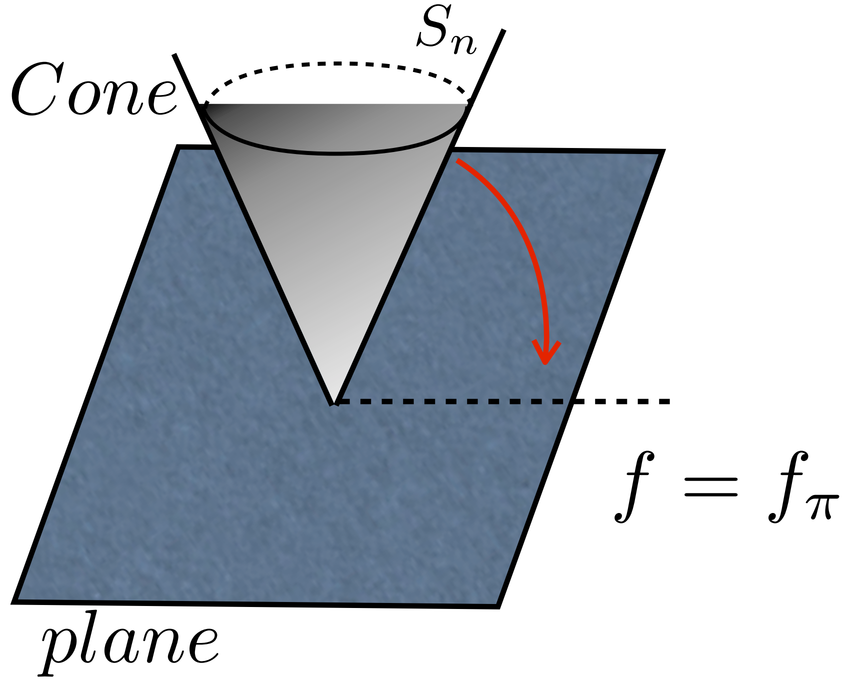
no reasons to expect cancellations

$$\boxed{A(\pi\pi\to\pi\pi)\sim E^4}$$

Accidental Symmetry



plane: invariant ISO(n+1)=SO(n+1)+translations

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial_\mu\varphi^a\partial_\mu\varphi^a \qquad \varphi^a \rightarrow \varphi^a + c^a$$

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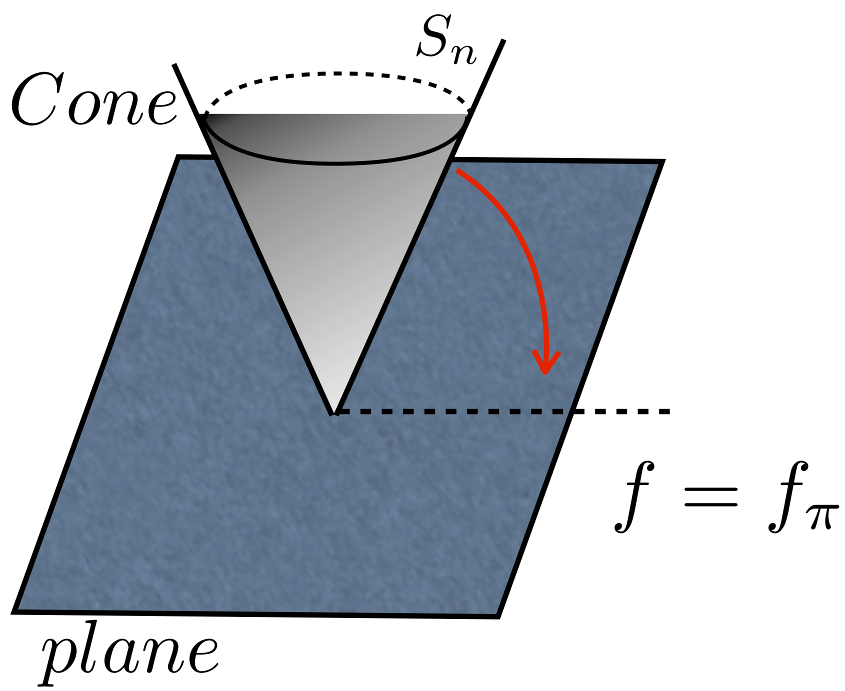

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(true sym. only SO x CFT)

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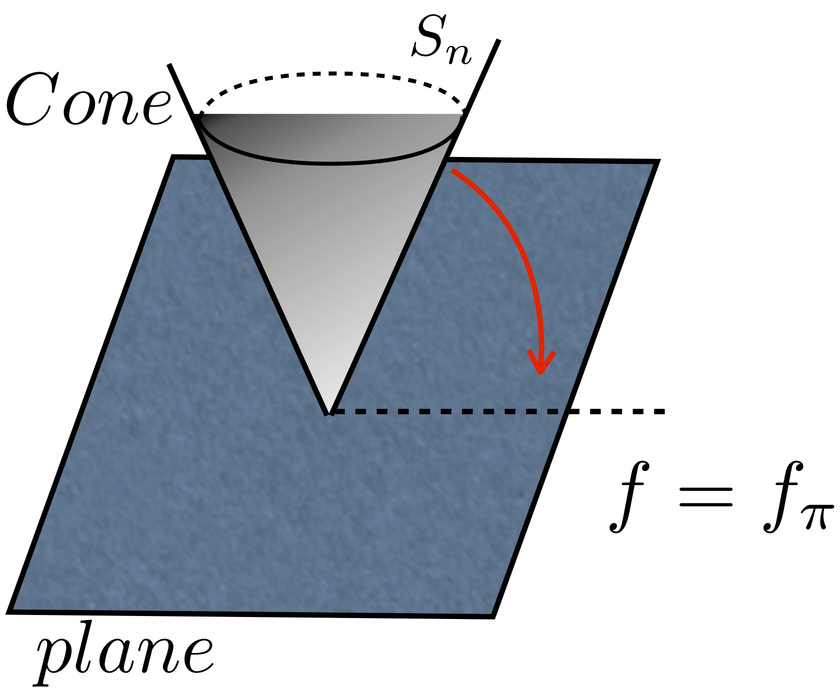

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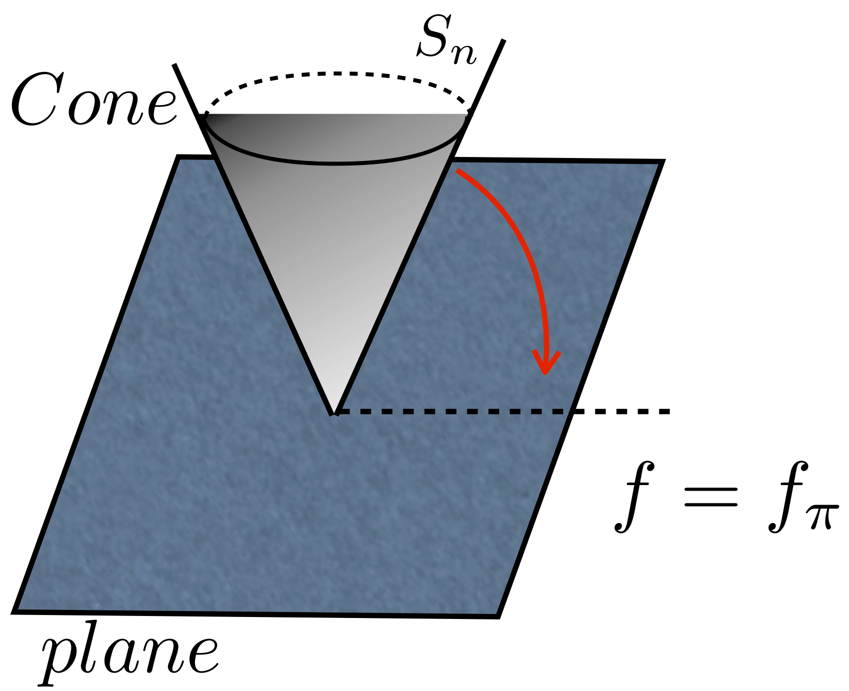

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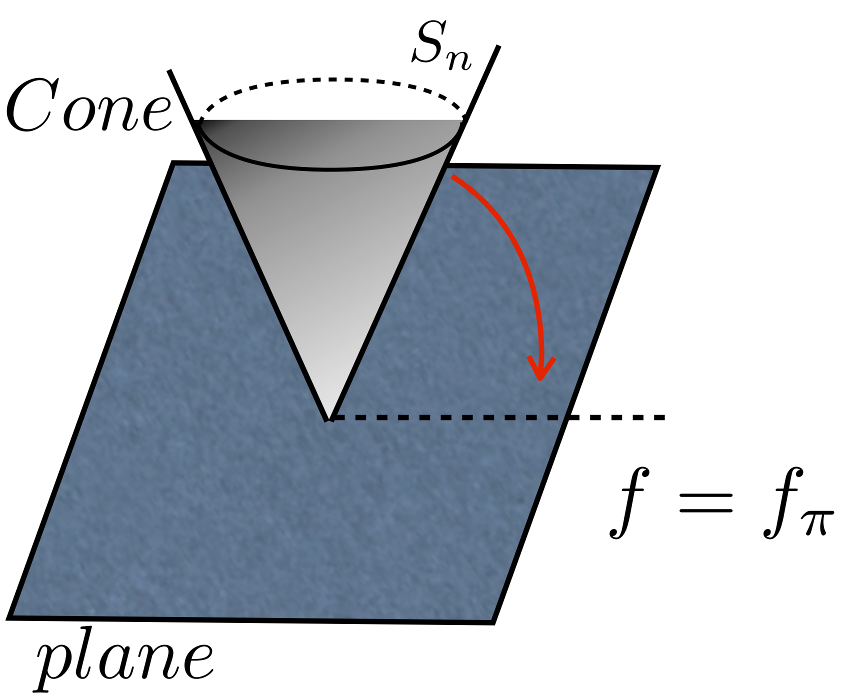
step 1) $\mathcal{L}^{(4)} = a(\partial_\mu\varphi^a)^4 + b(\partial_\mu\varphi^i\partial_\nu\varphi^i)^2 + \ldots$

step 2) make it marginal: divide by~ $\varphi^a\Box\varphi^a$?

Accidental Symmetry



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$$\mathcal{L}^{(4)} \sim \frac{a}{\varphi^a\Box\varphi^a}(\partial_\mu\varphi^a)^4 + \frac{b}{(\ldots)}(\partial_\mu\varphi^i\partial_\nu\varphi^i)^2 + \ldots$$

non-locality forced by translations+dilations!

Accident vs Symmetry

Accident vs Symmetry

Accident

$$SO(n+1) \times CFT$$

$$\downarrow$$

$$SO(n) \times Poincare'$$

$$\mathcal{A}(\pi\pi \to \pi\pi) \sim E^4$$

resonances at $\Lambda = 4\pi f$

strongly coupled

ACCIDENT VS SYMMETRY

barring non-locality (=no extra massless fields)

Accident

$$SO(n+1) \times CFT$$

$$\downarrow$$

$$SO(n) \times Poincare'$$

$$\mathcal{A}(\pi\pi \to \pi\pi) \sim E^4$$

resonances at $\Lambda = 4\pi f$

strongly coupled

Symmetry

$$ISO(n+1) \times CFT$$

$$\downarrow$$

$$SO(n) \times Poincare'$$

$$\mathcal{A}(\pi\pi \to \pi\pi) \sim E^0$$

cut-off can be at $\Lambda = \infty$

weakly coupled

Hierarchy problem?

Symmetry

$$ISO(n+1) \times CFT$$

$$\Downarrow$$

$$SO(n) \times Poincare'$$

weakly coupled

ISO breakings

new scale breaking CFT

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi^a\partial_\mu\varphi^a + \epsilon_{I\not{S}O} \times M^2_{C\not{F}T}\,\varphi^a\varphi^a + \ldots$$

the relevant operator is small by symmetry

Hierarchy problem?

Symmetry

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but the whole potential is suppressed by translations

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi^a \partial_\mu \varphi^a + \epsilon_{I\not{S}O} \cdot \lambda^2 \cdot \left(\frac{M^2_{C\not{F}T}}{4\lambda^2} - \varphi^a \varphi^a \right)^2 + \ldots$$

Hierarchy problem?

