Interplay of flavour and CP symmetries

C. Hagedorn

EC 'Universe', TUM, Munich, Germany

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Outline

- lepton mixing: parametrization and data
- combination of flavour and CP symmetries
  - general idea
  - examples: $G_f = S_4$ and $G_f = \Delta(48)$
  - predictions for leptogenesis
- conclusions & outlook

Parametrization of lepton mixing

- charged lepton and (Majorana) neutrino mass terms

$$e^c_a\, m_{e,ab}\, l_b \quad \text{and} \quad \nu_a\, m_{\nu,ab}\, \nu_b$$

  cannot be diagonalized simultaneously

- going to the mass basis

$$U_e^\dagger m_e^\dagger m_e U_e = \text{diag}(m_e^2, m_\mu^2, m_\tau^2) \quad \text{and} \quad U_\nu^T m_\nu U_\nu = \text{diag}(m_1, m_2, m_3)$$

  leads to non-diagonal charged current interactions

$$\bar{l} \not{W}^- U_{PMNS}\, \nu \quad \text{with} \quad U_{PMNS} = U_e^\dagger U_\nu$$

Parametrization of lepton mixing

Parametrization ***(PDG)***

$$U_{PMNS} = \tilde{U}\ \mathrm{diag}(1, e^{i\alpha/2}, e^{i(\beta/2+\delta)})$$

with

$$\tilde{U} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

and $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

Jarlskog invariant J_{CP}

$$\begin{aligned} J_{CP} &= \mathrm{Im}\left[U_{PMNS,11}U^*_{PMNS,13}U^*_{PMNS,31}U_{PMNS,33}\right] \\ &= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta \end{aligned}$$

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Majorana invariant I_1

$$\begin{aligned} I_1 &= \mathrm{Im}\left[U^2_{PMNS,12}\left(U^*_{PMNS,11}\right)^2\right] \\ &= \sin^2\theta_{12}\cos^2\theta_{12}\cos^4\theta_{13}\sin\alpha \end{aligned}$$

Parametrization of lepton mixing

Parametrization (PDG)

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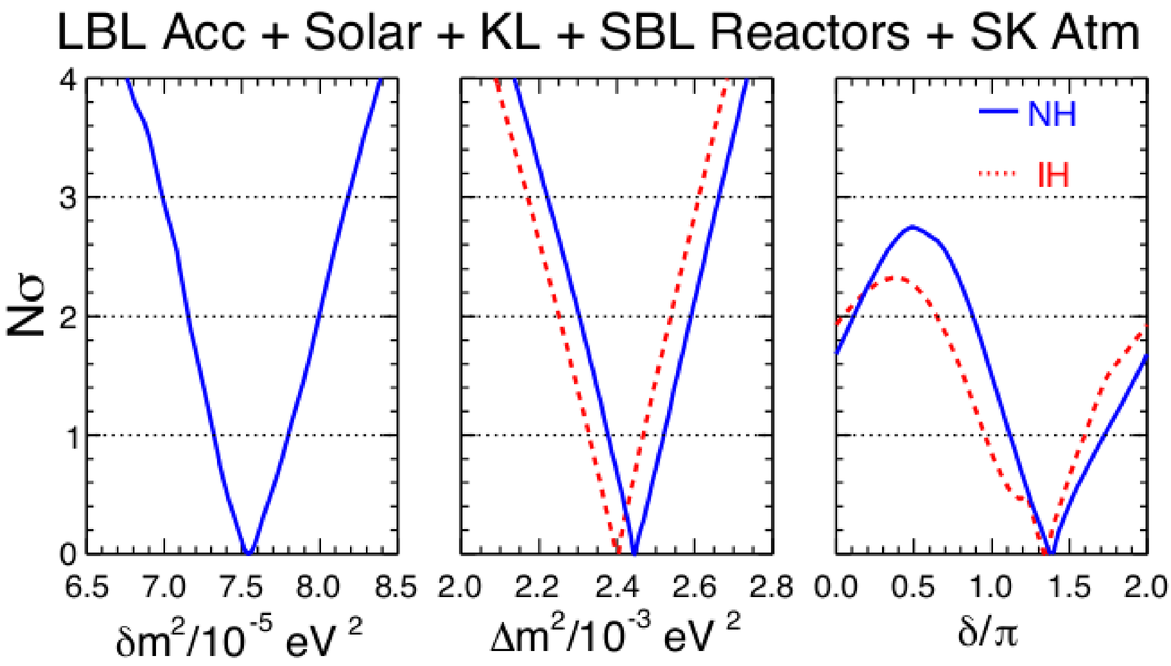
and $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$

Majorana invariant I_2

$$\begin{aligned} I_2 &= \mathrm{Im}\left[U^2_{PMNS,13}\left(U^*_{PMNS,11}\right)^2\right] \\ &= \sin^2\theta_{13}\cos^2\theta_{12}\cos^2\theta_{13}\sin\beta \end{aligned}$$

Data on lepton mixing

Latest global fits ***(Capozzi et al. ('13))***



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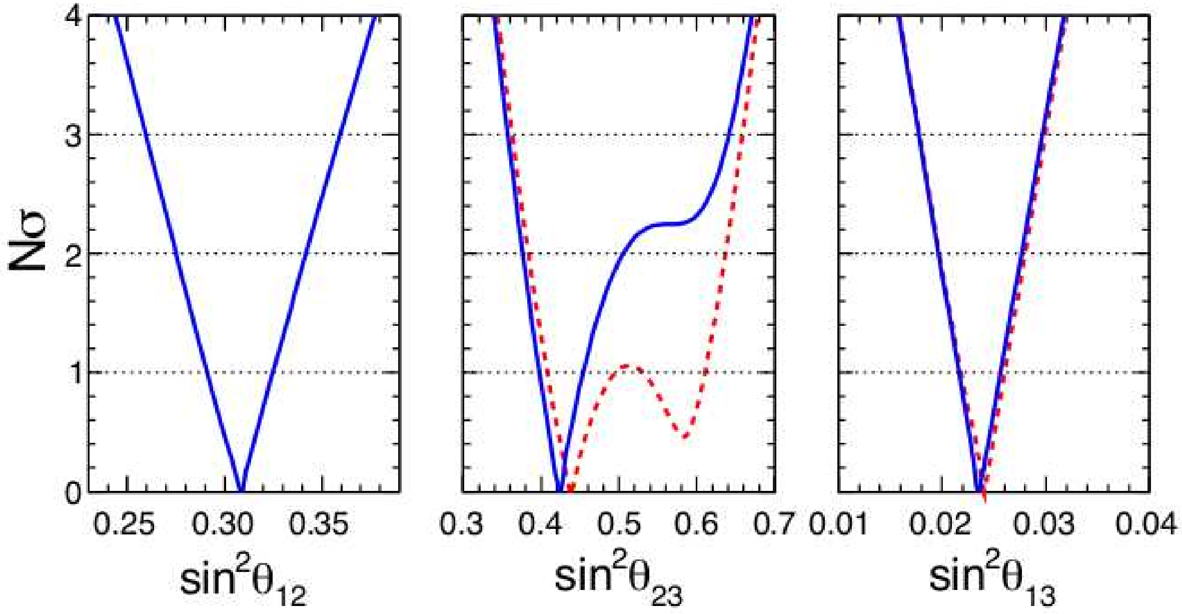

