The FLAG review: determination of the light quark masses

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Bruxelles, June 10, 2011

Outline

Introduction

Quark mass ratios from CHPT

Sum rules

Lattice determination of m_u, m_d and m_s
- FLAG
- Isospin limit
- Isospin breaking

Summary and Outlook

Quantum Gluodynamics

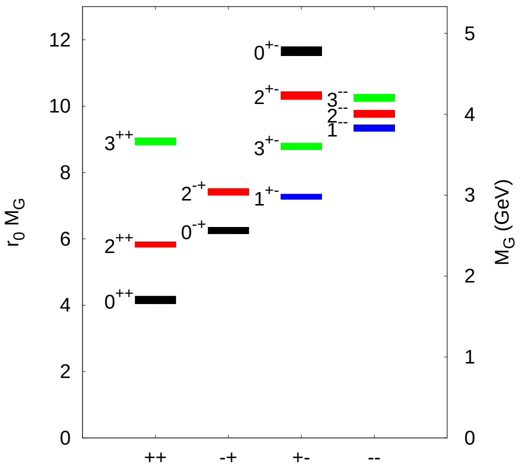
SU(3) pure-gauge theory:

$$\mathcal{L}_{\mathrm{SU(3)-glue}} = -\frac{1}{4g^2}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu}$$

$G_{\mu\nu}$ describes eight massless spin-1 bosons (gluons)

- the fundamental degrees of freedom are massless
- the spectrum has a mass gap

Quantum Gluodynamics



Recent numerical study

Chen et al. PRD (05)

Quantum Gluodynamics

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$G_{\mu\nu}$ describes eight massless spin-1 bosons (gluons)

- the fundamental degrees of freedom are massless
- the spectrum has a mass gap
- the theory has no free parameters
  (g vanishes at short distances)

Add quarks...

...but no free parameters yet

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4g^2}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} + \sum_i \bar{q}_i(i\not{D} - m_{q_i})q_i + \sum_j \bar{Q}_j(i\not{D} - m_{Q_j})Q_j$$

$$m_{q_i} \to 0 \qquad m_{Q_j} \to \infty$$

A theoretician's paradise (H. Leutwyler):

- reasonably good approximation to the real world
- no free parameters

Add quarks...

...but no free parameters yet

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4g^2}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} + \sum_i \bar{q}_i(i\not{D} - m_{q_i})q_i + \sum_j \bar{Q}_j(i\not{D} - m_{Q_j})Q_j$$

$$m_{q_i} \to 0 \qquad m_{Q_j} \to \infty$$

What can be studied

- spectrum of hadrons made of light quarks only
- bound states of one heavy and one or more light quarks
- potential between two heavy quarks
  (string formation and breaking)

Symmetries of the paradise

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4g^2}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} + \sum_i \left[\bar{q}_{i\,L} i\not{D}\, q_{i\,L} + \bar{q}_{i\,R} i\not{D}\, q_{i\,R}\right]$$

- gluons are flavour blind
- left- and right-handed components of the quark fields interact separately
- $\Rightarrow$ large global symmetry group G:

$$G = U(1)_V \times U(1)_A \times SU(N_\ell)_L \times SU(N_\ell)_R$$

- $U(1)_V$ is a good symmetry
  $U(1)_A$ is anomalous
  the vacuum is symmetric only under $SU(N_\ell)_V$

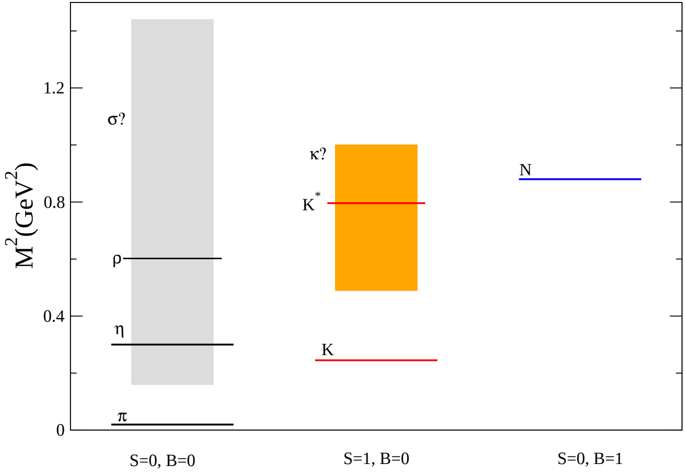
Symmetries of the paradise

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4g^2}{\rm Tr}G_{\mu\nu}G^{\mu\nu} + \sum_i \left[\bar{q}_{i\,L} i\not{D}\, q_{i\,L} + \bar{q}_{i\,R} i\not{D}\, q_{i\,R}\right]$$

Symmetry $\Rightarrow$ qualitative understanding of the spectrum

- spectrum organized in $SU(N_\ell)_V$ multiplets
- $N_\ell^2 - 1$ massless pseudoscalar mesons

Symmetries of the paradise



The real world

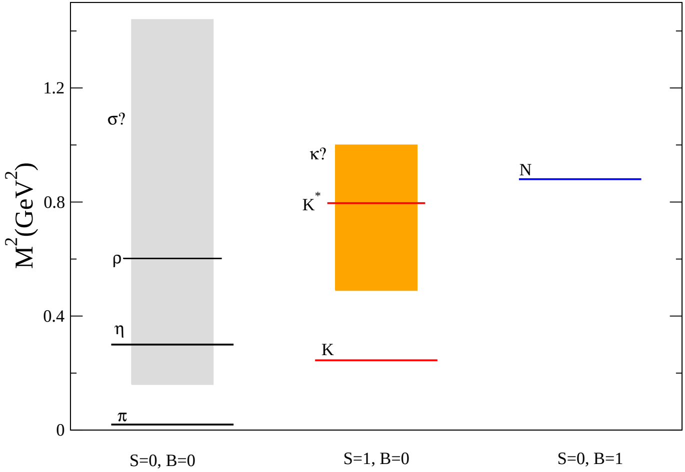
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4g^2}\mathrm{Tr}G_{\mu\nu}G^{\mu\nu} + \sum_i \bar{q}_i(i\not{D} - m_{q_i})q_i + \sum_j \bar{Q}_j(i\not{D} - m_{Q_j})Q_j$$

- Keep m_{q_i} and m_{Q_j} finite
- Observe that $m_{q_i} \ll \Lambda$ while $m_{Q_j} \gg \Lambda$ $[\Lambda \sim m_N]$
- Real world as a perturbation around the paradise!

$$\text{real world} - \text{paradise} \sim \mathcal{O}\left(\frac{m_{q_i}}{\Lambda}\right) \text{ or } \mathcal{O}\left(\frac{\Lambda}{m_{Q_i}}\right)$$

- the strange and charm quark are problematic...