Symmetry

$$ISO(n+1) \times CFT$$

$$\Big\downarrow$$

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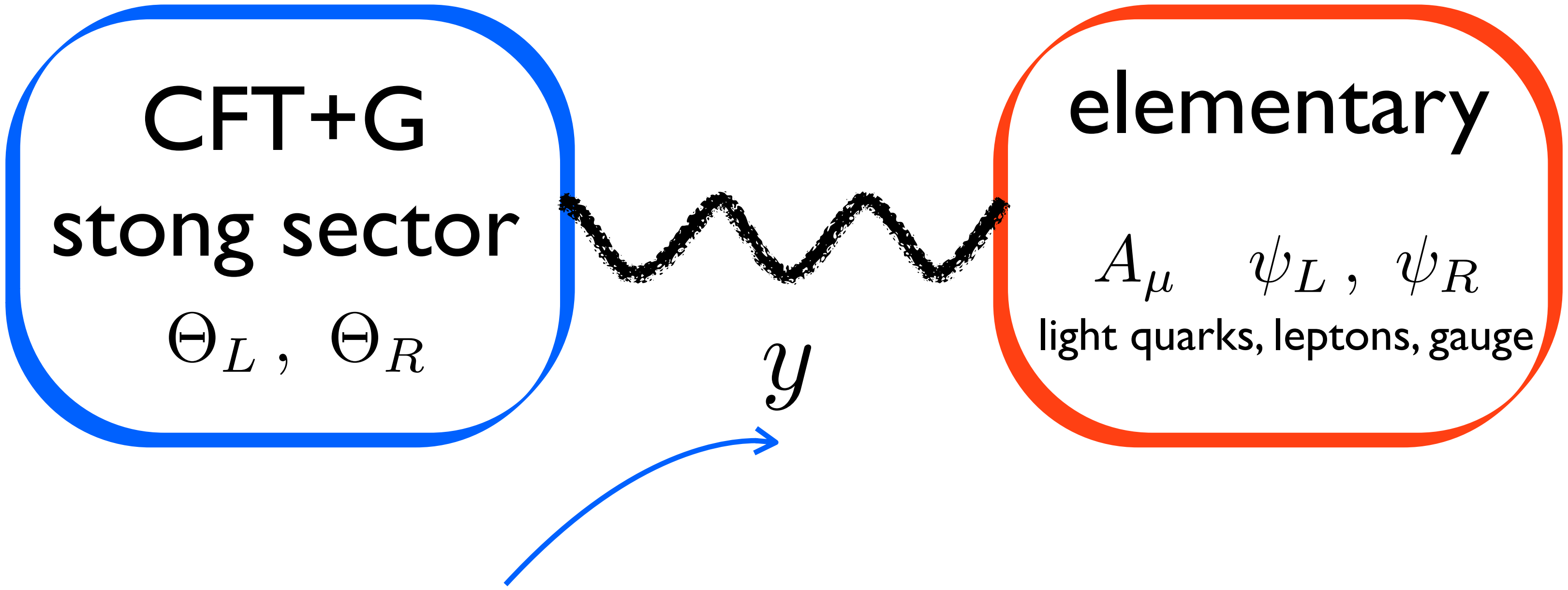
$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi^a \partial_\mu \varphi^a + \epsilon_{I\not{S}O} \cdot \lambda^2 \cdot \left(\frac{M^2_{C\not{F}T}}{4\lambda^2} - \varphi^a \varphi^a\right)^2 + \dots$$

$$\boxed{f^2 = \frac{M^2_{C\not{F}T}}{4\lambda^2} \qquad m^2_\sigma \propto \epsilon f^2 \qquad \Delta = \frac{f^2}{v^2} \gg 1}$$

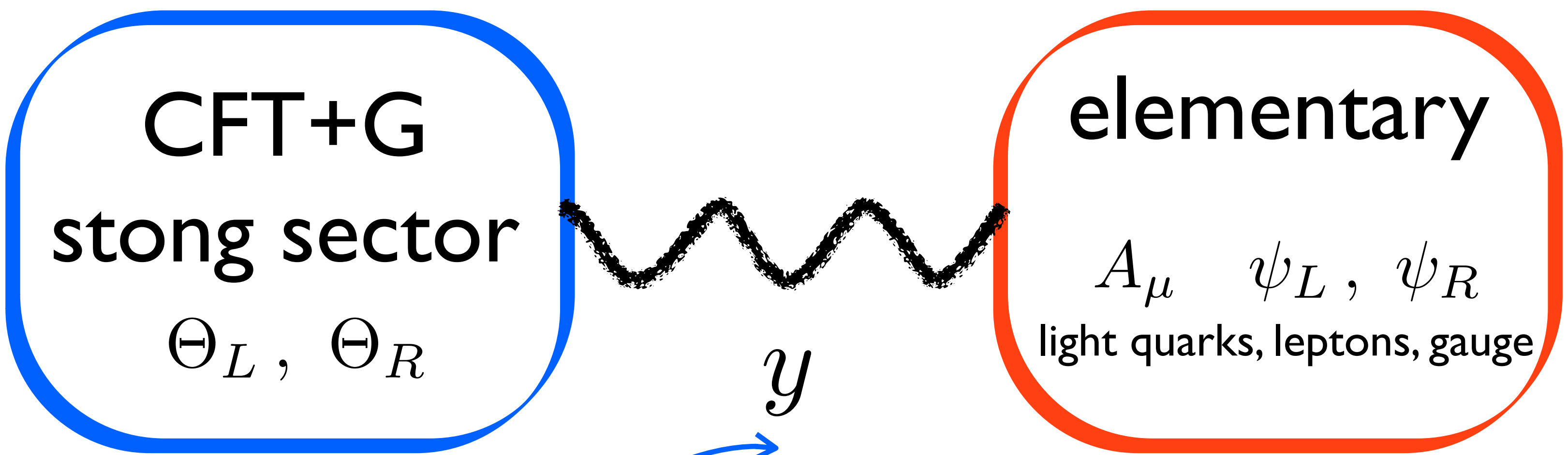
generically big tuning!

Dilaton & Higgs potentials



Partial Compositeness: explicitly break CFT & G

Dilaton & Higgs potentials

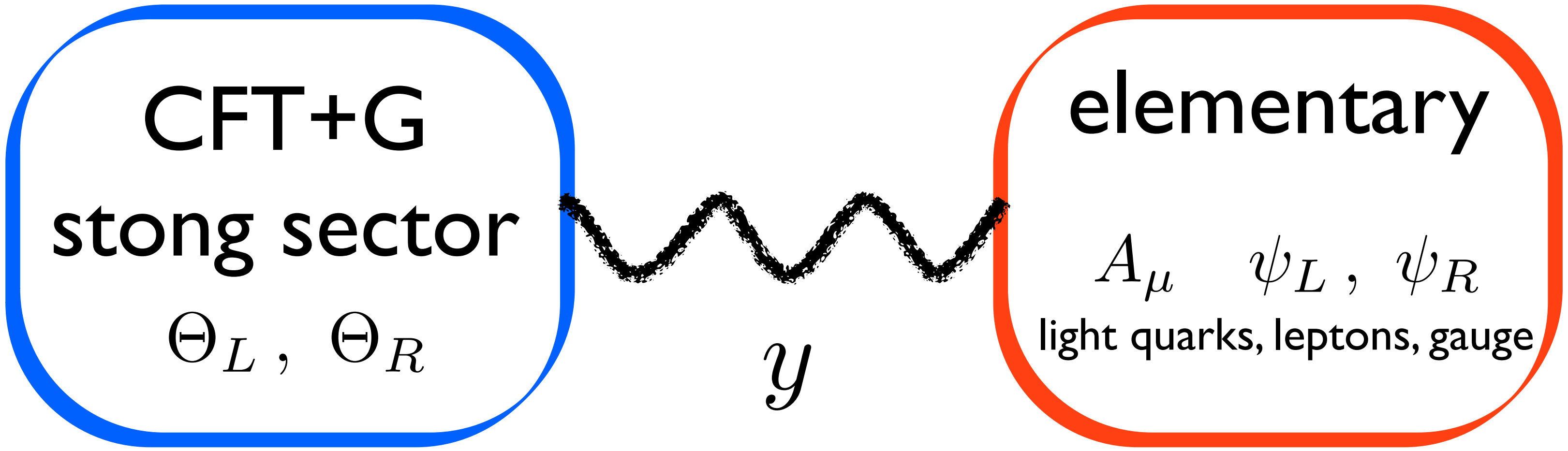


Partial Compositeness: explicitly
break CFT & G

spurions carry both G-indexes and scale dimension

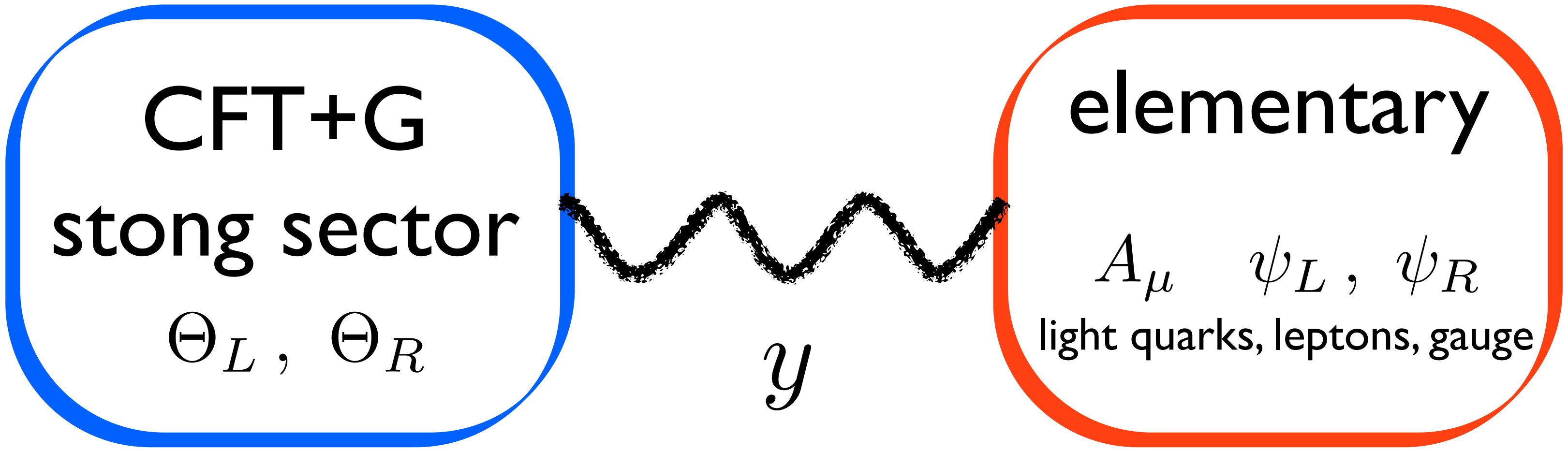
$$\mathcal{L}_{mix} = y_L\psi_L\Theta_R + \underbrace{y_R}_{[y_{R,L}] = -\gamma_{L,R}}\underbrace{\psi_R}_{3/2}\underbrace{\Theta_L}_{5/2+\gamma_R}$$

Dilaton & Higgs potentials



Integrate-out CFT (all orders)

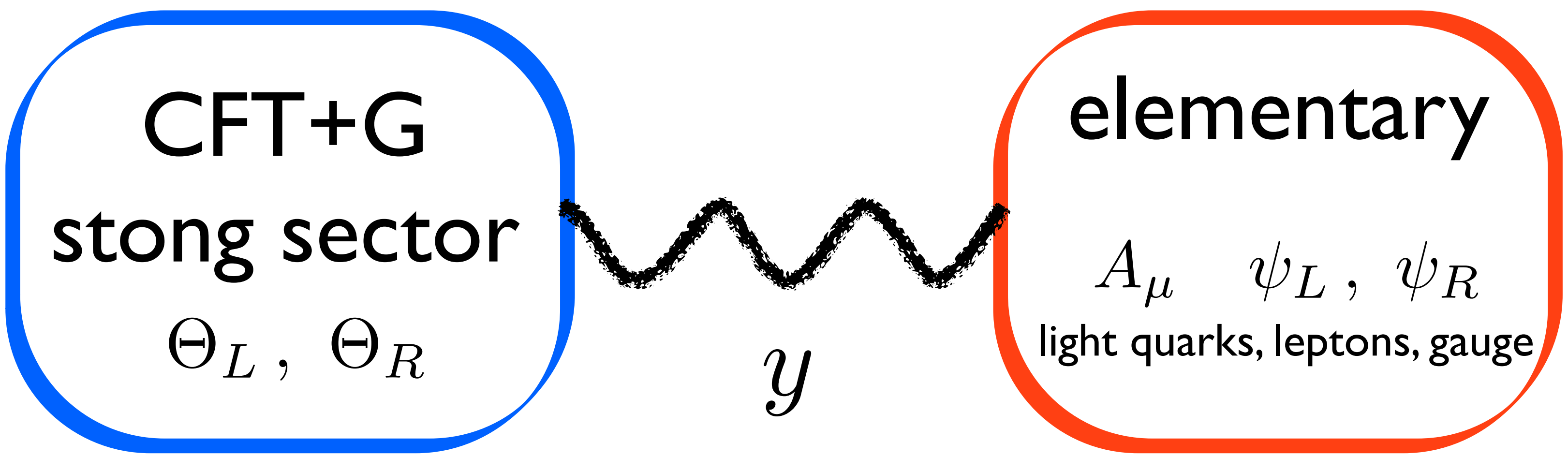
Dilaton & Higgs potentials



Integrate-out CFT (all orders) $\mathcal{L}_{eff}^{gauge} = \frac{1}{2}\left[\Pi_0(p)\mathrm{Tr}[A_\mu A_\mu] + \Pi_i(p)\Phi^T A_\mu A_\mu \Phi\right] P_{\mu\nu}^{\perp}$