Data on lepton mixing

Latest global fits NH [IH] ***(Capozzi et al. ('13))***

best fit and $1\,\sigma$ error	$3\,\sigma$ range
$\sin^2\theta_{13} = 0.0234[9]^{+0.0022[1]}_{-0.0018[21]}$	$0.0177[8] \leq \sin^2\theta_{13} \leq 0.0297[300]$
$\sin^2\theta_{12} = 0.308^{+0.017}_{-0.017}$	$0.259 \leq \sin^2\theta_{12} \leq 0.359$
$\sin^2\theta_{23} = \begin{cases} 0.425[37]^{+0.029[59]}_{-0.027[9]} \\ [0.531 \leq \sin^2\theta_{23} \leq 0.610] \end{cases}$	$0.357[63] \leq \sin^2\theta_{23} \leq 0.641[59]$
$\delta = 1.39[5]\,\pi^{+0.33[24]\,\pi}_{-0.27[39]\,\pi}$	$0 \leq \delta \leq 2\,\pi$
$\alpha\,,\ \ \beta$	unconstrained

Data on lepton mixing

Latest global fits NH [IH] ***(Capozzi et al. ('13))***

$$||U_{PMNS}|| \approx \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.40[39] & 0.65 & 0.64[5] \\ 0.40[2] & 0.52 & 0.75[4] \end{pmatrix}$$

and no information on Majorana phases

$$\Downarrow$$

Mismatch in lepton flavour space is large!

General idea

- interpret this mismatch in lepton flavour space as mismatch of residual symmetries G_ν and G_e
- if we want to predict lepton mixing, we have to derive this mismatch
- let us assume that there is a symmetry, broken to G_ν and G_e
- this symmetry is in the following a combination of a

finite, discrete, non-abelian symmetry G_f and CP

(Feruglio et al. ('12,'13), Holthausen et al. ('12), Grimus/Rebelo ('95))

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General idea

Idea:

Relate lepton mixing to how G_f and CP are broken
Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f and CP

$$
\begin{array}{ccc}
 & G_f \text{ \& CP} & \\
\swarrow & & \searrow \\
\text{neutrinos} & & \text{charged leptons} \\
G_\nu & & G_e
\end{array}
$$

General idea

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G_f & CP

$\swarrow \qquad \searrow$

neutrinos
assume 3 generations
of Majorana neutrinos

charged leptons
distinguish 3 generations

General idea

Idea:

Relate lepton mixing to how G_f and CP are broken

Interpretation as mismatch of embedding of different subgroups G_ν and G_e into G_f and CP

$$\begin{array}{ccc} & G_f \text{ \& CP} & \\ \swarrow & & \searrow \\ \text{neutrinos} & & \text{charged leptons} \\ G_\nu = Z_2 \times \text{CP} & & G_e = Z_N \quad \text{with} \quad N \geq 3 \end{array}$$

An example: $\mu\tau$ reflection symmetry ***(Harrison/Scott ('02,'04), Grimus/Lavoura ('03))***

General idea

G_f & CP

$\swarrow \qquad \searrow$

neutrinos

$G_\nu = Z_2 \times$ CP

charged leptons

$G_e = Z_N$ with $N \geq 3$

Further requirements

- two/three non-trivial mixing angles $\Rightarrow$ irred 3-dim rep of G_f
- "maximize" predictability of approach

General idea

Definition of generalized CP transformation ***(see e.g. Branco et al. ('11))***

$$\phi_i \xrightarrow{\text{CP}} X_{ij}\phi_j^\star$$

with X is unitary and symmetric

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Definition of generalized CP transformation ***(see e.g. Branco et al. ('11))***

$$\phi_i \xrightarrow{\text{CP}} X_{ij}\phi_j^\star$$

with X is unitary and symmetric;
apply CP twice

$$\phi \xrightarrow{\text{CP}} X\phi^\star \xrightarrow{\text{CP}} XX^\star\phi = \phi$$

General idea

Definition of generalized CP transformation ***(see e.g. Branco et al. ('11))***

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with X is unitary and symmetric.
Realize direct product of $Z_2 \subset G_f$ and CP

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$$\phi_i \xrightarrow{\text{CP}} X_{ij}\phi_j^\star$$

with X is unitary and symmetric.

Realize direct product of $Z_2 \subset G_f$ and CP; Z generates Z_2

$$\phi \xrightarrow{\text{CP}} X\phi^\star \xrightarrow{Z_2} XZ^\star\phi^\star \quad \text{and} \quad \phi \xrightarrow{Z_2} Z\phi \xrightarrow{\text{CP}} ZX\phi^\star$$

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with X is unitary and symmetric.

Realize direct product of $Z_2 \subset G_f$ and CP; Z generates Z_2

$$XZ^\star - ZX = 0$$

General idea

- neutrino sector: $Z_2 \times$ CP preserved

  neutrino mass term $\nu_a \, m_{\nu,ab} \, \nu_b$

  is invariant under $\nu_\alpha \rightarrow Z_{\alpha\beta} \, \nu_\beta$

  is invariant under generalized CP transformation $\nu_\alpha \rightarrow X_{\alpha\beta} \, \nu^\star_\beta$

- charged lepton sector: Z_N, $N \geq 3$, preserved

  charged lepton mass term $e^c_a \, m_{e,ab} \, l_b$

  is invariant under $l_\alpha \rightarrow Q_{e,\alpha\beta} \, l_\beta$

General idea

- neutrino sector: $Z_2 \times$ CP preserved

$\rightarrow$ neutrino mass matrix m_ν fulfills

$$Z^T m_\nu Z = m_\nu \quad \text{and} \quad X m_\nu X = m_\nu^\star$$

- charged lepton sector: Z_N, $N \geq 3$, preserved

$\rightarrow$ charged lepton mass matrix m_e fulfills

$$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$

General idea

- neutrino sector: $Z_2 \times$ CP preserved and generated by ($\nu = \Omega_\nu \, \nu'$)

$$X = \Omega_\nu \Omega_\nu^T \quad \text{and} \quad Z = \Omega_\nu Z^{diag} \Omega_\nu^\dagger$$

$$Z^{diag} = \text{diag}\,(-1, 1, -1) \quad \text{and} \quad \Omega_\nu \quad \text{unitary}$$