The real world



How to determine quark masses

- From their influence on the spectrum
  - $m_Q \gg \Lambda$

$$M_{\bar{Q}q_i} = m_Q + \mathcal{O}(\Lambda)$$

  - $m_q \ll \Lambda$

$$M_{\bar{q}_i q_j} = M_{0\,ij} + \mathcal{O}(m_{q_i}, m_{q_j}) \qquad M_{0\,ij} = \mathcal{O}(\Lambda)$$

In both cases need to understand the $\mathcal{O}(\Lambda)$ term

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In both cases need to understand the $\mathcal{O}(\Lambda)$ term

- From their influence on any other observable

Quark masses are coupling constants
$\Rightarrow$ exploit the sensitivity to them of any observable
[e.g. η and τ decays]

How to determine quark masses

Approaches

1. Chiral perturbation theory — spectrum + low energy observables
2. Sum rules — spectral functions
3. Lattice QCD — spectrum

How to determine quark masses

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1. Chiral perturbation theory — spectrum + low energy observables
2. Sum rules — spectral functions
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Technical remark:
quark masses are coupling constants! (not observables)
they depend on the renormalization scale μ (like α_s)
for light quarks by convention: $\mu = 2\ \mathrm{GeV}$

In the following

$$m_q \equiv m_q(2\ \mathrm{GeV})$$

Expansion around the chiral limit

$$\mathcal{H}_{\mathrm{QCD}} = \mathcal{H}^0_{\mathrm{QCD}} + \mathcal{H}_m \qquad \mathcal{H}_m := \bar{q}\mathcal{M}q \quad \mathcal{M} = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

$$\bar{q} = (\bar{u}, \bar{d}, \bar{s})$$

the mass term $\mathcal{H}_m$ will be treated as a small perturbation

Expansion around the chiral limit

$$\mathcal{H}_{\rm QCD} = \mathcal{H}^0_{\rm QCD} + \mathcal{H}_m \qquad \mathcal{H}_m := \bar{q}\mathcal{M}q \quad \mathcal{M} = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

Expansion around $\mathcal{H}^0_{\rm QCD} \equiv$ expansion in powers of m_q

General quark mass expansion for a particle P:

$$M^2 = M_0^2 + (m_u + m_d)\langle P|\bar{q}q|P\rangle + O(m_q^2)$$

Expansion around the chiral limit

$$\mathcal{H}_{\rm QCD} = \mathcal{H}^0_{\rm QCD} + \mathcal{H}_m \qquad \mathcal{H}_m := \bar{q}\mathcal{M}q \quad \mathcal{M} = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

Expansion around $\mathcal{H}^0_{\rm QCD}$ $\equiv$ expansion in powers of m_q

For a Goldstone bosons $M_0^2 = 0$:

$$M_\pi^2 = -(m_u + m_d)\frac{1}{F_\pi^2}\langle 0|\bar{q}q|0\rangle + O(m_q^2)$$

where we have used a Ward identity: Gell-Mann, Oakes and Renner (68)

$$\langle \pi|\bar{q}q|\pi\rangle = -\frac{1}{F_\pi^2}\langle 0|\bar{q}q|0\rangle =: B_0$$

Quark masses

Consider the whole pseudoscalar octet:

$$M_\pi^2 = B_0(m_u + m_d) \quad M_{K^+}^2 = B_0(m_u + m_s) \quad M_{K^0}^2 = B_0(m_d + m_s)$$

Quark masses

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Quark mass ratios:

$$\frac{m_u}{m_d} \simeq \frac{M_{\pi^+}^2 - M_{K^0}^2 + M_{K^+}^2}{M_{\pi^+}^2 + M_{K^0}^2 - M_{K^+}^2} \simeq 0.67$$

$$\frac{m_s}{m_d} \simeq \frac{M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} \simeq 20$$

Quark masses

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$$\frac{m_s}{m_d} \simeq \frac{M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} \simeq 20$$

$\hat{m} \equiv (m_u + m_d)/2 \simeq 5.4$ MeV

SU(6) relation, Leutwyler (75)

$m_u \simeq 4$ MeV $\quad m_d \simeq 6$ MeV $\quad m_s \simeq 135$ MeV

Gasser and Leutwyler (75)

Electromagnetic corrections to the masses

According to Dashen's theorem

$$
\begin{aligned}
M^2_{\pi^0} &= B_0(m_u + m_d) \\
M^2_{\pi^+} &= B_0(m_u + m_d) + \Delta_{\text{em}} \\
M^2_{K^0} &= B_0(m_d + m_s) \\
M^2_{K^+} &= B_0(m_u + m_s) + \Delta_{\text{em}}
\end{aligned}
$$

Extracting the quark mass ratios gives

Weinberg (77)

$$
\begin{aligned}
\frac{m_u}{m_d} &= \frac{M^2_{K^+} - M^2_{K^0} + 2M^2_{\pi^0} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 0.56 \\
\frac{m_s}{m_d} &= \frac{M^2_{K^0} + M^2_{K^+} - M^2_{\pi^+}}{M^2_{K^0} - M^2_{K^+} + M^2_{\pi^+}} = 20.1
\end{aligned}
$$

Weinberg (77) estimated m_s from the splitting in baryon octet

$$m_u = 4.2 \text{ MeV} \qquad m_d = 7.5 \text{ MeV} \qquad m_s = 150 \text{ MeV}$$

Higher order chiral corrections

Mass formulae to second order

Gasser-Leutwyler (85)

$$\frac{M_K^2}{M_\pi^2} = \frac{m_s + \hat{m}}{2\hat{m}} \left[1 + \Delta_M + \mathcal{O}(m^2)\right]$$

$$\frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2 - M_\pi^2} = \frac{m_d - m_u}{m_s - \hat{m}} \left[1 + \Delta_M + \mathcal{O}(m^2)\right]$$

$$\Delta_M = \frac{8(M_K^2 - M_\pi^2)}{F_\pi^2}(2L_8 - L_5) + \chi\text{-logs}$$

The same $\mathcal{O}(m)$ correction appears in both ratios
$\Rightarrow$ this double ratio is free from $\mathcal{O}(m)$ corrections

$$Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} = \frac{M_K^2}{M_\pi^2} \frac{M_K^2 - M_\pi^2}{M_{K^0}^2 - M_{K^+}^2} \left[1 + \mathcal{O}(m^2)\right]$$