form factors

Dilaton & Higgs potentials



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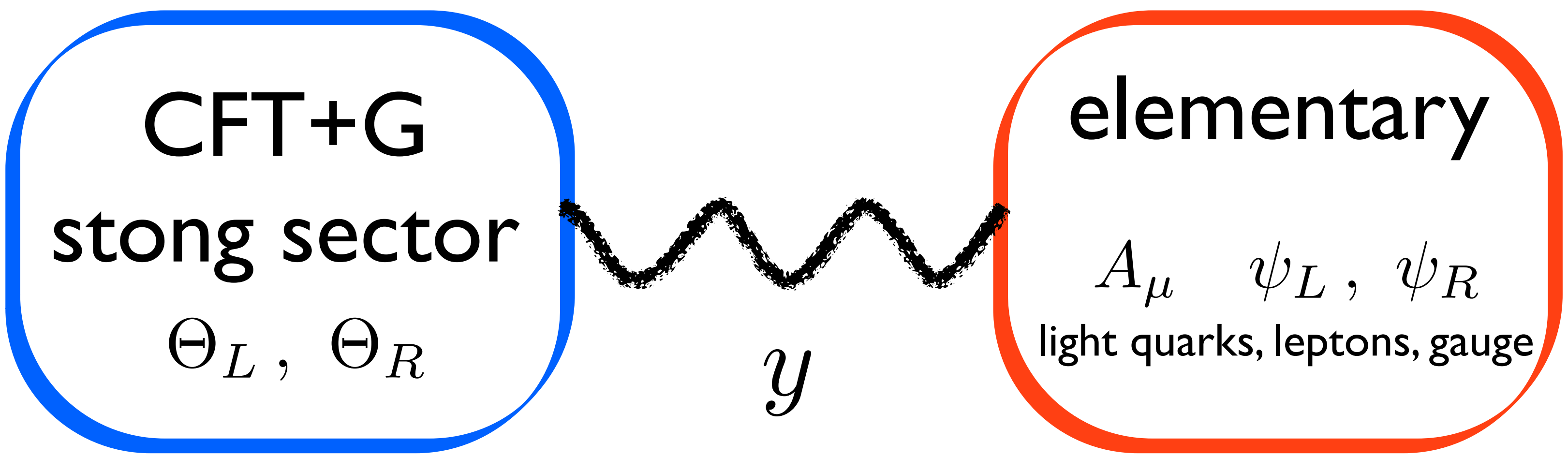
form factors

1-loop of elem. fields: Coleman-Weinberg! $V(\pi,\chi) = \sum_i \int \frac{d^4p}{(2\pi)^4} \log \Pi_i(p^2,\Phi)$

Dilaton & Higgs potentials



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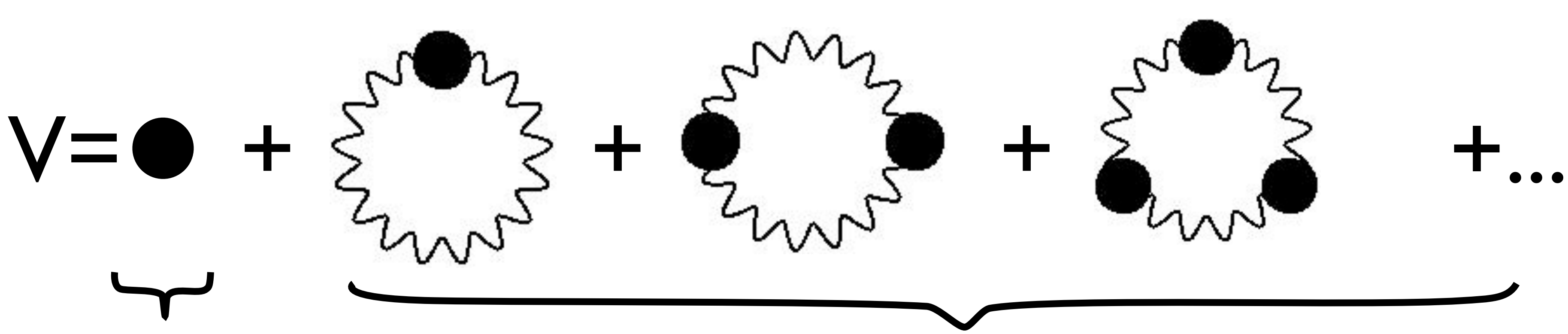
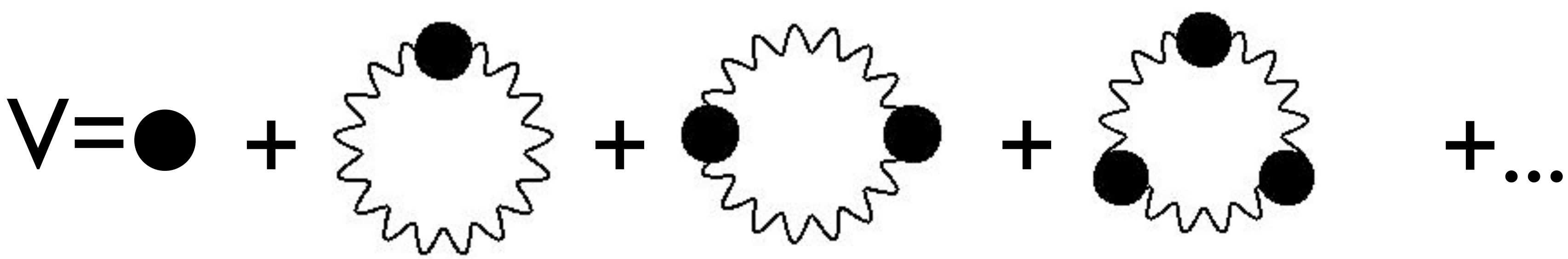


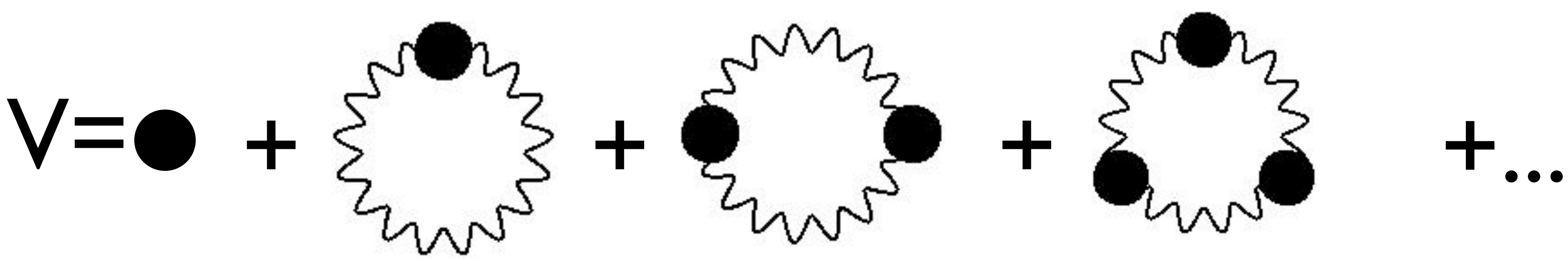
symmetric

symmetry breaking

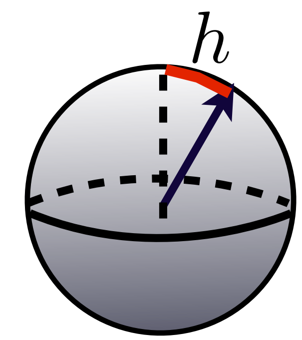
Dilaton & Higgs potentials



Dilaton & Higgs potentials

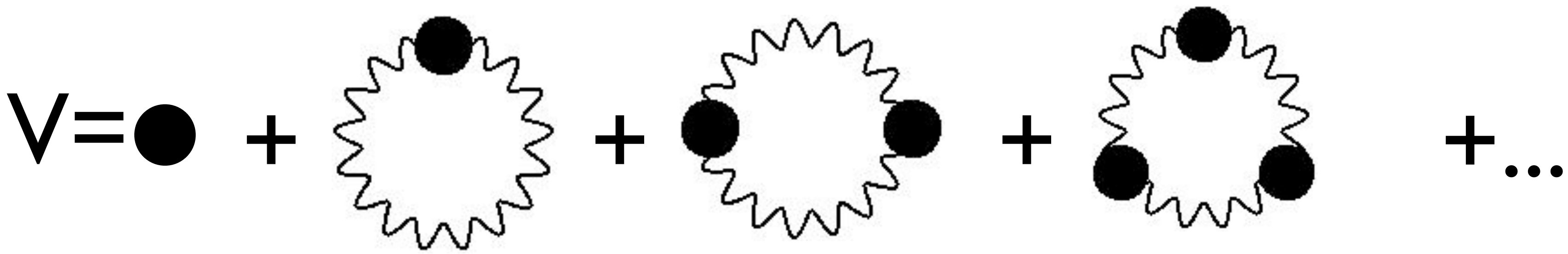


Potential on the sphere



$$v = f_\pi \sin h$$

Dilaton & Higgs potentials



$$V = \kappa + y^2 \left(\Lambda_1 + A \sin^2 h + B \sin^4 h\right)$$

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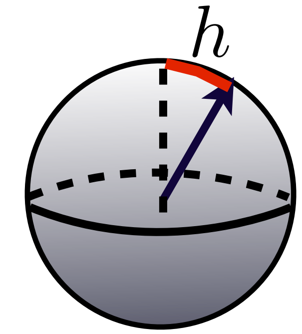

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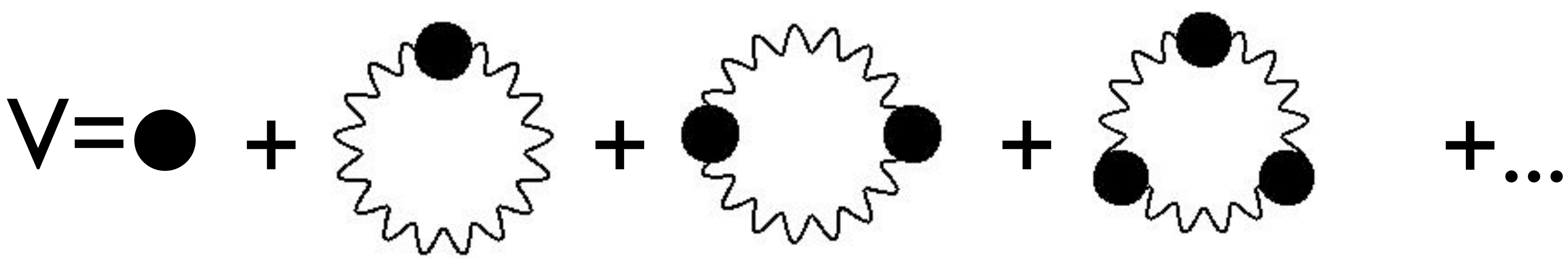

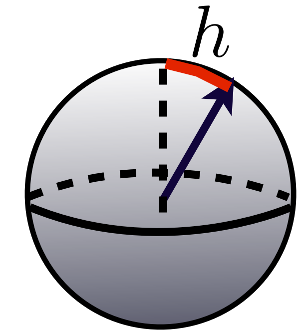

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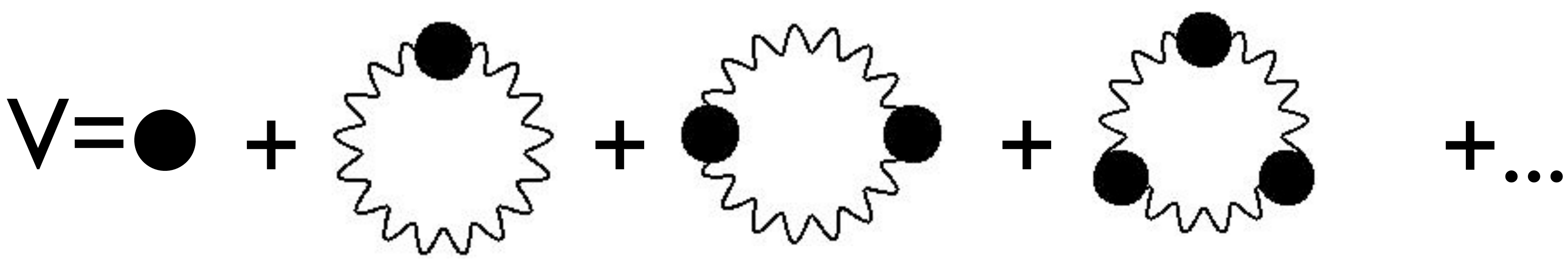
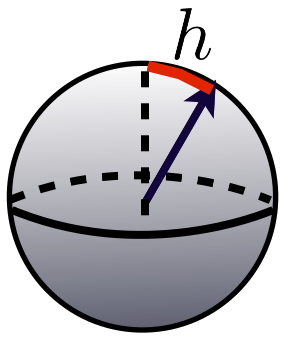
dress with the dilaton

$$V = \left(\frac{\chi}{f}\right)^4 \left[\kappa + y^2 \left(\frac{\chi}{f}\right)^{2\gamma} \left(\Lambda_1 + A\sin^2 h + B\sin^4 h\right)\right] = \chi^4 F(y(\chi), \sin h)$$