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- neutrino sector: $Z_2 \times$ CP preserved

$\rightarrow$ neutrino mass matrix m_ν fulfills

$$Z^{diag}[\Omega_\nu^T m_\nu \Omega_\nu] Z^{diag} = [\Omega_\nu^T m_\nu \Omega_\nu] \quad \text{and} \quad [\Omega_\nu^T m_\nu \Omega_\nu] = [\Omega_\nu^T m_\nu \Omega_\nu]^\star$$

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- neutrino sector: $Z_2 \times$ CP preserved

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  $$\Omega_\nu(X, Z) R(\theta) K_\nu$$

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General idea

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$$\Omega_\nu(X,Z)R(\theta)K_\nu$$

- charged lepton sector: Z_N, $N \geq 3$, preserved and generated by

$$Q_e = \Omega_e Q_e^{diag} \Omega_e^\dagger \quad \text{with} \quad \Omega_e \quad \text{unitary}$$

$$Q_e^{diag} = \text{diag}\left(\omega_N^{n_e}, \omega_N^{n_\mu}, \omega_N^{n_\tau}\right)$$

$$\text{and} \quad n_e \neq n_\mu \neq n_\tau \quad \text{and} \quad \omega_N = e^{2\pi i/N}$$

General idea

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  $\rightarrow$ neutrino mass matrix m_ν is diagonalized by

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$$\Omega_e^\dagger(Q_e) m_e^\dagger m_e \Omega_e(Q_e) \quad \text{is diagonal}$$

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- conclusion: PMNS mixing matrix reads

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu R(\theta) K_\nu \quad \text{in} \quad \bar{l} \not{W}^- U_{PMNS} \, \nu$$

General idea

$$U_{PMNS} = \Omega_e^\dagger \Omega_\nu R(\theta) K_\nu$$

- 3 unphysical phases are removed by $\Omega_e \to \Omega_e K_e$
- U_{PMNS} contains one parameter θ
- permutations of rows and columns of U_{PMNS} possible

$$\Downarrow$$

Predictions:

Mixing angles and CP phases are predicted
in terms of one parameter θ only,
up to permutations of rows/columns

General idea: consistency conditions

We want to consistently combine G_f and the generalized CP transformation $\phi_i \xrightarrow{\text{CP}} X_{ij}\phi_j^\star$

$$\Downarrow$$

"closure" relations have to hold

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"closure" relations have to hold:

assume ϕ transforms as 3-dim rep of G_f, then

$$\phi \xrightarrow{\text{CP}} X\phi^\star \xrightarrow{G_f} XA^\star\phi^\star \xrightarrow{\text{CP}} XA^\star X^\star\phi = (X^\star AX)^\star\,\phi$$

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$(X^\star A X)^\star = A'$ with in general $A \neq A'$ and $A,\ A' \in G_f$

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compare to relation for having direct product of Z_2 and CP

$$X Z^\star - Z X = 0$$

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$$(X^\star Z X)^\star = Z$$

General idea: consistency conditions

- fulfilling these conditions ensures a consistent theory, but can lead to enlarged symmetry group
- additional requirement in order not to change representation content of G_f ***(Chen et al. ('14))***:

  all representations transform into
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  [mathematically: mapping induced via X has to be 'class-inverting' automorphism ($A' \sim A^{-1}$)]

General idea: consistency conditions

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- if not fulfilled or not possible to fulfill for G_f
  $\Rightarrow$ constraints on representations
  [S_4 fulfilled;
  $\Delta(48)$ not fulfilled in general, only for certain representations]

Study of S_4 and CP

Generators in rep. $\mathbf{3}'$: $\left(\omega = e^{2\pi i/3}\right)$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \; , \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \; , \quad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which fulfill

$$S^2 = \mathbb{1} \; , \;\; T^3 = \mathbb{1} \; , \;\; U^2 = \mathbb{1} \; ,$$
$$(ST)^3 = \mathbb{1} \; , \;\; (SU)^2 = \mathbb{1} \; , \;\; (TU)^2 = \mathbb{1} \; , \;\; (STU)^4 = \mathbb{1}$$

Study of S_4 and CP

A transformation X in rep. $\mathbf{3}'$ for $Z = S$ is

$$X_{\mathbf{3}'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which fulfills

$$XX^\dagger = XX^\star = \mathbb{1}$$

$$(X^\star A X)^\star = A' \;\;,\quad XZ^\star - ZX = 0$$

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Residual symmetry G_e is generated by T.

Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and $X_{\mathbf{3}'}$

(Harrison/Scott ('02,'04), Grimus/Lavoura ('03), Feruglio et al. ('12,'13))

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2\cos\theta & \sqrt{2} & 2\sin\theta \\ -\cos\theta + i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta - i\sqrt{3}\cos\theta \\ -\cos\theta - i\sqrt{3}\sin\theta & \sqrt{2} & -\sin\theta + i\sqrt{3}\cos\theta \end{pmatrix} K_\nu$$

$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\theta \;, \quad \sin^2\theta_{12} = \frac{1}{2+\cos 2\theta} \;, \quad \sin^2\theta_{23} = \frac{1}{2}$$

and

$$|\sin\delta| = 1 \;\;, \quad |J_{CP}| = \frac{|\sin 2\theta|}{6\sqrt{3}} \;\;, \quad \sin\alpha = 0 \;\;, \quad \sin\beta = 0$$