Higher order chiral corrections

Mass formulae to second order

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$$\frac{M_K^2}{M_\pi^2} = \frac{m_s + \hat{m}}{2\hat{m}}\left[1 + \Delta_M + \mathcal{O}(m^2)\right]$$

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The same $\mathcal{O}(m)$ correction appears in both ratios

$\Rightarrow$ this double ratio is free from $\mathcal{O}(m)$ and em corrections

$$Q_D^2 \equiv \frac{(M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2 + M_{\pi^0}^2)(M_{K^0}^2 + M_{K^+}^2 - M_{\pi^+}^2 - M_{\pi^0}^2)}{4M_{\pi^0}^2(M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2 - M_{\pi^0}^2)} = 24.2$$

Leutwyler's ellipse

Information on Q amounts to an elliptic constraint in the plane of $\frac{m_s}{m_d}$ and $\frac{m_u}{m_d}$

Leutwyler

$$\left(\frac{m_s}{m_d}\right)^2 \frac{1}{Q^2} + \left(\frac{m_u}{m_d}\right)^2 = 1$$

Leutwyler's ellipse

Information on Q amounts to an elliptic constraint in the plane of $\frac{m_s}{m_d}$ and $\frac{m_u}{m_d}$

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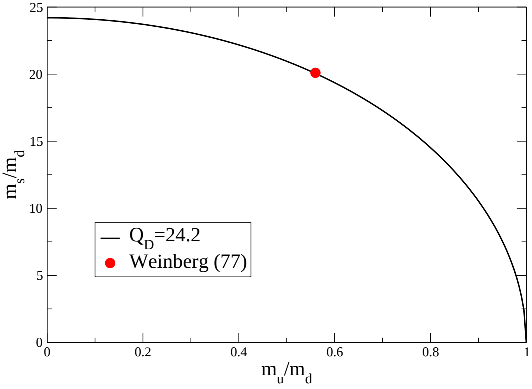
$$\left(\frac{m_s}{m_d}\right)^2 \frac{1}{Q^2} + \left(\frac{m_u}{m_d}\right)^2 = 1$$



Estimate of Q: violation of Dashen's theorem

$$\left(M_{K^+}^2 - M_{K^0}^2\right)_{\rm em} = \left(M_{\pi^+}^2 - M_{\pi^0}^2\right)_{\rm em} \Rightarrow (M_{K^+} - M_{K^0})_{\rm em} = 1.3 \text{ MeV}$$

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Higher order corrections change the numerical value

$$(M_{K^+} - M_{K^0})_{\rm em} = \begin{cases} 1.9 \text{ MeV} & \text{Duncan et al. (96)} & Q = 22.8 \\ & \text{(Lattice)} & \\ 2.3 \text{ MeV} & \text{Bijnens-Prades (97)} & Q = 22 \\ & \text{(ENJL model)} & \\ 2.6 \text{ MeV} & \text{Donoghue-Perez (97)} & Q = 21.5 \\ & \text{(VMD)} & \\ 3.2 \text{ MeV} & \text{Anant-Moussallam (04)} & Q = 20.7 \\ & \text{(Sum rules)} & \end{cases}$$

Most recent evaluation: Kastner-Neufeld (08): $Q = 20.7 \pm 1.2$

Estimate of Q: the decay $\eta \to \pi^0\pi^+\pi^-$

Decay amplitude at leading order

$$\mathcal{A}(\eta \to \pi^0\pi^+\pi^-) = -\frac{\sqrt{3}}{4}\frac{m_u - m_d}{m_s - \hat{m}}\frac{s - 4M_\pi^2/3}{F_\pi^2}$$

Estimate of Q: the decay $\eta \to \pi^0\pi^+\pi^-$

Decay amplitude

$$\mathcal{A}(\eta \to \pi^0\pi^+\pi^-) = -\frac{1}{Q^2}\frac{M_K^2(M_K^2 - M_\pi^2)}{3\sqrt{3}M_\pi^2 F_\pi^2} M(s,t,u)$$

The decay width can be written as

$$\Gamma(\eta \to \pi^0\pi^+\pi^-) = \Gamma_0 \left(\frac{Q_D}{Q}\right)^4 = (295 \pm 20)\ \text{eV} \quad \text{PDG (08)}$$

- isospin-breaking sensitive process
- em contributions suppressed (Sutherland's theorem)
  $\Rightarrow$ mainly sensitive to $m_u - m_d$
- strong decay width Γ_0 difficult to estimate

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$$\Gamma_0 = \begin{cases} (167 \pm 50)\ \text{eV} & \text{Gasser-Leutwyler (85)} & Q = 21.1 \pm 1.6 \\ (219 \pm 22)\ \text{eV} & \text{Anisovich-Leutwyler (96)} & Q = 22.6 \pm 0.7 \\ (209 \pm 20)\ \text{eV} & \text{Kambor et al (96)} & Q = 22.3 \pm 0.6 \end{cases}$$

Gasser Leutwyler (85) based on one-loop CHPT

The other two evaluations based on dispersion relations

See also: analysis of KLOE data on $\eta \to 3\pi$ Martemyanov-Sopov (05)

$$Q = 22.8 \pm 0.4$$

Estimate of Q: the decay $\eta \to \pi^0\pi^+\pi^-$

Decay amplitude

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The other two evaluations based on dispersion relations

See also: full two-loop calculation of $\eta \to 3\pi$ Bijnens-Ghorbani (07)

$$Q = 23.2$$

Estimate of Q: the decay $\eta \to \pi^0\pi^+\pi^-$

A new analysis is in progress

GC, Lanz, Leutwyler, Passemar

- recent measurements of the Dalitz plot
  $\Rightarrow$ test the calculation of the strong dynamics of the decay
- dispersive analysis based on $\pi\pi$ scattering phases
  recent improvements must be taken into account