Dilaton & Higgs potentials



Potential on the sphere



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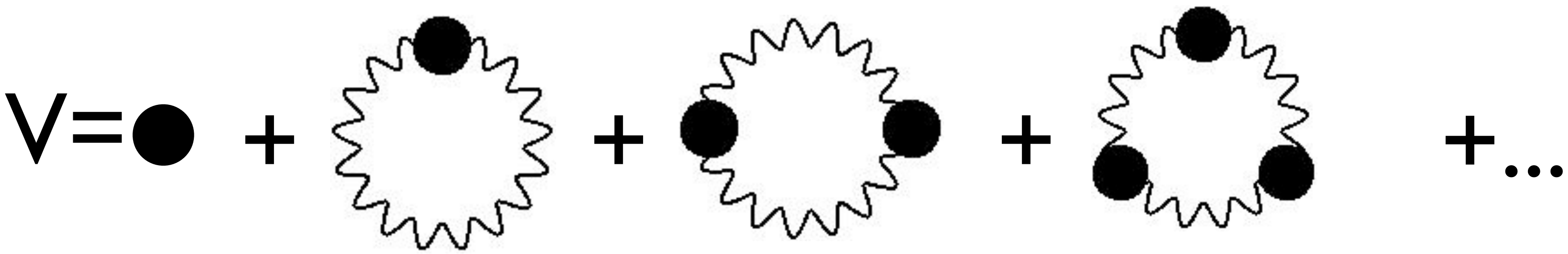

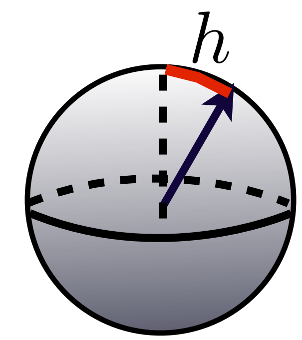

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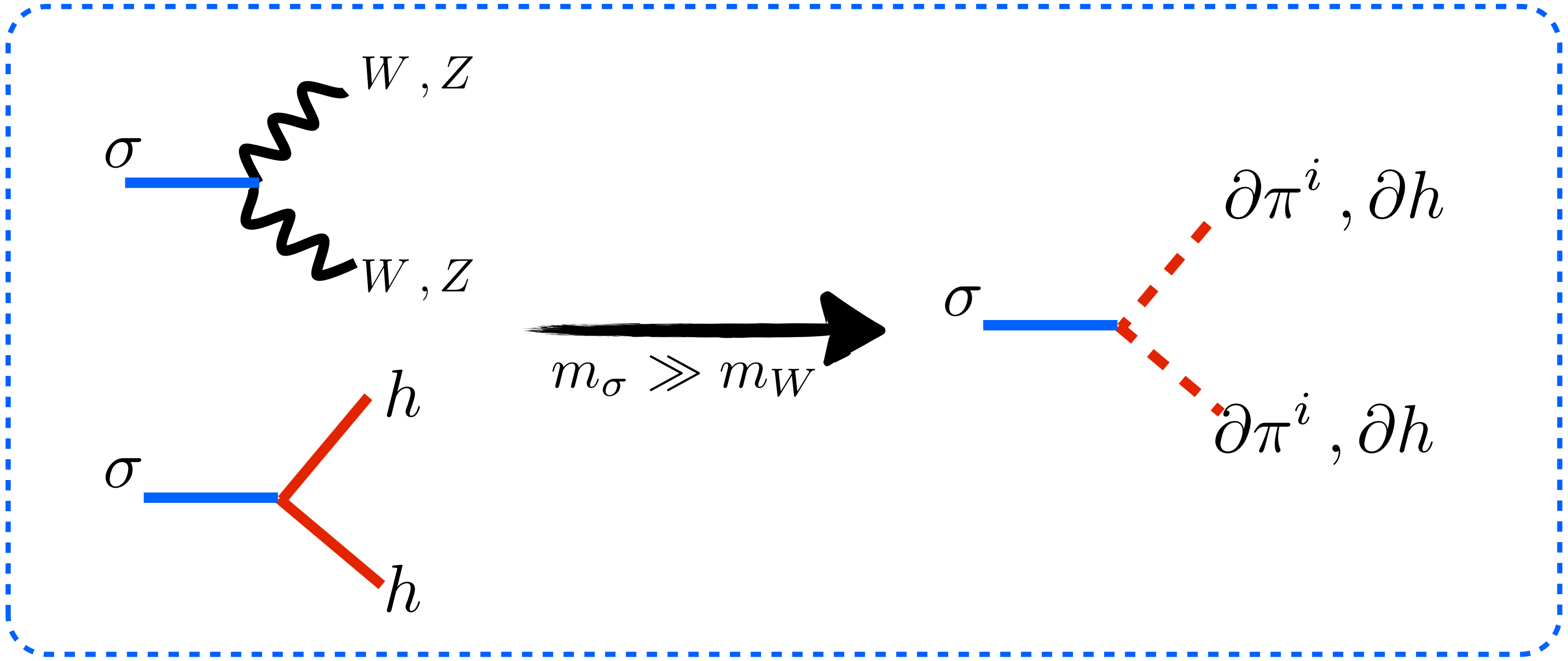
5 parameters: trade for $m_\sigma \quad m_h \quad v/f_\pi \quad f \quad m_t$

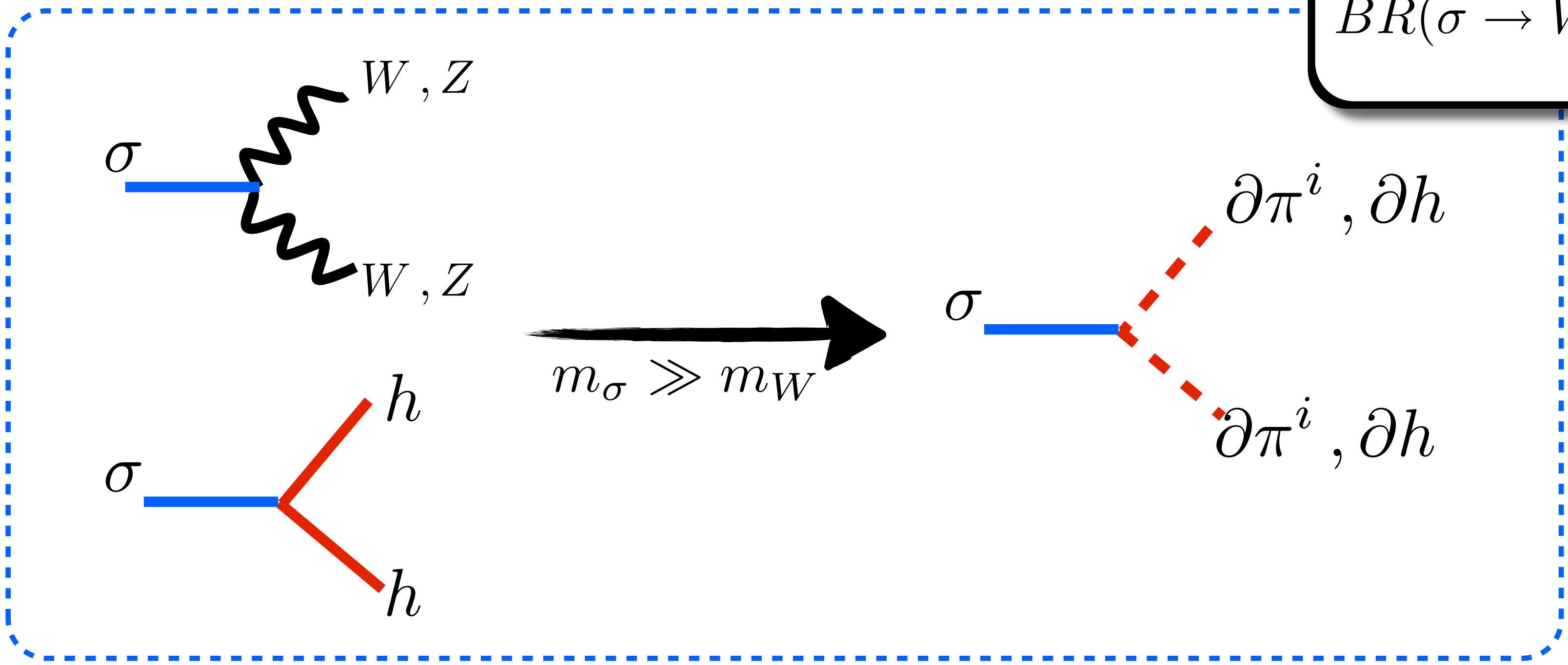
Predictions (e.g. amplitudes) all in terms of physical quantities

Dilaton decays



Dilaton decays

$$BR(\sigma \to WW) \simeq 2BR(\sigma \to ZZ) \simeq 2BR(\sigma \to hh)$$



SO(4) restored

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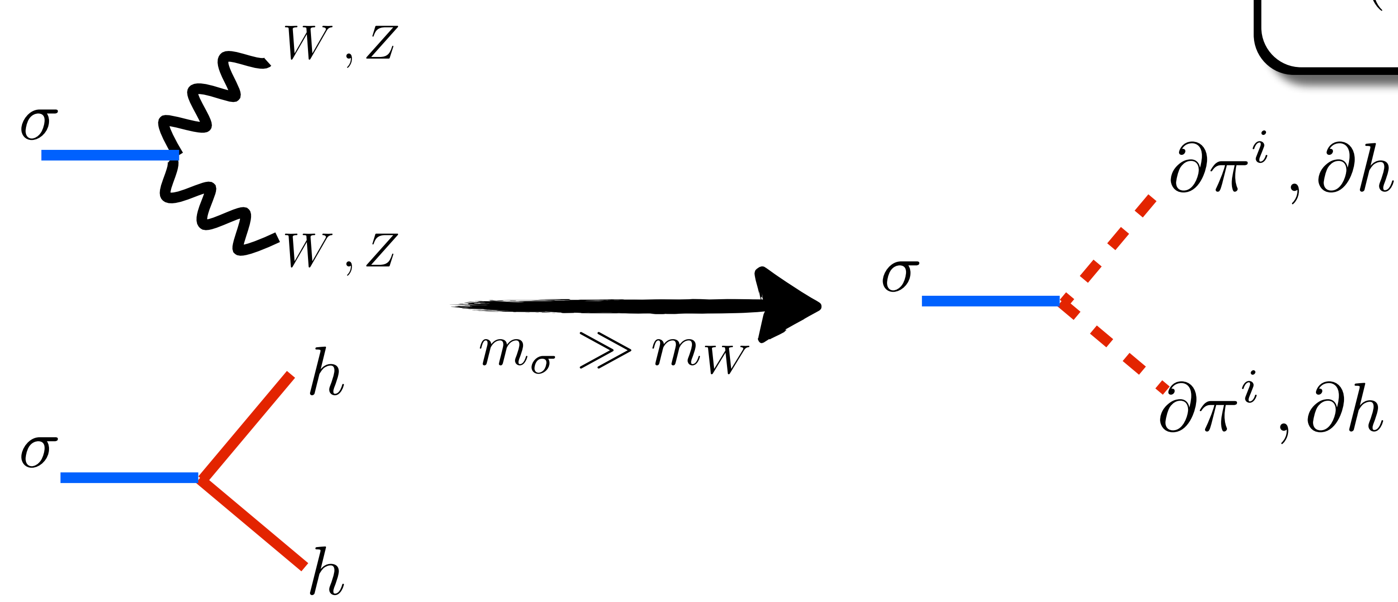

dominate because of longitudinal boost

$$BR(\sigma \to \pi\pi) \gg BR(\sigma \to XX)$$

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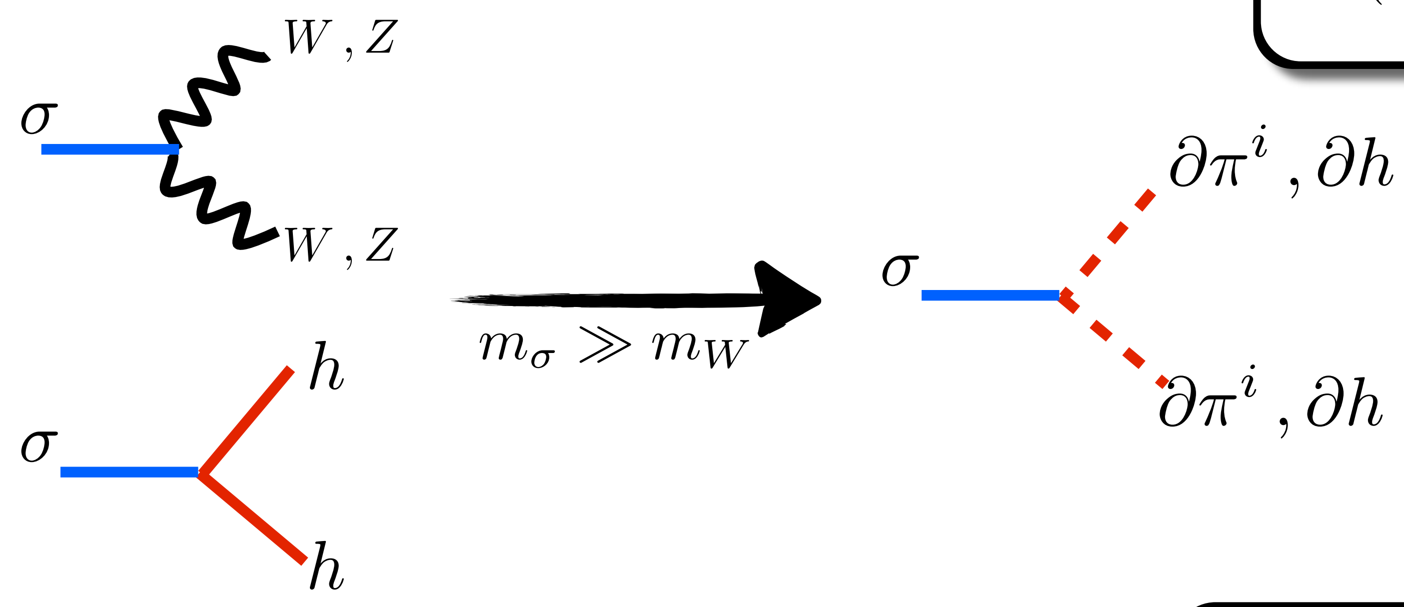