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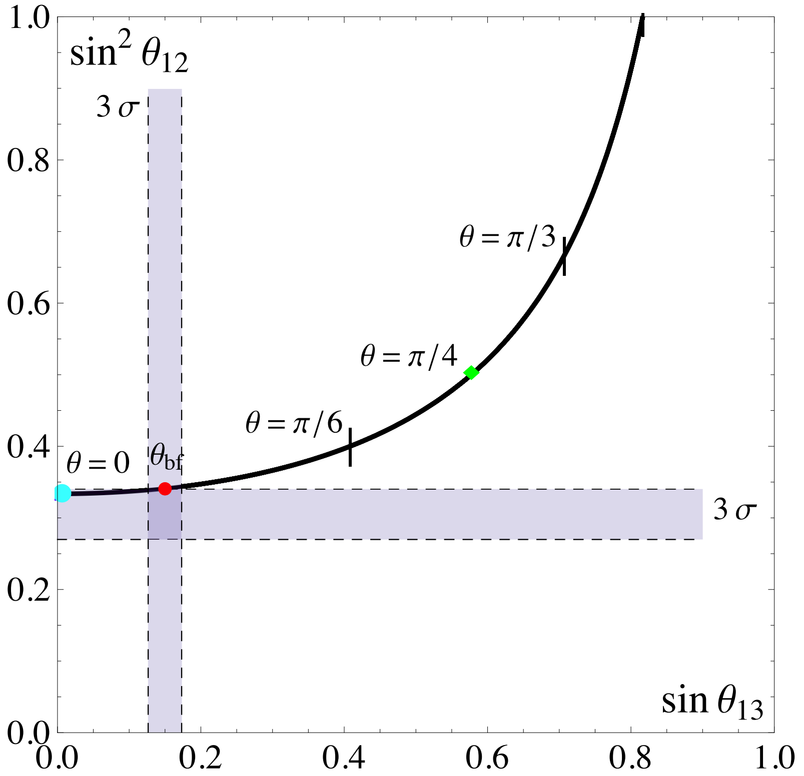
$$\sin^2\theta_{13} \approx 0.023\ , \quad \sin^2\theta_{12} \approx 0.341\ , \quad \sin^2\theta_{23} = \frac{1}{2}$$

and

$$|\sin\delta| = 1\ \ , \quad |J_{CP}| \approx 0.0348\ \ , \quad \sin\alpha = 0\ \ , \quad \sin\beta = 0 \quad \text{for} \quad \theta \approx 0.185$$

Study of S_4 and CP

Maximal θ_{23} and δ from $G_e = Z_3$, $Z = S$ and $X_{\mathbf{3}'}$ ***(Feruglio et al. ('12,'13))***



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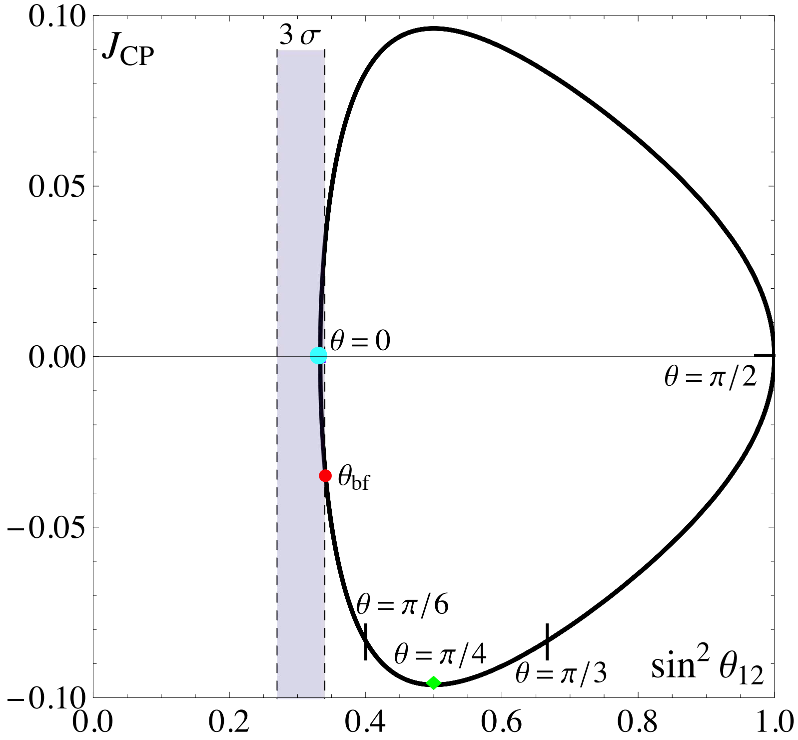

Study of $\boldsymbol{S_4}$ and $\boldsymbol{CP}$

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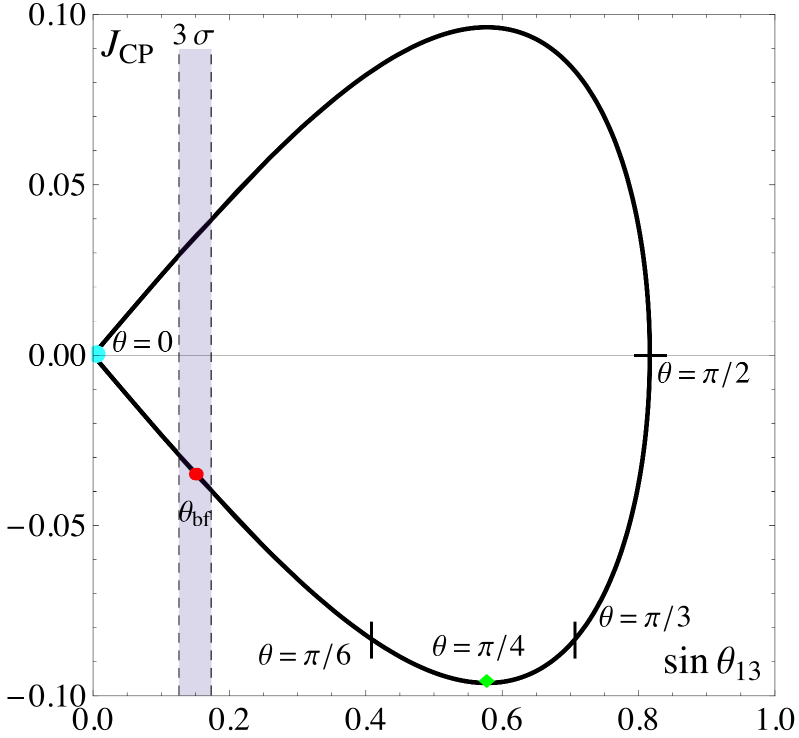

Study of $\mathbf{\Delta(48)}$ and $\boldsymbol{CP}$

Generators in rep. $\mathbf{3}$: $\left(\omega = e^{2\pi i/3}\right)$

(Miller et al. ('16), Fairbairn et al. ('64), Luhn et al. ('07))

$$a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \;, \quad c = \frac{1}{3} \begin{pmatrix} 1 & 1-\sqrt{3} & 1+\sqrt{3} \\ 1+\sqrt{3} & 1 & 1-\sqrt{3} \\ 1-\sqrt{3} & 1+\sqrt{3} & 1 \end{pmatrix} \;, \quad d = a^{-1} c a$$

which satisfy

$$a^3 = \mathbb{1} \;, \;\; c^4 = \mathbb{1} \;, \;\; d^4 = \mathbb{1} \;,$$
$$cd = dc \;, \;\; a c a^{-1} = c^{-1} d^{-1}$$

Study of $\boldsymbol{\Delta(48)}$ and $\boldsymbol{CP}$

A transformation X in rep. $\mathbf{3}$ for $Z = c^2$ is ***(Ding/Zhou ('13))***

$$X_{\mathbf{3}} = d \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which fulfills

$$X X^\dagger = X X^\star = \mathbb{1}$$

$$(X^\star A X)^\star = A' \; , \quad X Z^\star - Z X = 0$$

Residual symmetry G_e is generated by a.