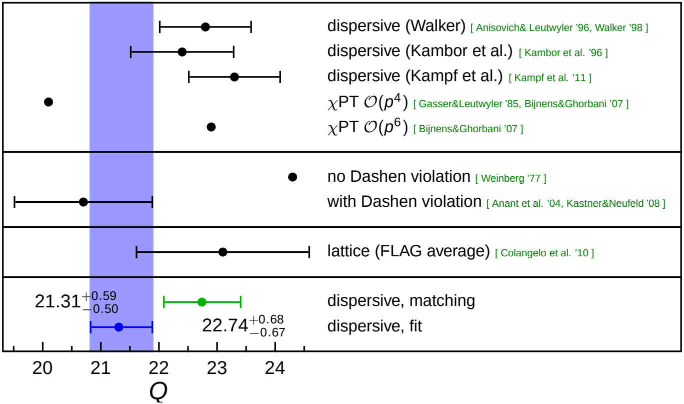
  GC, Gasser, Leutwyler (01)
- recent progress in dealing with isospin breaking (NREFT)

  Gasser, Rusetsky et al.

  can be applied also here

  Kubis, Schneider (09)

Estimates of Q: summary



S. Lanz, PhD thesis, work in progress GC, Lanz, Leutwyler Passemar

τ decays – basic notation

Total hadronic decay rate:

Braaten, Narison, Pich (92)

$$R_\tau = \frac{\Gamma(\tau \to \text{hadrons}\, \nu_\tau)}{\Gamma(\tau \to e\nu_e\nu_\tau)} = R_{\tau,V+A} + R_{\tau,S}$$

$$= 12\pi \int_0^1 dx(1-x)^2 \left[(1+2x)\mathrm{Im}\Pi^T + \mathrm{Im}\Pi^L\right] \quad \left[x = \frac{s}{M_\tau^2}\right]$$

Hadronic correlator:

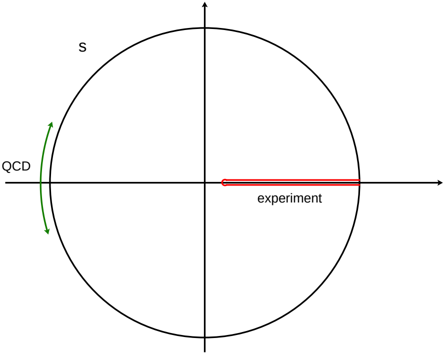
$$\Pi^I_{\mu\nu}(p) \equiv i\int dx\, e^{ipx}\langle 0|TJ^I_\mu(x)J^{I\,\dagger}_\nu(0)|0\rangle \quad I = V, A$$

$$= (p_\mu p_\nu - g_{\mu\nu}p^2)\Pi^{I,T}(p^2) + p_\mu p_\nu \Pi^{I,L}(p^2)$$

Separation *ud* and *us* currents

$$\Pi^J = |V_{ud}|^2\left[\Pi^{V,J} + \Pi^{A,J}\right] + |V_{us}|^2\Pi^J_S$$

τ decays – basic notation



Experiment = QCD $\Rightarrow$ determine QCD parameters

τ decays – basic notation

e.g. Gámiz, Jamin, Pich, Prades, Schwab (03)

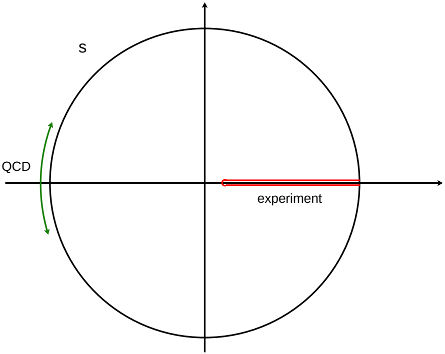
$$R_\tau^{kl} \equiv \int_0^1 dx\,(1-x)^k x^l \frac{dR_\tau}{dx} \qquad \delta R_\tau^{kl} = \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2}$$

These δR_τ^{kl} are:

- zero in the $SU(3)$ limit, i.e. sensitive to m_s
- calculable in perturbative QCD (in terms of m_s)
- measurable (provided one knows $V_{ud,s}$)

Experiment = QCD $\qquad \Rightarrow m_s$

m_s from τ decays



$$\Delta_{kl}^{L+T} m_s^2 = \frac{M_\tau^2}{18(1-\epsilon_d^2)} \left[\frac{\delta R_\tau^{kl}}{S_{EW}} - \delta R_{\tau,D\geq 4}^{kl,L+T} \right]$$

$$\epsilon_d = m_d/m_s$$

$\delta R_{\tau,D\geq 4}^{kl,L+T}$ are higher dim. operators

Summary of recent results

	$m_s(2\text{GeV})$ (MeV)
Jamin et al. (02)	99 ± 16
Kambor Maltman (02)	100 ± 12
Gamiz et al. (03)	103 ± 17
Jamin et al. (05)	81 ± 22
Gorbunov-Pivovarov (05)	125 ± 28
Baikov et al. (05)	96 ± 19
Narison (05)	89 ± 25
Jamin et al. (06)	92 ± 9
Dominguez et al. (08)	102 ± 8

The first set is mostly based on ALEPH
The second one also on newer data from CLEO and OPAL
The third set is based on analyses of the (pseudo)scalar channel

Lattice determinations of quark masses

First principle (and brute force) method:

- QCD Lagrangian as input
- calculate the spectrum of the low-lying states for different quark masses
- tune the values of the quark masses such that the QCD spectrum is reproduced
- the comparison is made for mass ratios — one mass or a dimensionful quantity (*e.g.* F_π) can be used to set the scale

Lattice determinations of quark masses

Systematic effects:

- finite volume
  exponentially suppressed effects, easy to correct for
- finite lattice spacing
  powerlike effects, extrapolate numerically
- unphysical quark masses
  extrapolate numerically with guidance from CHPT
  BMW and PACS-CS: simulations at the physical point!
- renormalization
  relation between bare (input parameter) and renormalized (physically relevant quantity) quark masses not easy

What/Who is FLAG?

FLAG = FLAVIAnet Lattice Averaging Group

FlaviA net

European network
on Flavour Physics
Start: 1.10.2006
End: 30.09.2010

“Entering the high-precision era of flavour physics through the alliance of lattice simulations, effective field theories and experiment”

What/Who is FLAG?