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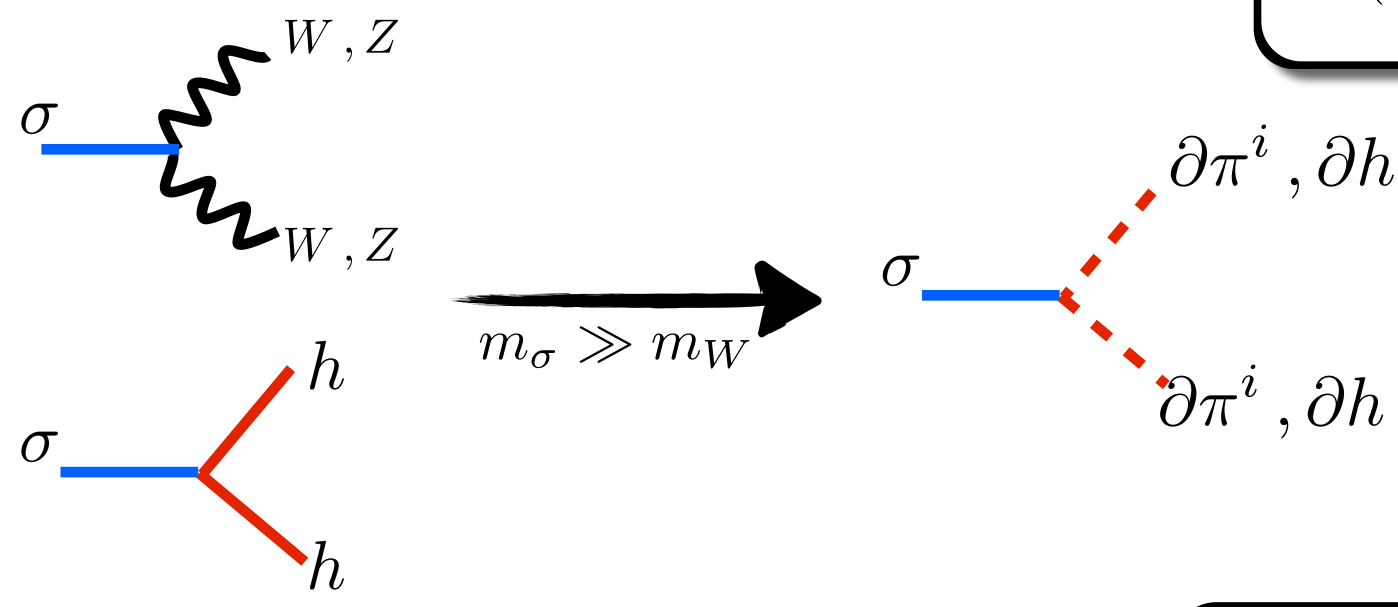
$$\mathcal{L}_{canon.} = \frac{\chi^2}{2f^2}(\partial_\mu \pi)^2 + \frac{\chi}{f} m_\Psi \bar{\Psi}\Psi + \dots$$

$$\downarrow$$

$$\mathcal{L} \sim \frac{m_\sigma^2}{f}(\sigma\, \pi^2)$$

Dilaton decays



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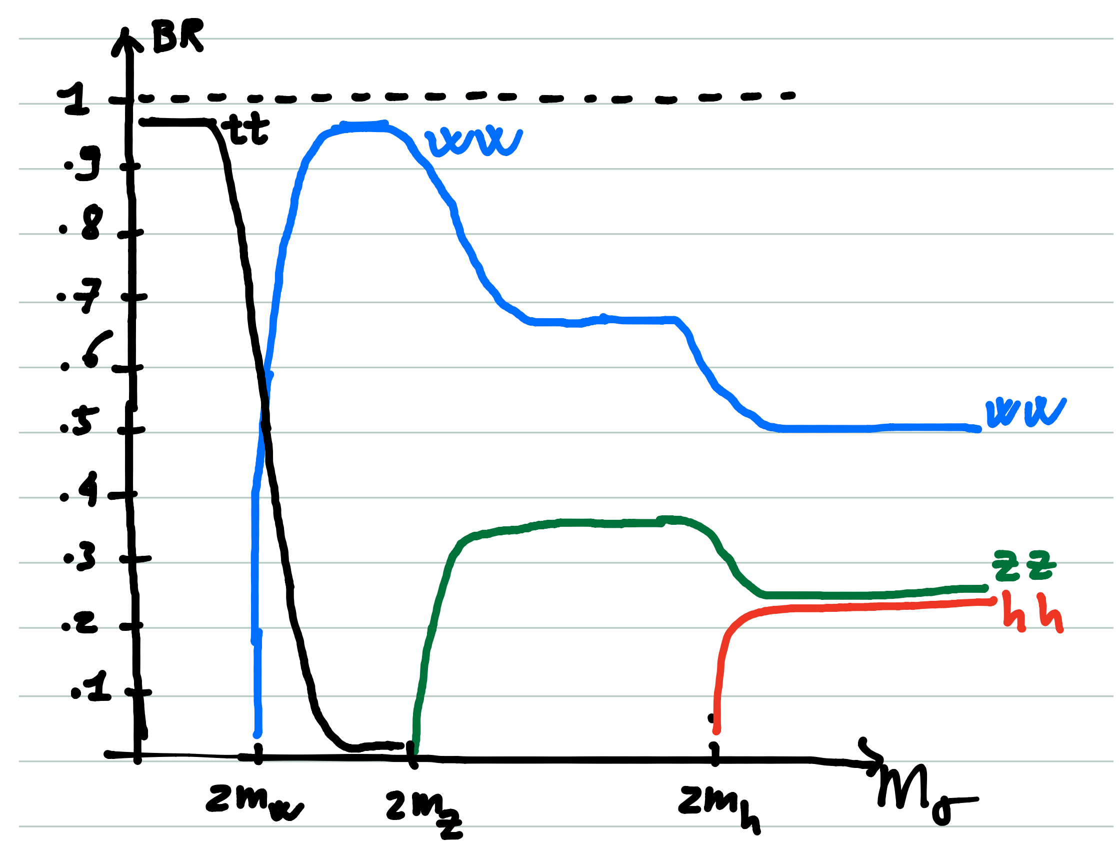

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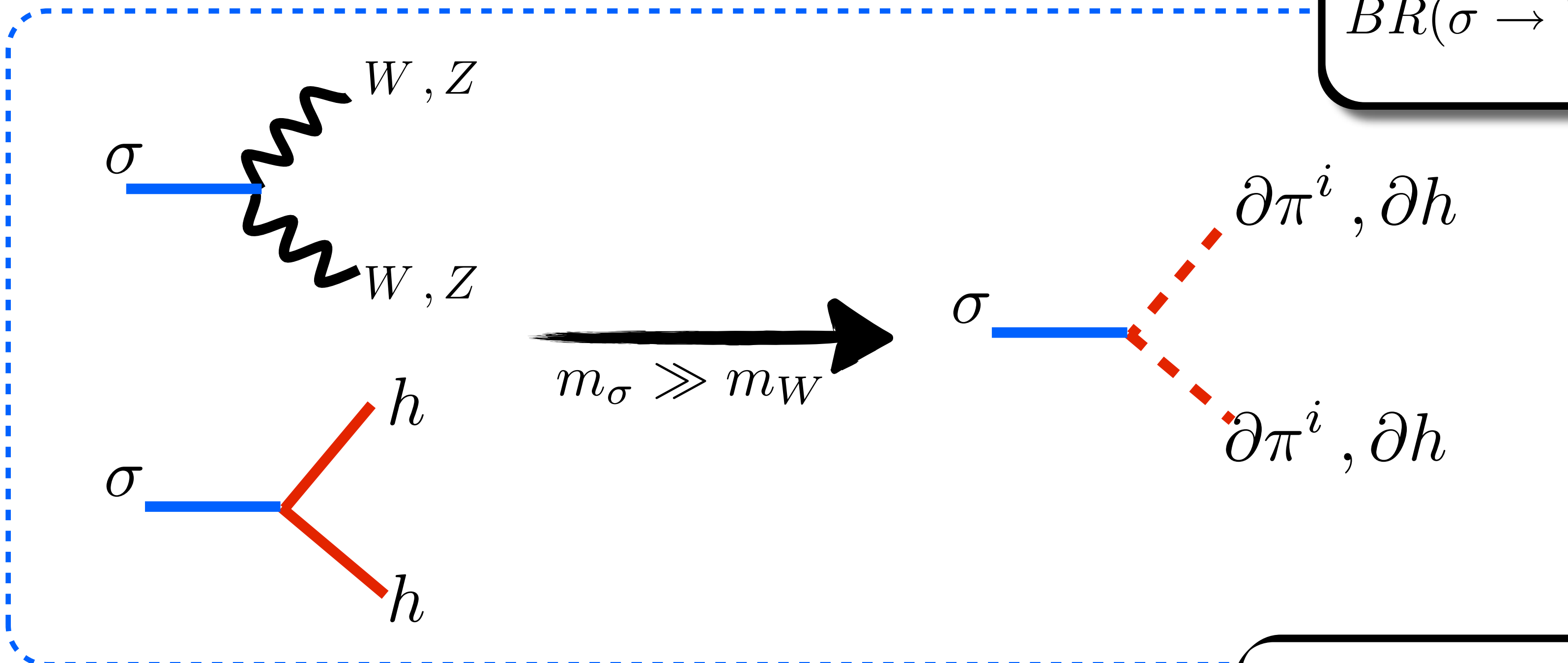

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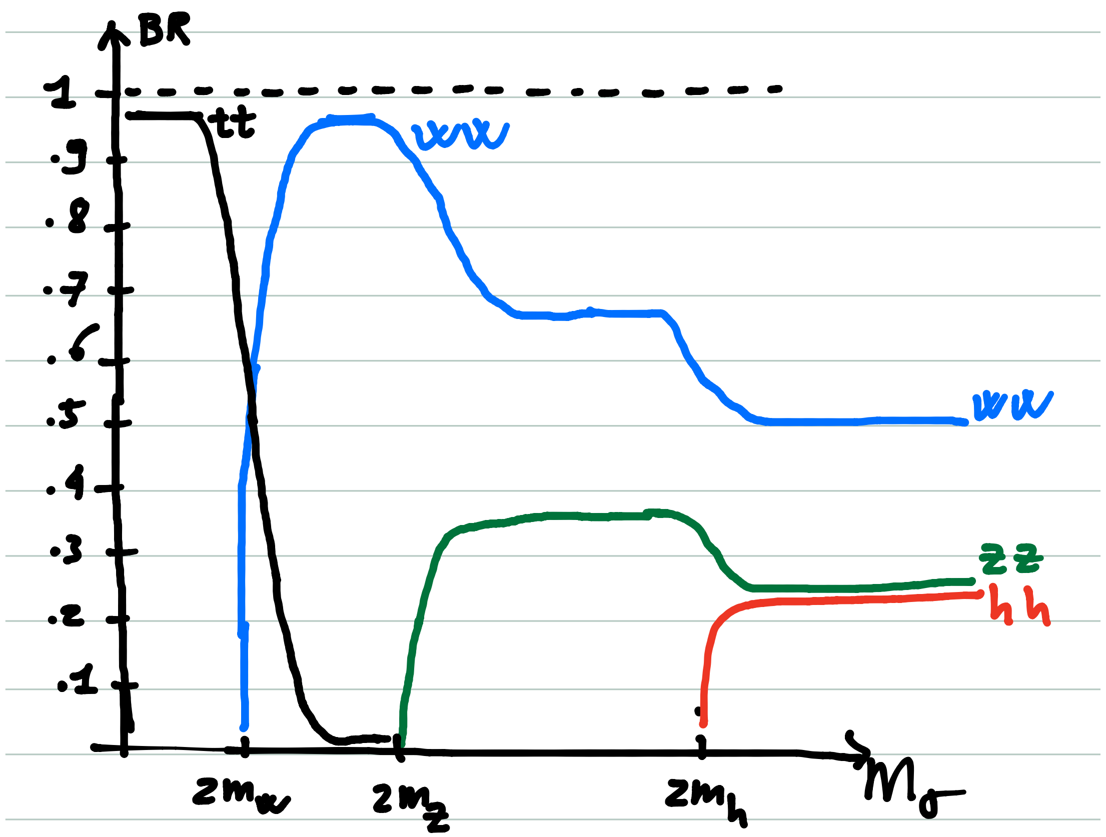
$$\mathcal{L} \sim \frac{m_\sigma^2}{f}(\sigma\,\pi^2)$$