Study of $\boldsymbol{\Delta}(48)$ and $\boldsymbol{CP}$

Angles and phases from $G_e = Z_3$, $Z = c^2$ and $X_{\mathbf{3}}$ ***(Ding/Zhou ('13))***

$$||U_{PMNS}|| = \frac{1}{\sqrt{6}} \begin{pmatrix} \frac{1}{\sqrt{2}}\sqrt{4-(\sqrt{2}+\sqrt{6})\cos 2\theta} & \sqrt{2} & \frac{1}{\sqrt{2}}\sqrt{4+(\sqrt{2}+\sqrt{6})\cos 2\theta} \\ \frac{1}{\sqrt{2}}\sqrt{4+(-\sqrt{2}+\sqrt{6})\cos 2\theta} & \sqrt{2} & \frac{1}{\sqrt{2}}\sqrt{4-(-\sqrt{2}+\sqrt{6})\cos 2\theta} \\ \sqrt{2+\sqrt{2}\cos 2\theta} & \sqrt{2} & \sqrt{2-\sqrt{2}\cos 2\theta} \end{pmatrix}$$

$$\sin^2\theta_{13} = \frac{1}{12}\left(4+(\sqrt{2}+\sqrt{6})\cos 2\theta\right) \;, \quad \sin^2\theta_{12} = \frac{4}{8-(\sqrt{2}+\sqrt{6})\cos 2\theta} \;,$$

$$\sin^2\theta_{23} = \frac{1}{2}\left(1+\frac{\sqrt{6}(\sqrt{3}-1)\cos 2\theta}{8-(\sqrt{2}+\sqrt{6})\cos 2\theta}\right) \;, \quad |J_{CP}| = \frac{|\sin 2\theta|}{6\sqrt{3}}$$

Study of $\mathbf{\Delta(48)}$ and $\boldsymbol{CP}$

Angles and phases from $G_e = Z_3$, $Z = c^2$ and $X_{\mathbf{3}}$ ***(Ding/Zhou ('13))***

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$$|\sin\alpha| = \left| \frac{1+\sqrt{3}-2\sqrt{2}\cos 2\theta + (-1+\sqrt{3})\sin 2\theta}{-4+(\sqrt{2}+\sqrt{6})\cos 2\theta} \right| ,$$

$$|\sin\beta| = \left| \frac{2\sin 2\theta}{-4+(2+\sqrt{3})\cos^2 2\theta} \right|$$

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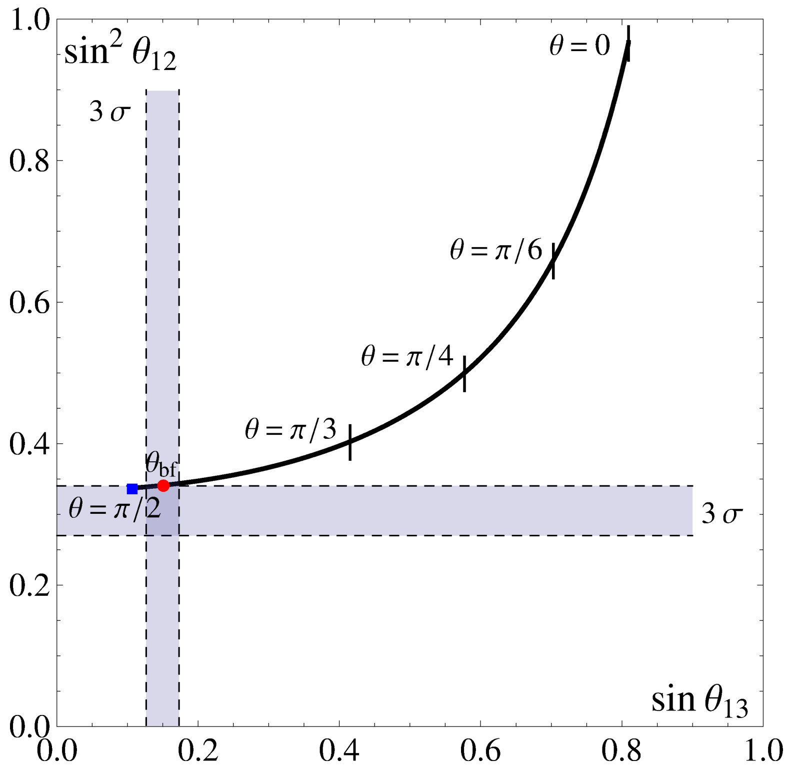
$$\sin^2\theta_{13} \approx 0.023\ , \quad \sin^2\theta_{12} \approx 0.341\ , \quad \sin^2\theta_{23} \approx 0.426\ , \quad |J_{CP}| \approx 0.0254\ ,$$

and

$$|\sin\delta| \approx 0.735\ , \quad |\sin\alpha| \approx 0.732\ , \quad |\sin\beta| \approx 1 \quad \text{for} \quad \theta \approx 1.437$$

Study of $\mathbf{\Delta(48)}$ and $\boldsymbol{CP}$

Angles and phases from $G_e = Z_3$, $Z = c^2$ and $X_\mathbf{3}$ ***(Ding/Zhou ('13))***