FLAG = FLAVIAnet Lattice Averaging Group

Members:

Gilberto Colangelo (Bern)
Stephan Dürr (Jülich, BMW)
Andreas Jüttner (Mainz, RBC/UKQCD)
Laurent Lellouch (Marseille, BMW)
Heiri Leutwyler (Bern)
Vittorio Lubicz (Rome 3, ETM)
Silvia Necco (CERN, Alpha)
Chris Sachrajda (Southampton, RBC/UKQCD)
Silvano Simula (Rome 3, ETM)
Tassos Vladikas (Rome 2, Alpha and ETM)
Urs Wenger (Bern, ETM)
Hartmut Wittig (Mainz, Alpha)

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History, status and future:

- FLAG start: Orsay, November 2007
- Meetings: Bern, March 2008 and April 2009
- paper and webpage were made public in November 2010
- paper will appear on EPJC (review section)
- we plan to make yearly updates

What exactly does FLAG offer?

An answer to the questions

- what is the best lattice value for quantity X?
- what is a reliable estimate of the uncertainty?

in a way easily accessible to non-experts

Quantities considered in the first edition:

- light quark masses
- LEC's
- decay constants
- form factors
- B_K

Color coding – our present definition

- chiral extrapolation
  ★ $M_{\pi,\mathrm{min}} < 250$ MeV
  ● 250 MeV $\leq M_{\pi,\mathrm{min}} \leq$ 400 MeV
  ■ $M_{\pi,\mathrm{min}} > 400$ MeV

Color coding – our present definition

- chiral extrapolation
  - ★ $M_{\pi,\mathrm{min}} < 250$ MeV
  - ● 250 MeV $\leq M_{\pi,\mathrm{min}} \leq$ 400 MeV
  - ■ $M_{\pi,\mathrm{min}} > 400$ MeV
- continuum extrapolation
  - ★ 3 or more lattice spacings, at least 2 points below 0.1 fm
  - ● 2 or more lattice spacings, at least 1 point below 0.1 fm
  - ■ otherwise

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- finite volume effects
  - ★ $(M_\pi L)_{\mathrm{min}} > 4$ or at least 3 volumes
  - ● $(M_\pi L)_{\mathrm{min}} > 3$ and at least 2 volumes
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  - ● $(M_\pi L)_{\text{min}} > 3$ and at least 2 volumes
  - ■ otherwise
- renormalization (where applicable)
  - ★ non-perturbative
  - ● 2-loop perturbation theory (converging series)
  - ■ otherwise

FLAG tables

Collaboration	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	m_{ud}
PACS-CS 10	P	★	■	■	★	2.78(27)
MILC 10A	C	●	★	★	●	3.19(4)(5)(16)
HPQCD 10	A	●	★	★	★	3.39(6)*
BMW 10A, 10B$^+$	P	★	★	★	★	3.469(47)(48)
RBC/UKQCD 10A	P	●	●	★	★	3.59(13)(14)(8)
PACS-CS 09	A	★	■	■	★	2.97(28)(03)
HPQCD 09	A	●	★	★	★	3.40(7)
MILC 09A	C	●	★	★	●	3.25 (1)(7)(16)(0)
MILC 09	P	●	★	★	●	3.2(0)(1)(2)(0)
PACS-CS 08	A	★	■	■	■	2.527(47)
RBC/UKQCD 08	A	●	■	●	★	3.72(16)(33)(18)
RBC 07	A	■	■	★	★	4.25(23)(26)
ETM 07	A	●	■	●	★	3.85(12)(40)
QCDSF/ UKQCD 04	A	■	●	★	★	4.7(2)(3)

FLAG tables

Collaboration	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	m_s
PACS-CS 10	P	★	■	■	★	86.7(2.3)
MILC 10A	C	●	★	★	●	–
HPQCD 10	A	●	★	★	★	92.2(1.3)
BMW 10A, 10B$^+$	P	★	★	★	★	95.5(1.1)(1.5)
RBC/UKQCD 10A	P	●	●	★	★	96.2(1.6)(0.2)(2.1)
PACS-CS 09	A	★	■	■	★	92.75(58)(95)
HPQCD 09	A	●	★	★	★	92.4(1.5)
MILC 09A	C	●	★	★	●	89.0(0.2)(1.6)(4.5)(0.1)
MILC 09	P	●	★	★	●	88(0)(3)(4)(0)
PACS-CS 08	A	★	■	■	■	72.72(78)
RBC/UKQCD 08	A	●	■	●	★	107.3(4.4)(9.7)(4.9)
RBC 07	A	■	■	★	★	119.5(5.6)(7.4)
ETM 07	A	●	■	●	★	105(3)(9)
QCDSF/UKQCD 04	A	■	●	★	★	119(5)(8)

FLAG tables

Collaboration	publication status	chiral extrapolation	continuum extrapolation	finite volume	m_s/m_{ud}
BMW 10A, 10B^{+}	P	★	★	★	27.53(20)(8)
RBC/UKQCD 10A	P	●	●	★	26.8(0.8)(1.1)
Blum 10†	P	●	■	●	28.31(0.29)(1.77)
PACS-CS 09	A	★	■	■	31.2(2.7)
MILC 09A	C	●	★	★	27.41(5)(22)(0)(4)
MILC 09	P	●	★	★	27.2(1)(3)(0)(0)
PACS-CS 08	A	★	■	■	28.8(4)
RBC/UKQCD 08	A	●	■	●	$28.8 \pm 0.4 \pm 1.6$
RBC 07	A	■	■	★	28.10(38)
ETM 07	A	●	■	●	$27.3 \pm 0.3 \pm 1.2$
QCDSF/UKQCD 04	A	■	●	★	27.2(3.2)

FLAG averages and estimates

Different lattice results are *averaged* *if*

- published
  [lattice proceedings not enough]
- no red tags
- same N_f
  [no average of $N_f = 2$ and $N_f = 3$ calculations]
- results are compatible
  otherwise we may offer an *estimate*

Final FLAG number:

- average or estimate or single *no-red-tag* $N_f = 3$ number
- average or estimate or single *no-red-tag* $N_f = 2$ number

If *both* $N_f = 3$ *and* $N_f = 2$ numbers available:

agreement $\Rightarrow$ more confidence in the final number

FLAG estimates of the light quark masses

$N_f = 2 + 1$:

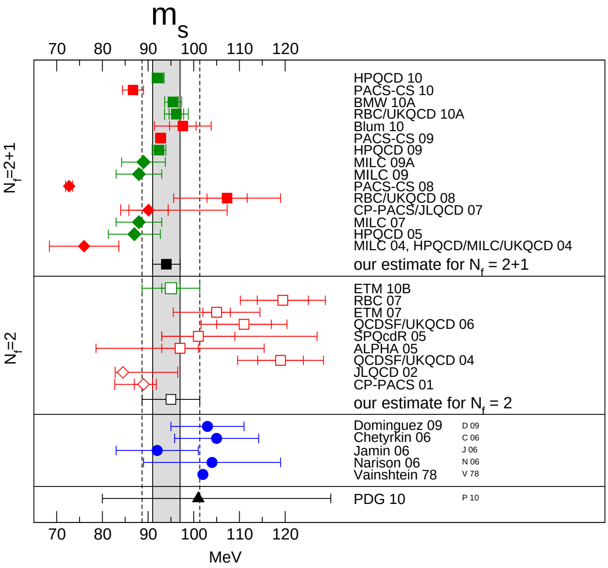
$$
\begin{aligned}
m_s &= 94 \pm 3 \ \mathrm{MeV} \\
m_{ud} &= 3.43 \pm 0.11 \ \mathrm{MeV} \\
\frac{m_s}{m_{ud}} &= 27.4 \pm 0.4
\end{aligned}
$$

$N_f = 2$:

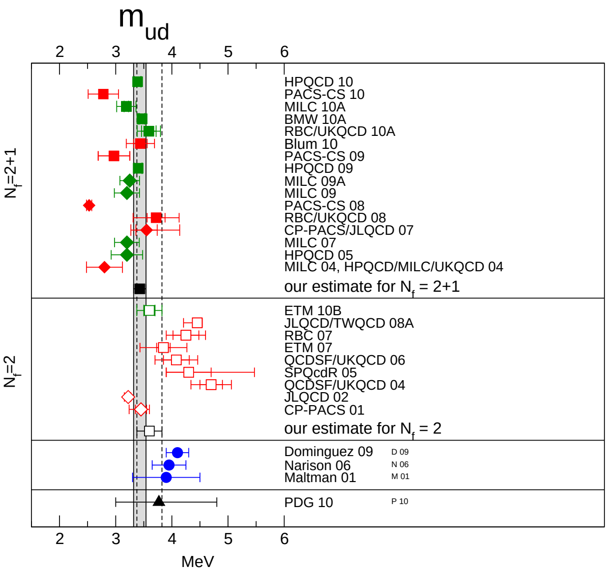
$$
\begin{aligned}
m_s &= 95 \pm 2 \pm 6 \ \mathrm{MeV} \\
m_{ud} &= 3.6 \pm 0.1 \pm 0.2 \ \mathrm{MeV} \\
\frac{m_s}{m_{ud}} &= 27.3 \pm 0.5 \pm 0.7
\end{aligned}
$$

PDG 2010: $m_s = 101^{+29}_{-21}$ $m_{ud} = 3.77^{+1.03}_{-0.77}$ $\frac{m_s}{m_{ud}} = 26 \pm 4$

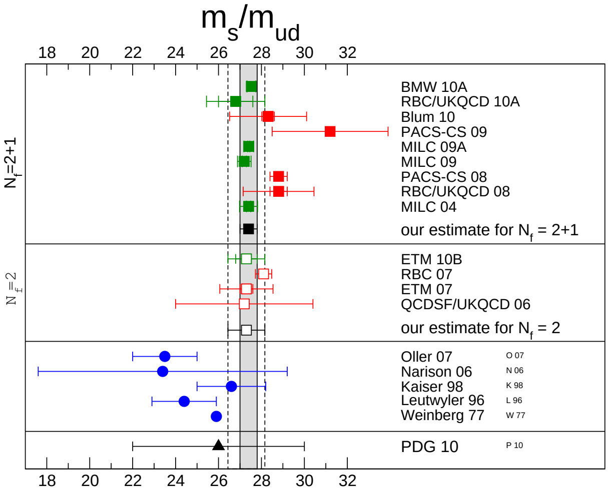
FLAG summary on m_{ud} and m_s



FLAG summary on m_{ud} and m_s



FLAG summary on m_{ud} and m_s



Isospin breaking on the lattice

- all lattice results described so far: isospin limit

$$m_u = m_d \qquad \text{and} \qquad \alpha_{em} = 0$$

- need as input M_π and M_K in this limit
  at the current level of precision

  the difference $\quad M_P - M_P(\alpha_{em} = 0) \quad$ matters!

  Remark: $M_{\pi^+}(\alpha_{em} = 0) \simeq M_{\pi^0}(\alpha_{em} = 0) \simeq M_{\pi^0}$
  $\Rightarrow$ need to know $(M_{K^+} - M_{K^0})_{em}$

- alternatively:
  simulate QCD+QED on the lattice and compare to the real-world spectrum

Electromagnetic effects in lattice calculations

Lattice calculations with QCD+QED

- Duncan, Eichten, Thacker (96)
  QCD quenched approximation, estimate for meson mass em splittings; systematic error not estimated

$$(M_{K^+} - M_{K^0})_{em} = 1.9\ \mathrm{MeV}$$

- Blum et al. (RBC 07)
  QCD with 2 dynamical quarks, but QED quenched

$$(M_{K^+} - M_{K^0})_{em} = 1.443(55)\ \mathrm{MeV}$$

- Blum et al. (10)
  QCD with 3 dynamical quarks (RBC/UKQCD configurations), but QED quenched

$$(M_{K^+} - M_{K^0})_{em} = 1.87(10)\ \mathrm{MeV}$$

Mixed approach

1. take as an input the phenomenological estimates of $(M_{K^+} - M_{K^0})_{em}$
2. use the kaon masses to determine m_u/m_d

MILC adopts the value Bijnens-Prades 97, Donoghue-Perez 97

$$(M_{K^+} - M_{K^0})_{em} = 2.8(6)\ \mathrm{MeV}$$

and obtains

$$\frac{m_u}{m_d} = 0.432(1)(9)(0)(39)$$

Lattice determinations of m_u/m_d

Collaboration	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	running	m_u	m_d
Blum et al. 10	P	●	■	●	★	●	2.24(10)(34)	4.65(15)(32)
MILC 09A	C	●	★	★	●	●	1.96(0)(6)(10)(12)	4.53(1)(8)(23)(12)
MILC 09	P	●	★	★	●	●	1.9(0)(1)(1)(1)	4.6(0)(2)(2)(1)
MILC 04, HPQCD/ MILC/UKQCD 04	A	●	●	●	■	■	1.7(0)(1)(2)(2)	3.9(0)(1)(4)(2)
RBC 07	A	■	■	★	★	●	3.02(27)(19)	5.49(20)(34)