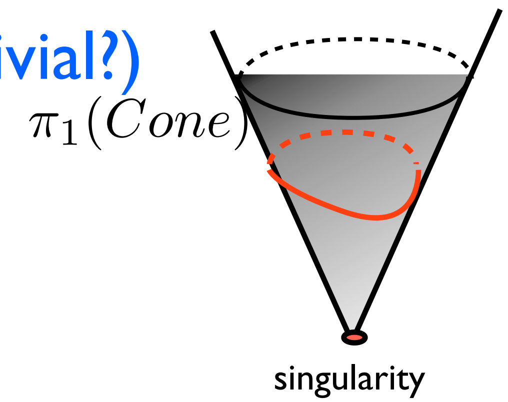
(and no kinetic mixing)

$$|\nabla_\mu H|^2 = |\partial_\mu - \Delta_H \partial_\mu \sigma) H|^2$$



CONCLUSIONS

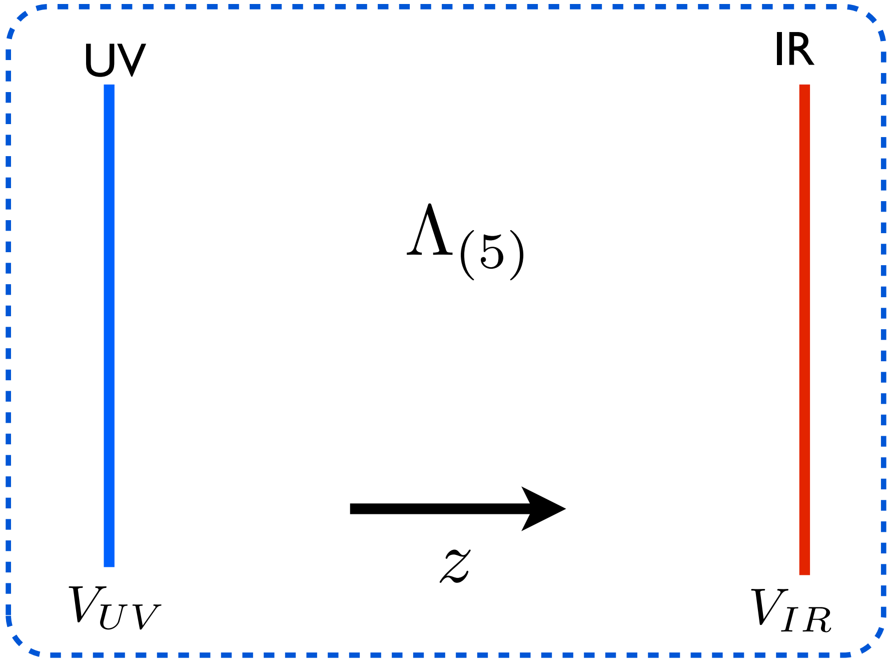
- A new scalar has been discovered: it can well be a pNGB as in CHM
- The EFT for a Composite Higgs+Dilaton is quite interesting and predictive
  - ★ Funny geometrical structure (btw, is the cone homotopy trivial?)
  - ★ f=fpi by symmetry ISO(n), but weakly coupled
  - ★ Clear Dilaton BRs
  - ★ Curious WW-scattering (can we see E^4 behavior? strong vs weak dynamics, dynamics vs symmetry)
  - ★ Higgs and Dilaton potential are related
  - ★ Can we distinguish it from another heavy H or pNGB? (not discussed)
  - ★ When f<fpi the curvature is negative and singular: does it imply ghost? (not discussed)



THANK YOU!

backup slides

The RS story



$$ds^2 = \frac{L^2}{z^2}\left(dx^2 - dz^2\right)$$

$x \to \lambda x\,,\; z \to \lambda z$

$z_{IR} \to \lambda z_{IR}$

breaks it spontaneously

$f = z_{IR}^{-1}$ the radion

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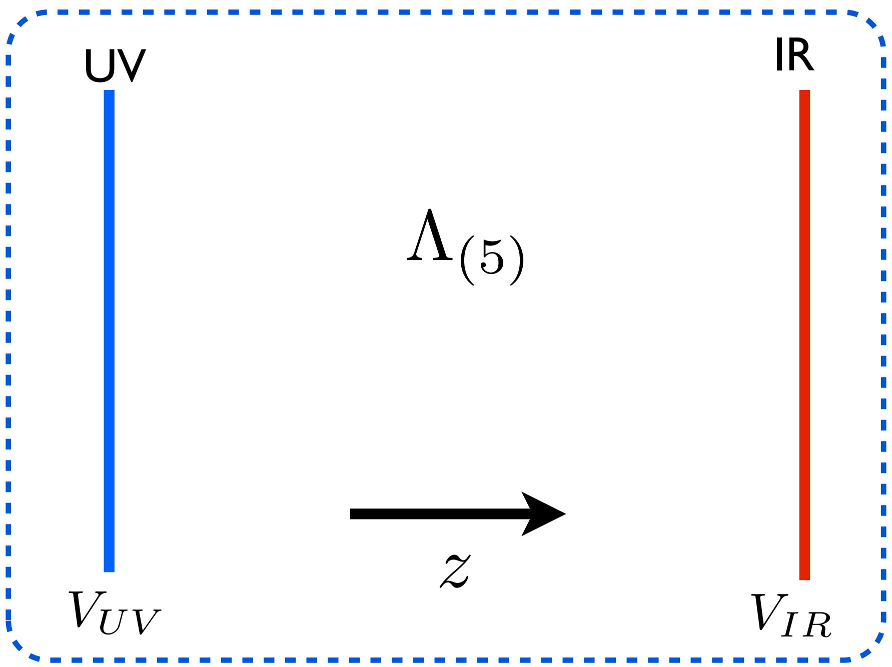

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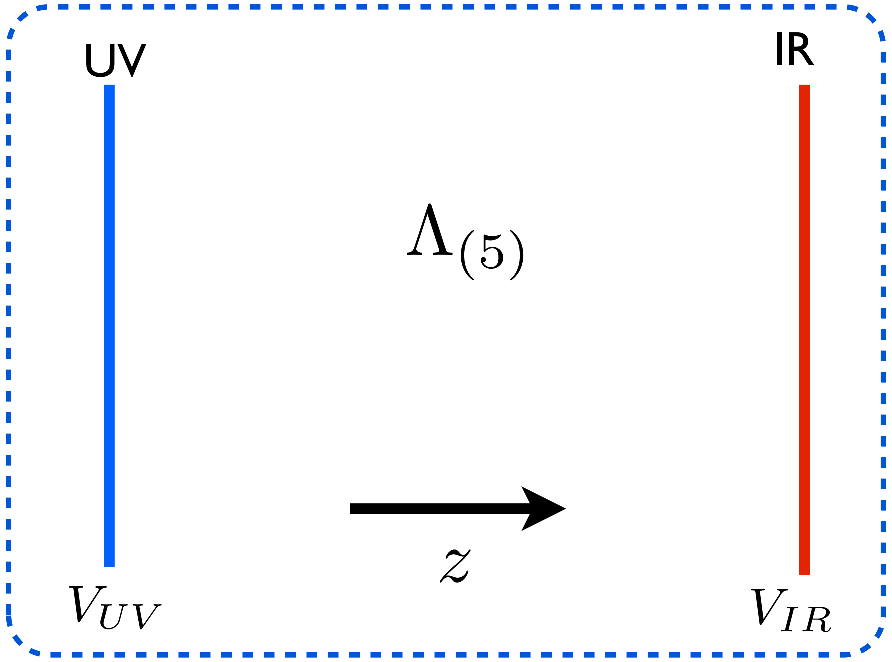

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floating IR-brane

$$V_{eff} = \overbrace{\left(V_{UV} + \Lambda_{(5)} L\right)}^{\Lambda_{(4)} = 0} + \frac{L^4}{z_{IR}^4}\overbrace{\left(-\Lambda_{(5)} L + V_{IR}\right)}^{a = 0} = \Lambda_{(4)} + a\chi^4$$

vanishing 4d CC
FT-1

vanishing quartic
FT-2

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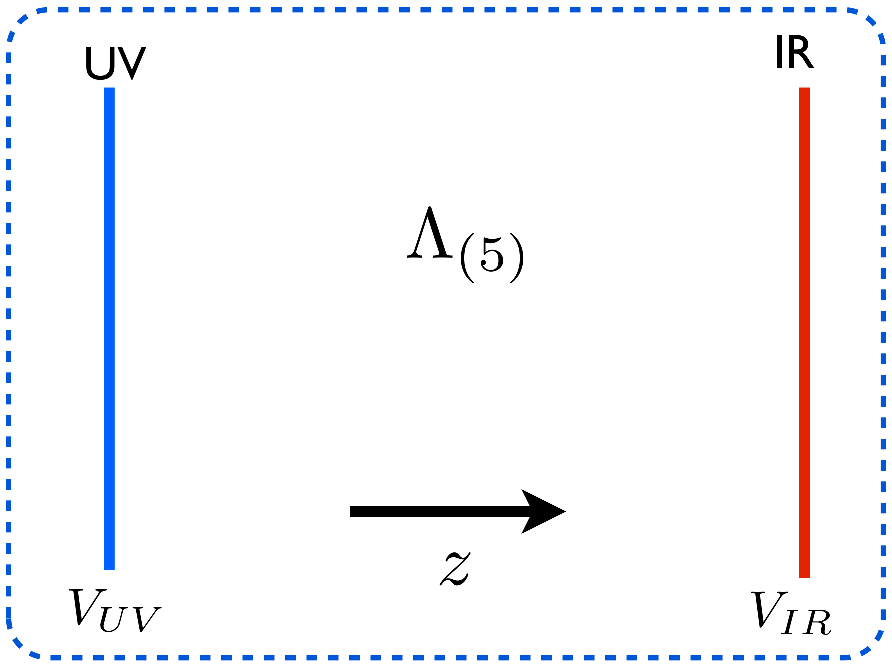

$$L_{eff} = -\Lambda_{(5)} L^5 (\partial\chi)^2/2 - \chi^4 \left(-\Lambda_{(5)} L^5 + V_{IR} L^4\right)$$

NDA: $$\begin{cases} \delta a_{bulk} = -\Lambda_{(5)} L^5 \sim \dfrac{12^{5/2}}{24\pi^3} = \mathcal{O}(1) \\ \delta a_{IR=} V_{IR} L^4 = V_{IR} \left(\dfrac{L}{z_{IR}}\right)^4 z_{IR}^4 \sim 16\pi^2 \end{cases}$$

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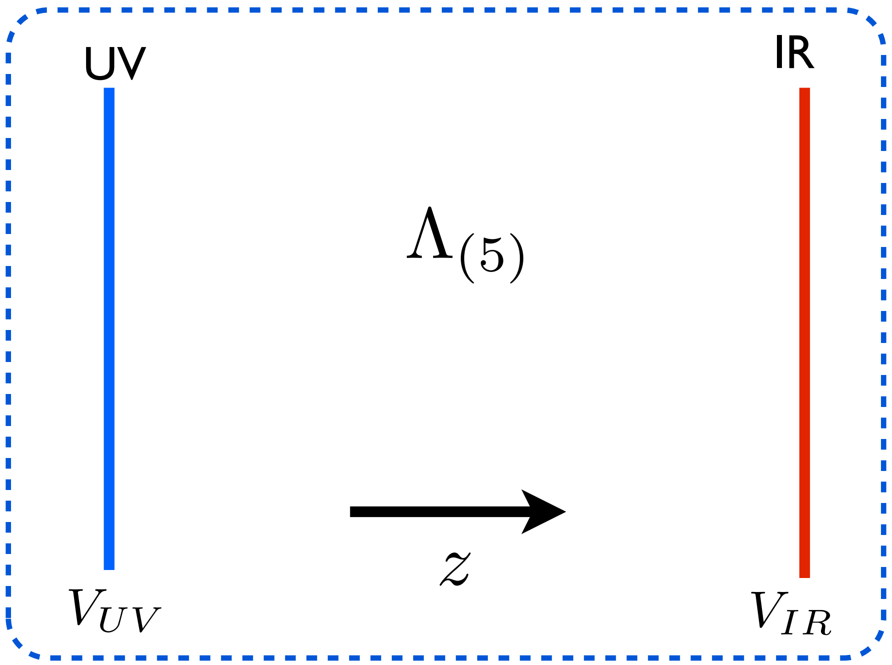