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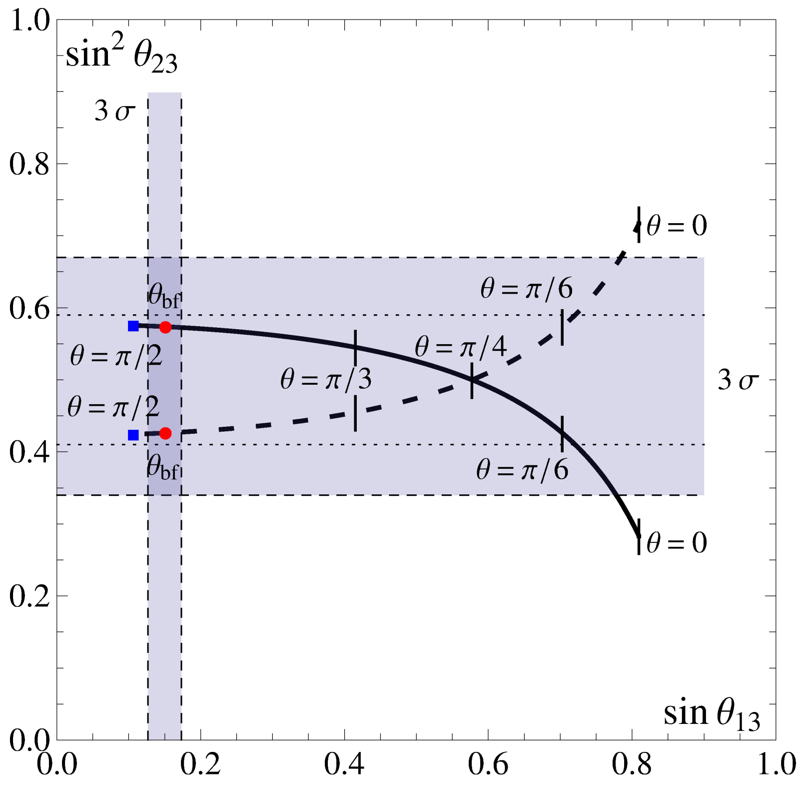

Study of $\mathbf{\Delta(48)}$ and $\boldsymbol{CP}$

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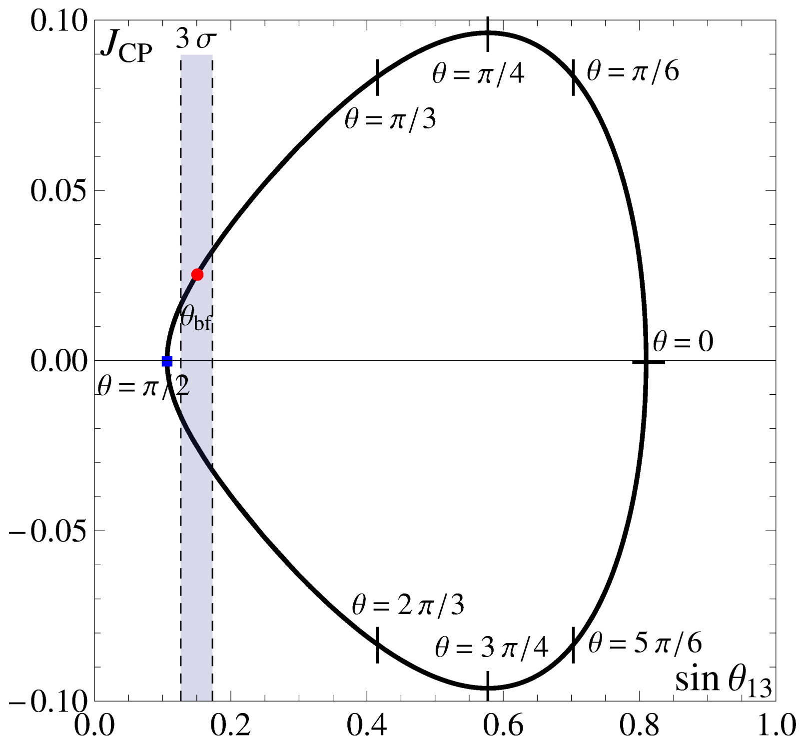
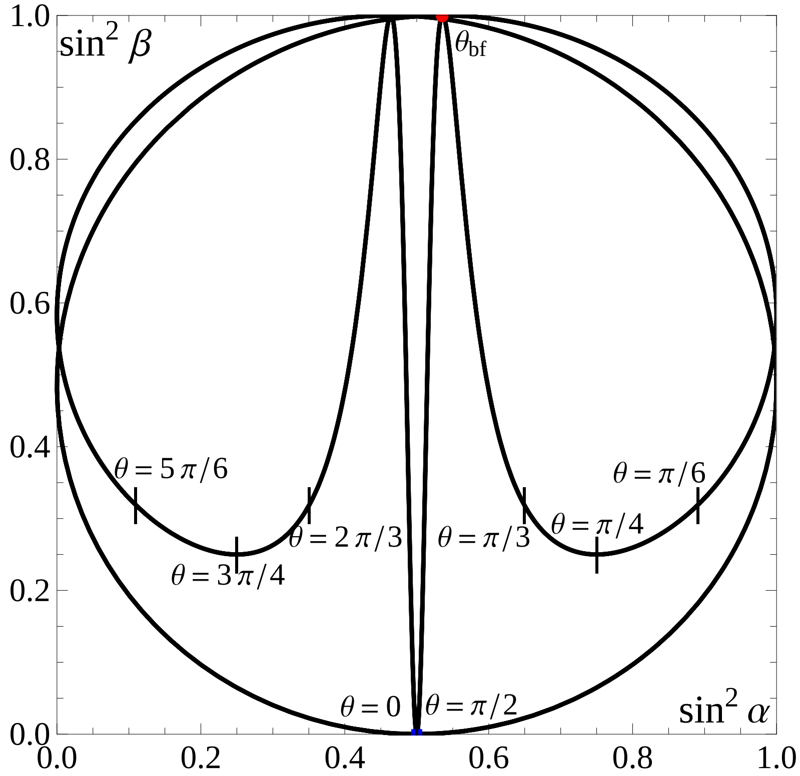


Study of $\Delta(48)$ and CP

Angles and phases from $G_e = Z_3$, $Z = c^2$ and $X_{\mathbf{3}}$ ***(Ding/Zhou ('13))***



Basics of leptogenesis

- baryon asymmetry of the Universe is measured well

$$Y_B = \left. \frac{n_B - n_{\bar{B}}}{s} \right|_0 = (8.77 \pm 0.24) \times 10^{-11}$$ ***(WMAP ('08), Planck ('13))***

- this asymmetry can be explained by decay of heavy right-handed neutrinos ***(Fukugita/Yanagida ('86))***
- the three Sakharov conditions are fulfilled ***(Sakharov ('67))***

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  - C and CP violation:
    Yukawa couplings of right-handed neutrinos provide source of CP violation

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- this asymmetry can be explained by decay of heavy right-handed neutrinos ***(Fukugita/Yanagida ('86))***
- the three Sakharov conditions are fulfilled ***(Sakharov ('67))***
  - departure from thermal equilibrium:
    Yukawa interactions of right-handed neutrinos have slow enough rate $\Gamma < H$

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- the three Sakharov conditions are fulfilled ***(Sakharov ('67))***
  - baryon number violation:
    Majorana masses violate lepton number so that lepton asymmetry is generated which is partially converted into baryon asymmetry via sphaleron processes