Collaborations in red (black): QCD+QED (mixed approach)

Quark masses from lattice + Q from $\eta \to 3\pi$

Best way to determine m_u, m_d and m_s today:

- determine m_{ud} and m_s on the lattice in the isospin limit
- determine Q from $\eta \to 3\pi$
- combine the two pieces of information

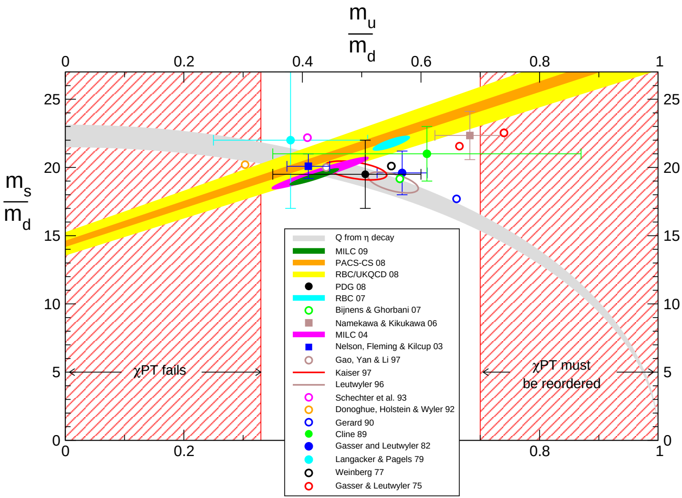
Example: BMW 10

$$m_s = 95.5(1.1)(1.5)\ \mathrm{MeV} \qquad m_{ud} = 3.469(47)(48)\ \mathrm{MeV}$$

Add Q from $\eta \to 3\pi$:

$$m_u = 2.15(03)(10)\ \mathrm{MeV} \qquad m_d = 4.79(07)(12)\ \mathrm{MeV}$$

Quark masses from lattice + Q from $\eta \to 3\pi$



Leutwyler PoS, CD09

Summary

- Quark masses are fundamental and yet unexplained parameters of the standard model
- their precision determination is an important goal of particle physics
- I have reviewed the three different methods to determine them
  - chiral perturbation theory
  - sum rules
  - lattice

  and shown that we are indeed entering the era of high precision lattice determinations
- ChPT+Lattice essential nowadays to cope with isospin breaking

Outlook

- FLAG aims to provide a summary of lattice results relevant for the phenomenology accessible to non-experts
- paper and webpage have been made public
  the paper will appear on EPJC Review section
- Future plans
  - yearly updates
  - extension to other quantities/sectors (heavy quarks)
  - let members from outside Europe join

Lattice calculations of $f_+(0)$ and f_K/f_π

$f_+(0)$	N_f		publication status	chiral extrapol.	finite volume	continuum extrapol.	action
$0.9599(34)(^{+31}_{-47})(14)$	2+1	RBC/UKQCD 10	A	●	★	■	DWF
0.9644(33)(34)(14)	2+1	RBC/UKQCD 07	A	●	★	■	DWF
$0.9544(68)_{stat}$	2	ETM 10D	C	●	★	●	max. tmQCD
0.9560(57)(62)	2	ETM 09A	P	●	●	●	max. tmQCD
$0.9647(15)_{stat}$	2	QCDSF 07	C	■	★	■	clover (NP)
0.968(9)(6)	2	RBC 06	A	■	★	■	DWF
0.967(6)	2	JLQCD 05	C	■	★	■	clover (NP)

Legenda publication status:

A = published article

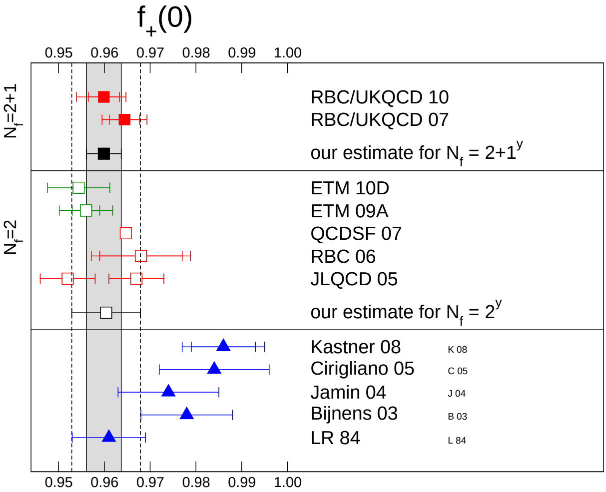
P = preprint

C = conference proceedings

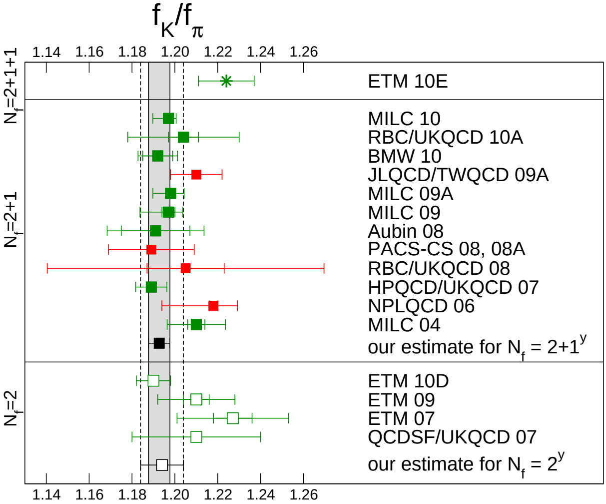
Lattice calculations of $f_+(0)$ and f_K/f_π