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Stabilization: Goldberger and Wise

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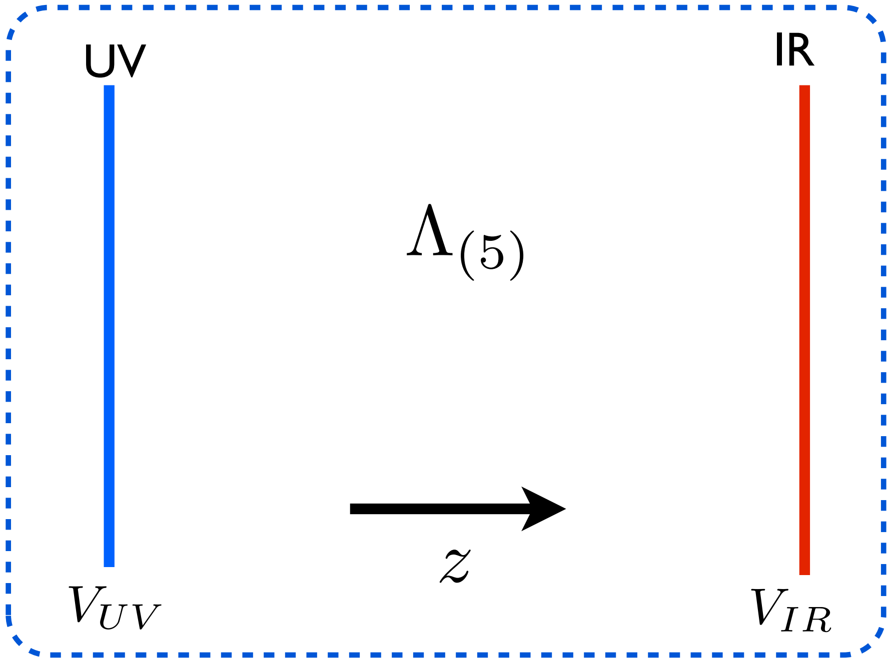

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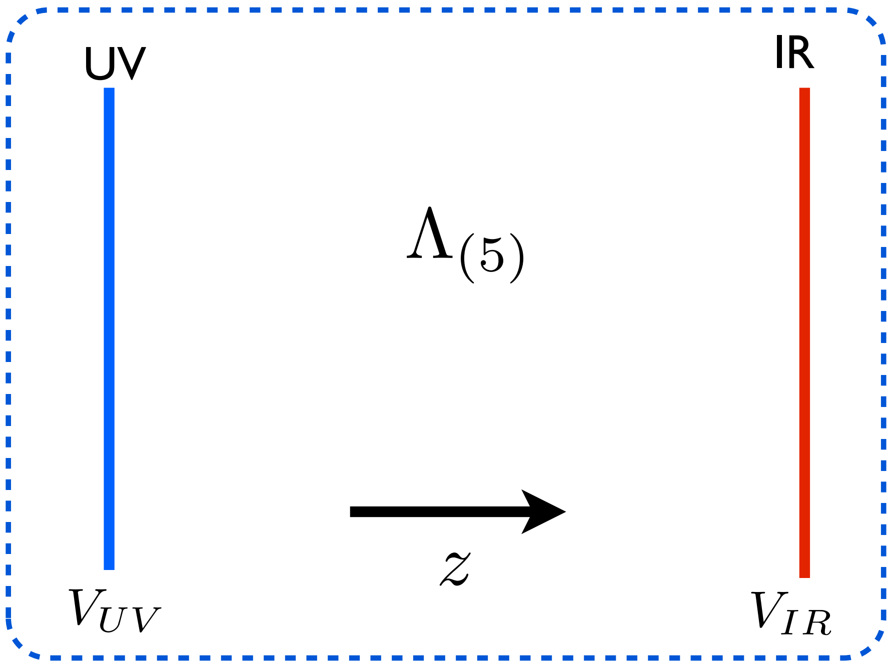

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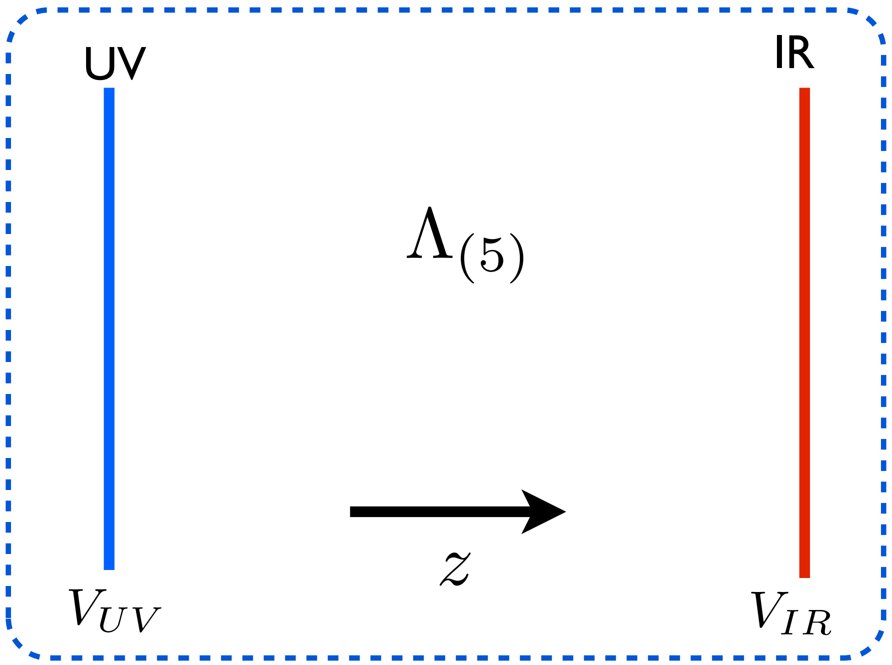

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from large K.T.

not a good candidate

The SM-Higgs is a fine-tuned dilaton

Higgs potential

$$V = \lambda \left(H^2 - \frac{v^2}{2} \right)^2$$

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only the *diagonal* symmetry survives

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non-linearly like a Goldstone boson

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sym.is restored!

SUSY Example: 3-2 model

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	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_R$
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runaway direction

push the fields to large vevs

$$V_{eff} \approx \frac{\Lambda_3^{14}}{f^{10}} + \lambda\frac{\Lambda_3^7}{f^3} + \lambda^2 f^4 \qquad f \approx \frac{\Lambda_3}{\lambda^{1/7}} \gg \Lambda_3$$

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$g_i(f) \ll 1$

no large breaking of conformality

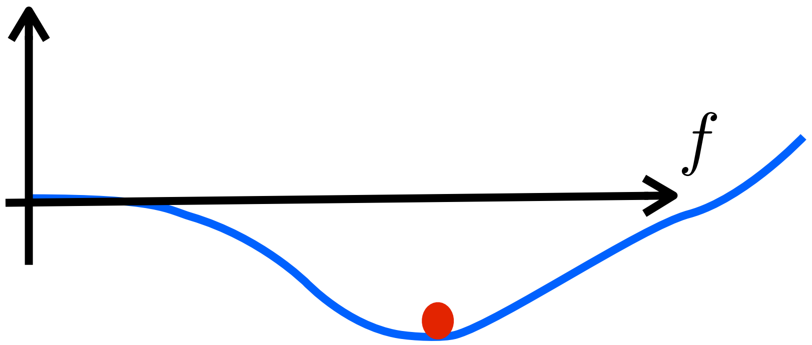
$V \approx \lambda^{10/7}\Lambda_3^4$ small quartic

$m_{dil} \approx \lambda f \approx \lambda^{6/7}\Lambda_3$ light dilaton

DILATON POTENTIAL

$$\mathcal{L}_{CFT} + \lambda \mathcal{O}$$

$$\boxed{V = \chi^4 F(\lambda(\chi))}$$



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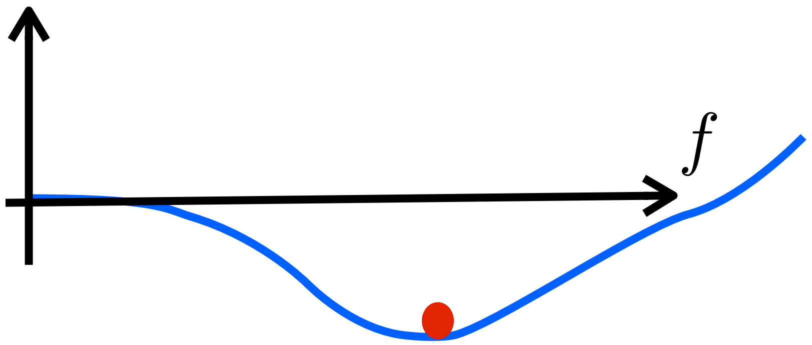

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$$V' = f^3[4F(\lambda(f)) + \beta F'(\lambda(f))] = 0$$

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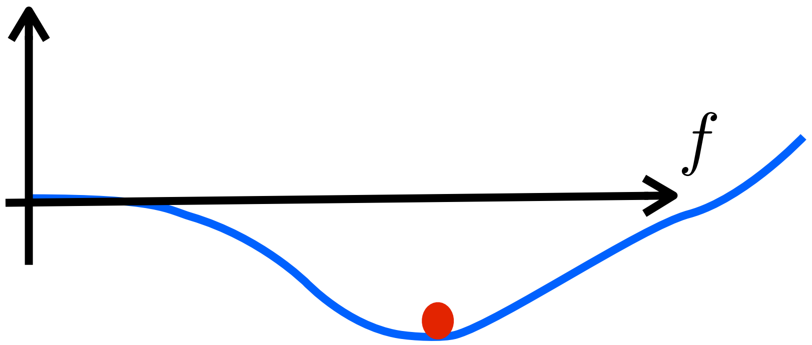

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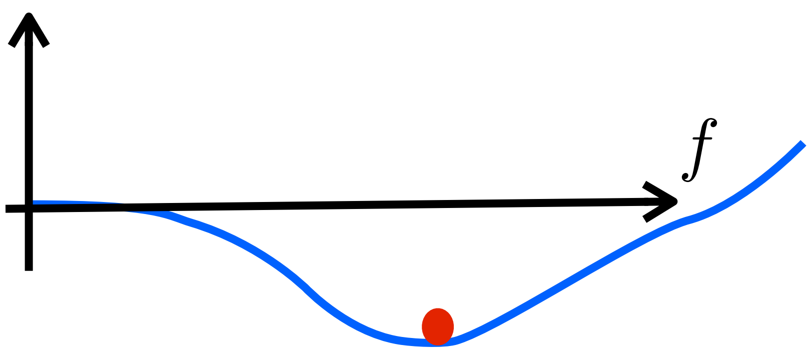

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is the dilaton naturally light?

not quite

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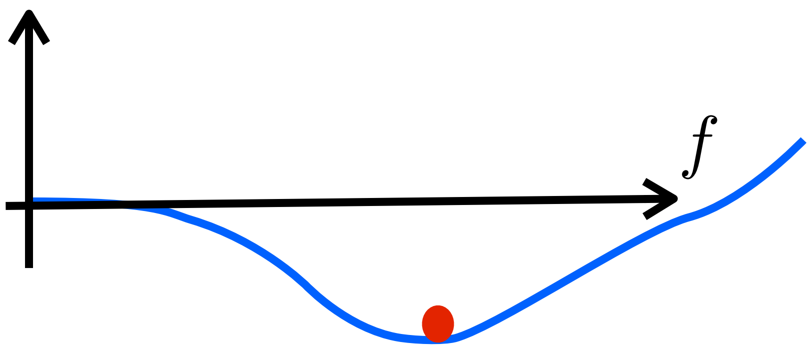

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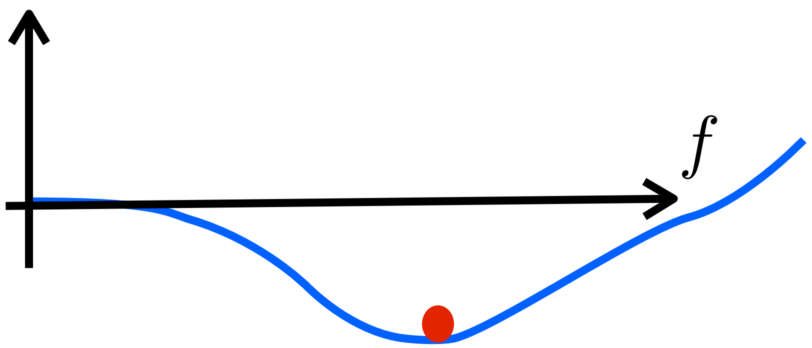

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generically very steep potential!