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- this asymmetry can be explained by decay of heavy right-handed neutrinos ***(Fukugita/Yanagida ('86))***
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- simplest scenario:

  thermal leptogenesis in which asymmetry stems from N_1 decay (with no flavour effects)

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- this asymmetry can be explained by decay of heavy right-handed neutrinos ***(Fukugita/Yanagida ('86))***
- the three Sakharov conditions are fulfilled ***(Sakharov ('67))***
- simplest scenario:

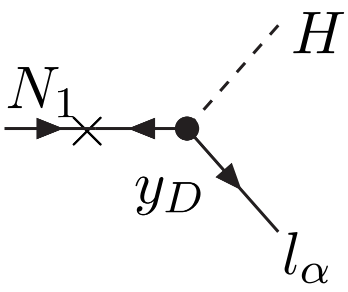
$Y_B \sim 10^{-3}\,\epsilon\,\eta$ with ϵ CP asymmetry , η washout factor

Basics of leptogenesis

- CP asymmetry ϵ

$$\epsilon_{\alpha\alpha} = \frac{\Gamma(N_1 \to H l_\alpha) - \Gamma(N_1 \to H^\star \bar{l}_\alpha)}{\Gamma(N_1 \to H l) + \Gamma(N_1 \to H^\star \bar{l})}$$

- diagrammatically: the CP asymmetry arises from interference of tree-level diagram



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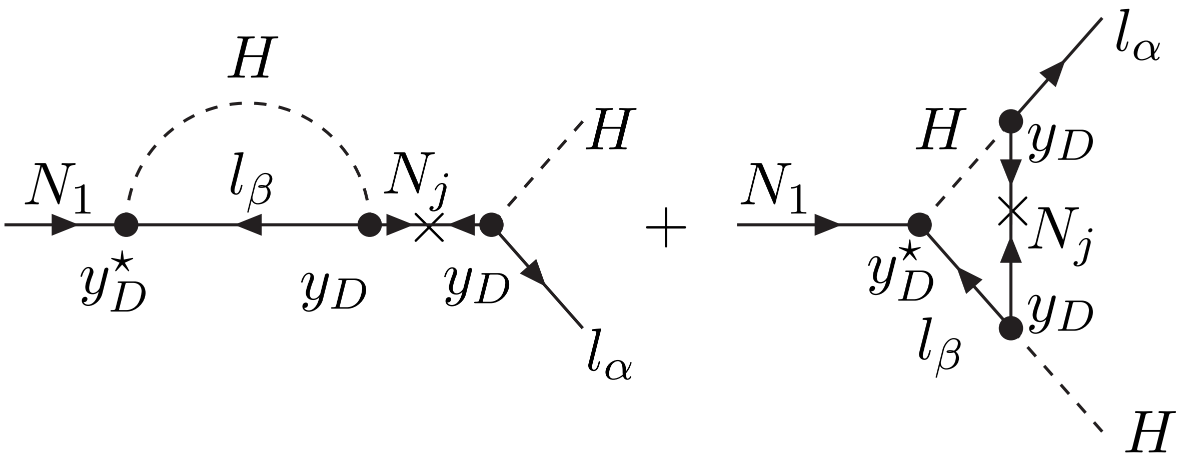
- diagrammatically: the CP asymmetry arises from interference of tree-level diagram and one-loop diagrams



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- computation of ϵ in case of unflavoured leptogenesis

$$\epsilon = \frac{1}{8\pi} \sum_{j \neq 1} \frac{\mathrm{Im}\left((\hat{Y}_D \hat{Y}_D^\dagger)^2_{j1} \right)}{(\hat{Y}_D \hat{Y}_D^\dagger)_{11}} f(x_j)$$

with $\hat{Y}_D = U_R^\dagger Y_D$ and $U_R^\dagger M_R U_R^\star = \mathrm{diag}(M_1, M_2, M_3)$

Leptogenesis in flavour models

- leptogenesis has been studied in several models with A_4 or S_4 flavour symmetry

  (Jenkins/Manohar ('08), H et al. ('09), Bertuzzo et al. ('09), Aristizabal Sierra et al. ('09))

- $G_f \to G_e$ in charged lepton sector and m_e is diagonal
- $G_f \to G_\nu = Z_2(\times Z_2)$ in neutrino sector and M_R encodes mixing, while Y_D has trivial flavour structure

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- for generations in $\mathbf{3}$ and Y_D invariant under G_f ϵ vanishes

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- if residual G_ν is broken at level ε,

$$\epsilon \propto \varepsilon^2 \quad \text{for unflavoured leptogenesis}$$

$$[\epsilon \propto \varepsilon \quad \text{for flavoured leptogenesis}]$$

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$$\epsilon \propto \varepsilon^2 \quad \text{for unflavoured leptogenesis}$$

- if CP is also a symmetry of the theory, constraints on phases and e.g. on sign of ϵ are expected

Leptogenesis in models with flavour and CP

Consider the following scenario

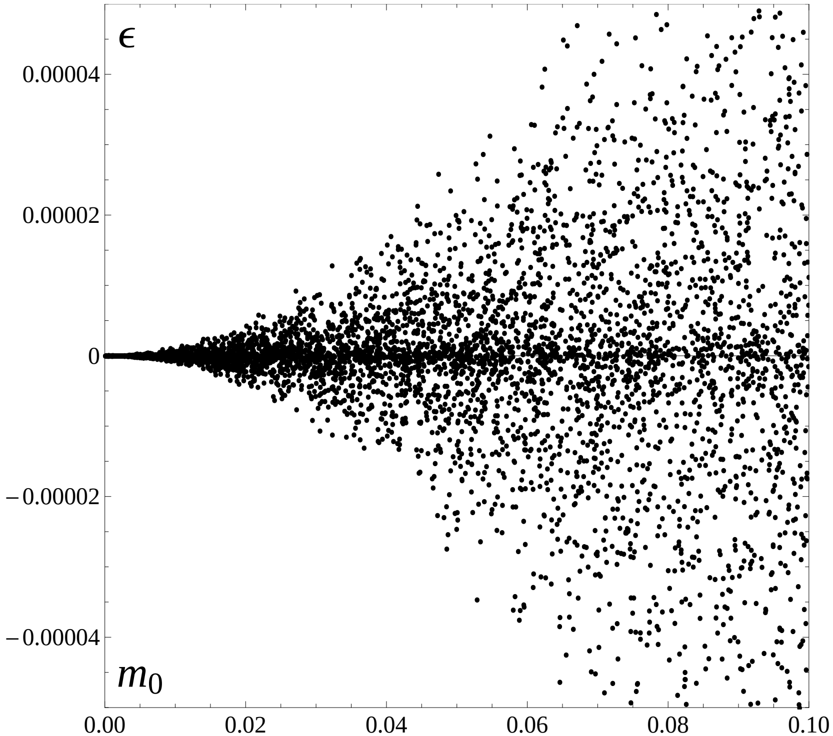
- G_f & CP $\to G_e$ in charged lepton sector and m_e is diagonal
- G_f & CP $\to G_\nu = Z_2 \times$ CP in neutrino sector and M_R encodes mixing, while Y_D has trivial flavour structure
- assume small breaking in Y_D at level ε which is real; e.g. for our example $G_f = S_4$