f_K/f_π	N_f		publication status	chiral extrapol.	finite volume	continuum extrapol.	action
$1.224(13)_{stat}$	2+1+1	ETM 10E	C	●	●	●	max. tmQCD
1.192(7)(6)	2+1	BMW 10	A	★	★	★	tlSW
$1.210(12)_{stat}$	2+1	JLQCD/TWQCD	C	●	■	■	overlap
$1.198(1)(^{+6}_{-8})$	2+1	MILC 09A	C	★	★	★	KS^{MILC}_{MILC}
1.191(16)(16)	2+1	ALVdW 08	C	★	●	●	KS_{MILC}/DWF
1.189(20)	2+1	PACS-CS 08	P	★	■	■	clover (NP)
1.18(1)(1)	2+1	BMW 08	C	★	★	★	impr. Wilson
1.189(2)(7)	2+1	HPQCD/UKQCD 08	A	★	●	★	KS^{HISQ}_{MILC}
1.205(18)(62)	2+1	RBC/UKQCD 07	A	●	★	■	DWF
$1.218(2)(^{+11}_{-24})$	2+1	NPLQCD 07	A	●	■	■	KS_{MILC}/DWF
$1.190(8)_{stat}$	2	ETM 10D	C	●	★	●	max. tmQCD
1.210(6)(15)(9)	2	ETM 09	A	●	●	★	max. tmQCD
1.21(3)	2	QCDSF/UKQCD 07	C	●	★	●	clover (NP)

Lattice calculations of $f_+(0)$ and f_K/f_π



Lattice calculations of $f_+(0)$ and f_K/f_π



Lattice calculations of $f_+(0)$ and f_K/f_π

Direct determinations of

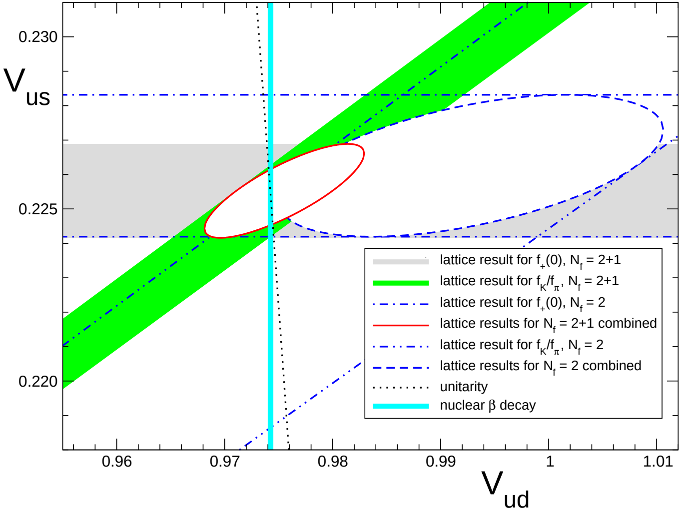
$f_+(0)$:

$$
\begin{aligned}
f_+(0) &= 0.9599(34)(^{+31}_{-47})(14) & (N_f = 2+1) \\
f_+(0) &= 0.956(6)(6) & (N_f = 2)
\end{aligned}
$$

f_K/f_π:

$$
\begin{aligned}
f_K/f_\pi &= 1.193(6) & (N_f = 2+1) \\
f_K/f_\pi &= 1.210(6)(17) & (N_f = 2)
\end{aligned}
$$

Unitarity test of the Standard Model



Analysis within the Standard Model

Unitarity + experiment: PDG (08)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \qquad \left[|V_{ub}| = 3.39(36) \cdot 10^{-3}\right]$$

Experiment: FLAVIAnet Kaon WG (08)

$$\begin{aligned} |V_{us} f_+(0)| &= 0.2163(5) \\ \left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| &= 0.2758(5) \end{aligned}$$

3 relations and 4 unknowns

determine anyone of V_{ud}, V_{us}, $f_+(0)$ or f_K/f_π

$\Rightarrow$ get the other three

Analysis within the Standard Model

collaboration	$\lvert V_{us}\rvert$	N_f	from
RBC/UKQCD 10	$0.2253(10)(^{+12}_{-8})$	2+1	$f_+(0)$
BMW 10	$0.2254(13)(11)$	2+1	f_K/f_π
MILC 09A	$0.2243(5)(^{+14}_{-11})$	2+1	f_K/f_π
HPQCD/UKQCD 08	$0.2260(5)(13)$	2+1	f_K/f_π
ETM 09	$0.2222(11)(31)$	2	f_K/f_π
ETM 09A	$0.2263(14)(15)$	2	$f_+(0)$

Other sources of information on V_{ud} and V_{us}

Super-allowed nuclear β decays

$|V_{ud}| = 0.97425(22)$ Hardy & Towner 08

$\Rightarrow |V_{us}| = 0.22544(95) \quad f_+(0) = 0.9608(46) \quad f_K/f_\pi = 1.1927(59)$

$\tau \to$ [hadrons($S = 1$)] $+ \nu$ decays

$|V_{us}| = 0.2165(26)_{\rm exp}(5)_{\rm th}$ Gamiz et al. 07

$\Rightarrow |V_{ud}| = 0.9763(6) \quad f_+(0) = 1.001(12) \quad f_K/f_\pi = 1.245(16)$

Problematic data: $\sum$ exclusive channels $\neq$ inclusive

Other sources of information on V_{ud} and V_{us}

Super-allowed nuclear β decays

$|V_{ud}| = 0.97425(22)$ Hardy & Towner 08

$\Rightarrow |V_{us}| = 0.22544(95) \quad f_+(0) = 0.9608(46) \quad f_K/f_\pi = 1.1927(59)$

$\tau \to [\text{hadrons}(S = 1)] + \nu$ decays + data on J_{em}

$|V_{us}| = 0.2208(39)$ Maltman 09

$\Rightarrow |V_{ud}| = 0.9753(9) \quad f_+(0) = 0.981(17) \quad f_K/f_\pi = 1.219(23)$

Problematic data: $\sum$ exclusive channels $\neq$ inclusive

Comparison between lattice and other determinations

	$\vert V_{us}\vert$	$\vert V_{ud}\vert$	$f_+(0)$	f_K/f_π
$N_f=2+1$	0.2253(10)	0.97427(23)	0.9598(41)	1.1924(54)
$N_f=2$	0.2251(18)	0.97433(42)	0.9604(75)	1.194(10)
β-dec.[1]	0.22544(95)	0.97425(22)	0.9595(46)	1.1919(57)
τ-dec.[2]	0.2165(26)	0.9763(6)	0.999(12)	1.244(16)
τ-dec.[3]	0.2208(39)	0.9753(9)	0.980(18)	1.218(23)

[1] Hardy & Towner
[2] Gamiz et al.
[3] Maltman