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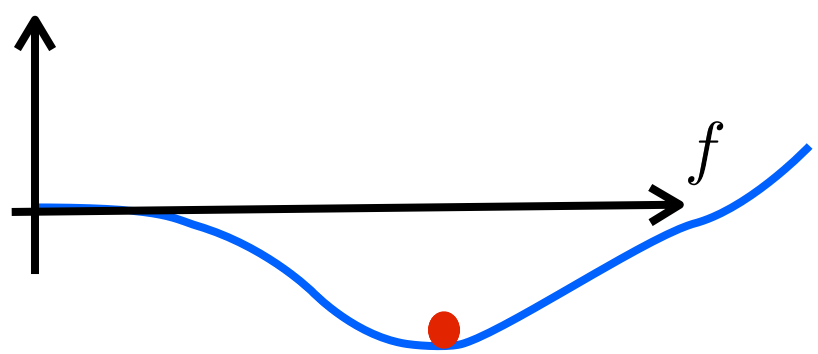

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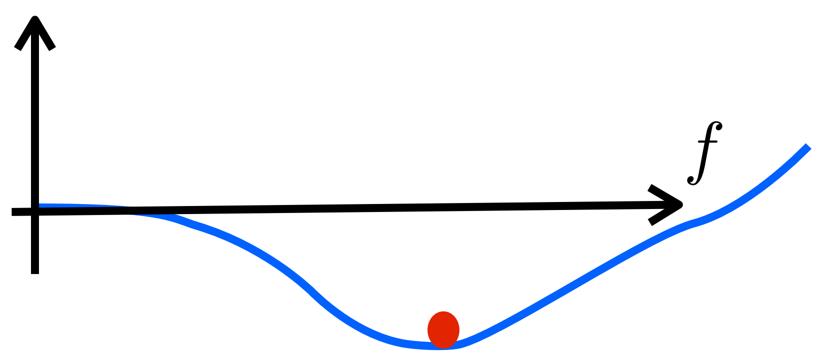

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to establish f<<UV-cutoff beta(IR) must be big
the CFT(IR) and the light dilaton are lost

or

start with a ~flat direction; no large vacuum energy
(natural only in SUSY?)

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$$\mathcal{L}_{CFT} + \lambda \mathcal{O} \longrightarrow \boxed{V = \chi^4 F(\lambda(\chi))}$$

$$F(\lambda) = \underset{\uparrow\ \text{sym}}{a} + \underset{\uparrow\ \text{sym breaking}}{\delta F(\lambda)} = 16\pi^2 \left[c_0 + c_1 \frac{\lambda}{4\pi} + \ldots \right]$$

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sym

sym breaking

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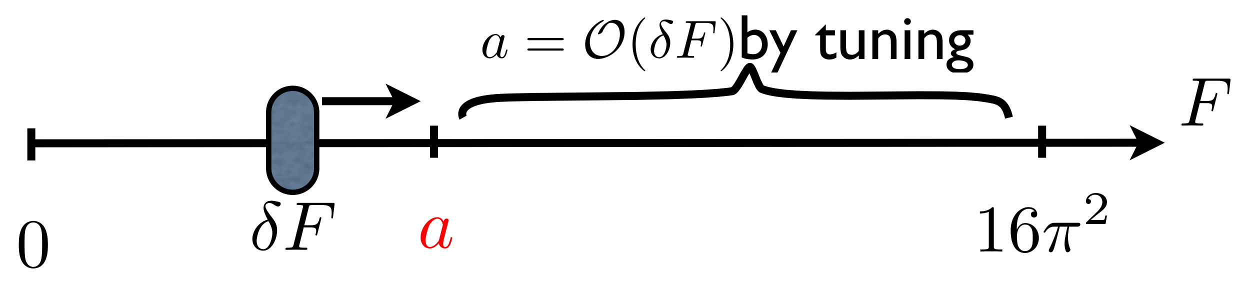

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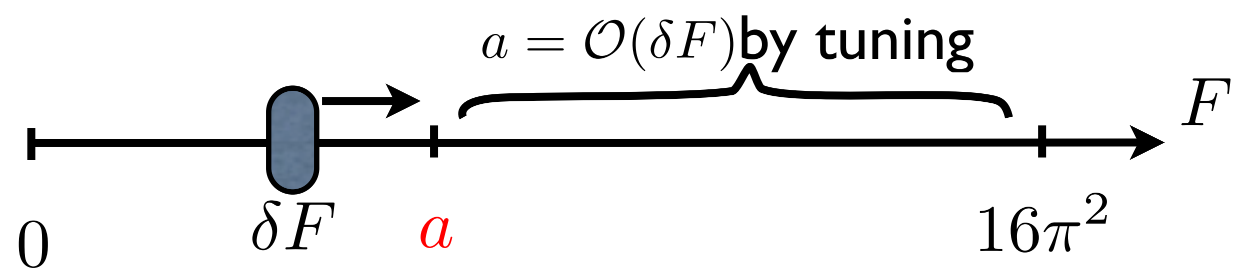

2) δF dynamically cancels vs a

$$a + \delta F(f) \simeq 0 \longrightarrow f = \Lambda_{UV}\left(\frac{-4\pi c_0}{\lambda(M)c_1}\right)^{1/\epsilon}$$

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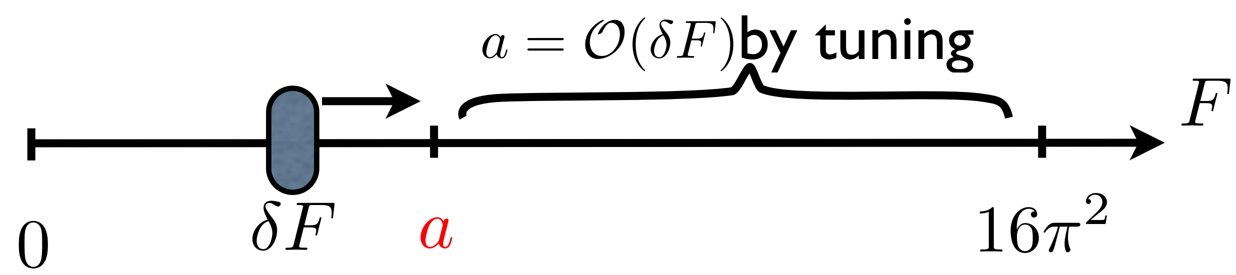

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2) δF dynamically cancels vs a $\quad a + \delta F(f) \simeq 0 \longrightarrow f = \Lambda_{UV}\left(\frac{-4\pi c_0}{\lambda(M)c_1}\right)^{1/\epsilon}$

3) $\mathcal{O}$ close to marginal at the scale f $\quad \beta = \epsilon\lambda + b_1\frac{\lambda^2}{4\pi} + \ldots \ll 1$

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LIGHT DILATON?

$$\mathcal{L}_{CFT} + \lambda\mathcal{O} \longrightarrow \boxed{V = \chi^4 F(\lambda(\chi))}$$

$$F(\lambda) = \underset{\text{sym}}{\underset{\uparrow}{a}} + \underset{\text{sym breaking}}{\underset{\uparrow}{\delta F(\lambda)}} = 16\pi^2 \left[c_0 + c_1 \frac{\lambda}{4\pi} + \dots\right]$$

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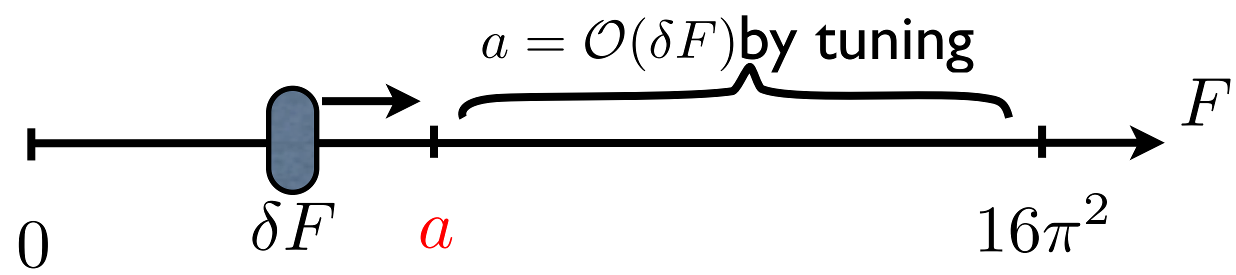

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$$m_{dil}^2 = 4f^2 \beta F'(\lambda(f)) \longrightarrow \boxed{\frac{m_{dil}^2}{\Lambda^2} \sim \frac{\beta(f)}{\pi} \simeq \epsilon \frac{\lambda(f)}{\pi}}$$

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way out? largish coupling but small beta $\quad \beta = \epsilon \left(\lambda + b_1 \frac{\lambda^2}{4\pi} + \ldots \right) \ll 1 \quad$ dual to a pNGB in 5D?

Contino, Pomarol, Rattazzi, at Planck09

Bellazzini, Csaki, Hubisz, Serra, Terning, to appear

G	H	N_G	NGBs rep.$[H]$ = rep.$[\mathrm{SU}(2)\times\mathrm{SU}(2)]$
SO(5)	SO(4)	4	$\mathbf{4}=(\mathbf{2},\mathbf{2})$
SO(6)	SO(5)	5	$\mathbf{5}=(\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{2})$
SO(6)	SO(4) × SO(2)	8	$\mathbf{4}_{+\mathbf{2}}+\bar{\mathbf{4}}_{-\mathbf{2}}=2\times(\mathbf{2},\mathbf{2})$
SO(7)	SO(6)	6	$\mathbf{6}=2\times(\mathbf{1},\mathbf{1})+(\mathbf{2},\mathbf{2})$
SO(7)	G_2	7	$\mathbf{7}=(\mathbf{1},\mathbf{3})+(\mathbf{2},\mathbf{2})$
SO(7)	SO(5) × SO(2)	10	$\mathbf{10}_{\mathbf{0}}=(\mathbf{3},\mathbf{1})+(\mathbf{1},\mathbf{3})+(\mathbf{2},\mathbf{2})$
SO(7)	$[\mathrm{SO}(3)]^3$	12	$(\mathbf{2},\mathbf{2},\mathbf{3})=3\times(\mathbf{2},\mathbf{2})$
Sp(6)	Sp(4) × SU(2)	8	$(\mathbf{4},\mathbf{2})=2\times(\mathbf{2},\mathbf{2}),(\mathbf{2},\mathbf{2})+2\times(\mathbf{2},\mathbf{1})$
SU(5)	SU(4) × U(1)	8	$\mathbf{4}_{-5}+\bar{\mathbf{4}}_{+\mathbf{5}}=2\times(\mathbf{2},\mathbf{2})$
SU(5)	SO(5)	14	$\mathbf{14}=(\mathbf{3},\mathbf{3})+(\mathbf{2},\mathbf{2})+(\mathbf{1},\mathbf{1})$

(infinitesimal) special conformal transformations

$$x^\mu \to x^{\mu\,\prime} = x^\mu + 2(b\cdot x)x^\mu - b^\mu x^2 + o(b^2)$$

$$J = |\partial x'/\partial x| = 1 + 8b\cdot x + o(b^2)$$

$$\chi(x) \to \chi'(x') = J^{-1/4}\chi(x) \tag{A.4}$$

$$\partial_\mu\chi(x) \to \frac{\partial x^\alpha}{\partial x'^{\,\mu}} J^{-1/4}\chi(x)\left(\partial_\alpha\sigma - 2b_\alpha\right) \tag{A.5}$$

$$\partial_\mu\sigma \to \frac{\partial x^\alpha}{\partial x'^{\,\mu}}\left(\partial_\alpha\sigma - 2b_\alpha\right) \tag{A.6}$$

$$\Box\sigma \to J^{-1/2}\left(\Box\sigma + 4b^\alpha\partial_\alpha\sigma\right) \tag{A.7}$$

$$(\partial_\mu\sigma)^2 \to J^{-1/2}\left[(\partial_\alpha\sigma)^2 - 4b^\alpha\partial_\alpha\sigma\right] \tag{A.8}$$

$$(\Box\sigma)^2 \to J^{-1}\left[(\Box\sigma) + 8b^\alpha\partial_\alpha\sigma\Box\sigma\right] \tag{A.9}$$

$$s^\mu_\mu \equiv (\Box\sigma + (\partial_\mu\sigma)^2) \to J^{-1/2}s^\mu_\mu \tag{A.10}$$

$$a^\mu_\mu \equiv (\Box\sigma - (\partial_\mu\sigma)^2) \to J^{-1/2}\left[a^\mu_\mu + 8b^\alpha\partial_\alpha\sigma\right] \tag{A.11}$$

$$s_{\mu\nu} \equiv (\partial_\mu\partial_\nu\sigma + \partial_\mu\sigma\partial_\nu\sigma) \to \frac{\partial x^\alpha}{\partial x'^{\,\mu}}\frac{\partial x^\beta}{\partial x'^{\,\nu}}\left[s_{\alpha\beta} - 4b_\alpha\partial_\beta\sigma - 4b_\beta\partial_\alpha\sigma + 2g_{\alpha\beta}b^\gamma\partial_\gamma\sigma\right] \tag{A.12}$$

$$a_{\mu\nu} \equiv (\partial_\mu\partial_\nu\sigma - \partial_\mu\sigma\partial_\nu\sigma) \to \frac{\partial x^\alpha}{\partial x'^{\,\mu}}\frac{\partial x^\beta}{\partial x'^{\,\nu}}\left[a_{\alpha\beta} + 2g_{\alpha\beta}b^\gamma\partial_\gamma\sigma\right]\,. \tag{A.13}$$

non-covariant transformations