$$Y_D^0 + \delta Y_D = \begin{pmatrix} y_0 + a\,\varepsilon & 0 & 0 \\ 0 & 0 & y_0 + b\,\varepsilon \\ 0 & y_0 + c\,\varepsilon & 0 \end{pmatrix}$$

- fit of reactor mixing angle requires $0.16 \lesssim \theta \lesssim 0.21$

Leptogenesis in models with flavour and $\boldsymbol{CP}$

Result for ϵ from N_1 decays vs lightest neutrino mass m_0
$\varepsilon = \lambda^4 \approx 1.6 \times 10^{-3}$; normal ordering and best fit values of Δm_{ij}^2
(Capozzi et al. ('13)) assumed



Leptogenesis in models with flavour and $\boldsymbol{CP}$

Consider the following scenario

- G_f & CP $\rightarrow G_e$ in charged lepton sector and m_e is diagonal
- G_f & CP $\rightarrow G_\nu = Z_2 \times$ CP in neutrino sector and M_R encodes mixing, while Y_D has trivial flavour structure
- assume small breaking in Y_D at level ε which is real; e.g. for our example $G_f = \Delta(48)$

$$Y_D^0 + \delta Y_D = \begin{pmatrix} y_0 + (s+2t)\,\varepsilon & 0 & 0 \\ 0 & y_0 + (s - t - \sqrt{3}u)\,\varepsilon & 0 \\ 0 & 0 & y_0 + (s - t + \sqrt{3}u)\,\varepsilon \end{pmatrix}$$

- fit of reactor mixing angle constrains θ: $1.40 \lesssim \theta \lesssim 1.48$

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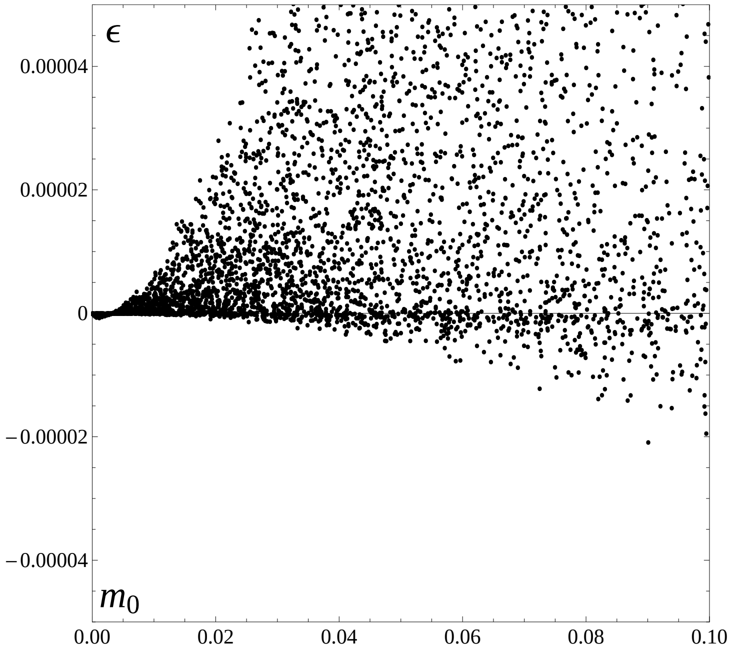

Notice: phases in K_ν can change sign of ϵ

Leptogenesis in models with flavour and $\boldsymbol{CP}$

We can understand this behaviour:

Look at $\mathrm{Im}\left((\hat{Y}_D \hat{Y}_D^\dagger)^2_{j1}\right)$; for $j = 2$

$$2\,(-1)^{k_1}\,y_0^2\,\varepsilon^2\,\left(-t^2 - 2tu + u^2 - \sqrt{2}(t^2+u^2)\cos 2\theta + (t^2 - 2tu - u^2)\sin 2\theta\right) + \mathcal{O}(\varepsilon^3)$$

and for $j = 3$

$$4\,(-1)^{k_2}\,y_0^2\,\varepsilon^2\,\left(-t^2 + u^2\right)\sin 2\theta + \mathcal{O}(\varepsilon^3)$$

Leptogenesis in models with flavour and CP

We can understand this behaviour:

Now expand for $\theta = \pi/2 + \kappa$ up to κ; for $j = 2$

$$2\,(-1)^{k_1} y_0^2\,\varepsilon^2\,\left(t^2(-1+\sqrt{2}-2\kappa) + 2\,t\,u\,(-1+2\kappa) + u^2(1+\sqrt{2}+2\kappa)\right) + \mathcal{O}(\kappa^2)$$

and for $j = 3$

$$8\,(-1)^{k_2}\,y_0^2\,\varepsilon^2(t-u)(t+u)\,\kappa$$

Leptogenesis in models with flavour and $\boldsymbol{CP}$

We can understand this behaviour:

Now expand for $\theta = \pi/2 + \kappa$ up to κ; for $j = 2$

$$2\,(-1)^{k_1}\,y_0^2\,\varepsilon^2\,\left(t\,\sqrt{-1+\sqrt{2}-2\kappa} - u\,\sqrt{1+\sqrt{2}+2\kappa}\right)^2 + \mathcal{O}(\kappa^2)$$

and for $j = 3$

suppressed by κ

The loop function $f(x_j)$ acts as weighting factor of the different contributions.

Conclusions & outlook

- approach with flavour and CP symmetry strongly constrains lepton mixing
- results for $G_f = S_4$ or $G_f = \Delta(48)$ are encouraging
- leptogenesis can be studied in this approach

Conclusions & outlook

- continue study of different groups G_f ($\Delta(3n^2)$ and $\Delta(6n^2)$) and CP:
  new mixing patterns, consistent definition of CP, ...
- explore more phenomena which involve CP phases:
  $0\nu\beta\beta$, electric dipole moments, phases of soft supersymmetry breaking terms, CKM phase, ...

Thank you for your attention.