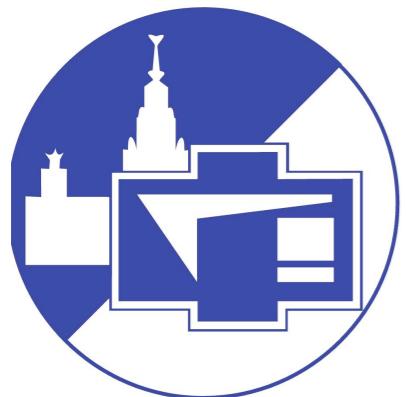


# Testing Lorentz Invariance of the Universe

Université libre Bruxelles

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Institute for Nuclear Research RAS)



in collaboration with Benjamin Audren, Diego Blas,  
Julien Lesgouques, Sergey Sibiryakov

JCAP 1210 (2012) 057 [[arXiv:1209.0464](https://arxiv.org/abs/1209.0464)]  
[arXiv:1409.xxxx](https://arxiv.org/abs/1409.xxxx)

# Problems in theoretical physics

Field theory in UV:

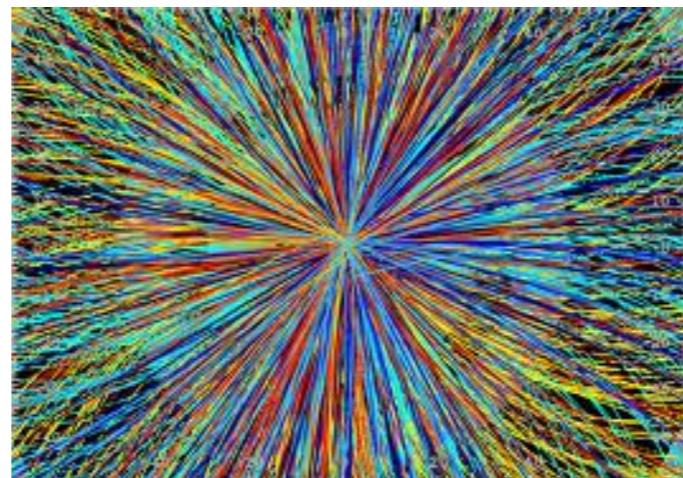
- 1) Quantum gravity
- 2) GUT
- 3) SUSY/strings
- 4) ...

$$\begin{aligned} \langle \Psi_{n_x, n_y, n_z} \rangle &= (n_x + n_y + n_z) \hbar \omega |\Psi_{n_x, n_y, n_z}\rangle \quad V(x) = e^{-\frac{m\omega}{2}} \\ \langle \Psi(t) \rangle &= U(t, t_0) |\Psi_0(t_0)\rangle \quad i\hbar \frac{d}{dt} A(t) = [A_H(t), H_H(t)] + i\hbar \left( \frac{d}{dt} A_S(t) \right) \\ U(t, t') &= U(t, t') U(t', t'') \quad \frac{d}{dt} P_H(t) = -\frac{\partial V}{\partial x}(x_H, t) \quad \frac{d}{dt} x_H(t) = \frac{1}{m} p_H(t) \\ \varphi_n(x) &= \left[ \frac{1}{2^n n!} \left( \frac{\hbar}{m\omega} \right)^n \right] \left( \frac{m\omega}{\pi\hbar} \right)^{\frac{n}{2}} \left[ \frac{m\omega}{\hbar} x - \frac{d}{dx} \right]^n e^{-\frac{1}{2} \frac{m\omega}{\hbar} x^2} \quad p_n(z) = \langle z \rangle \\ &\quad E = mc^2 \quad \frac{d^2 z}{dt^2} = -2 \frac{d}{dx} \end{aligned}$$

More theorists!

Particle physics:

- 1) Neutrino masses
- 2) Hierarchy problem
- 3) BSM
- 4) ...



LHC new runs  
other experiments

Cosmology:

- 1) Dark Matter
- 2) Dark Energy
- 3) Inflation
- 4) Baryon asymmetry



Euclid, LiteBird..

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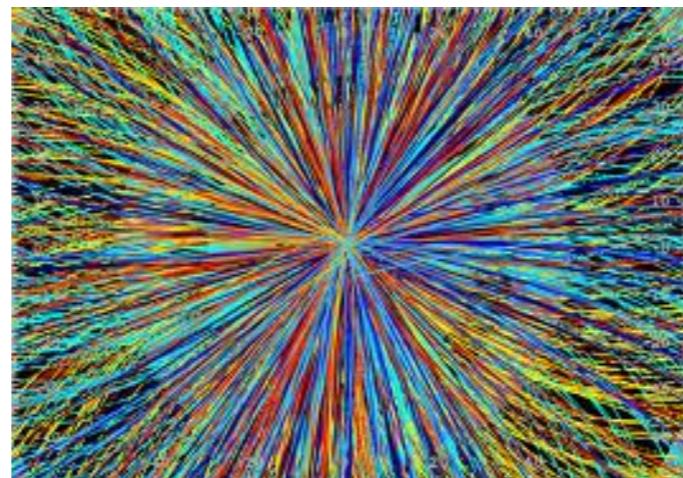
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# Why Lorentz Invariance?



All current data are compatible with  
the  $\Lambda CDM$  model

(assumes Lorentz Invariance as a fundamental property  
of Nature)



GR is a unique LI theory of gravity with EOM  
of the second order



Very precise tests and tight bounds  
on LI in the SM sector

$$< 10^{-20}$$

# Why not Lorentz Invariance?



For other sectors  
bounds are milder or even don't exist!

Gravity

$< 10^{-7}$

Dark Matter

???

Dark Energy

$< 10^{-2}?$

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Addressing dark energy and inflation:  
massive gravity, ghost condensate,...

# Why not Lorentz Invariance?



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Given the key role played by LI in  
modelling Nature,  
it is essential to test it to the best  
possible accuracy in all the sectors



# Outline of my talk:



**Lorentz-violating gravity: from UV to IR**



**LV in dark matter**



**Physical effects of LV on cosmological  
observables**



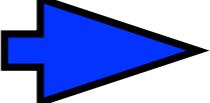
**Current constraints on LV  
in gravity and dark matter**

# Anisotropic scaling: idea

Lifshitz, 1941

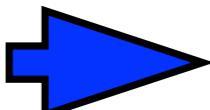
$$S = \int dt d^3x (\dot{\varphi}^2 - \varphi(-\Delta)^z \varphi - V(\varphi))$$

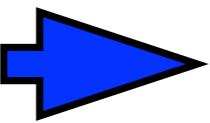
$$\mathbf{x} \mapsto b^{-1}\mathbf{x}, \quad t \mapsto b^{-z}t, \quad \varphi \mapsto b^{(3-z)/2}\varphi$$

$z = 3$    $\varphi$  is dimensionless

The most general renormalizable action:

$$S = \int dt d^3x \left[ \dot{\varphi}^2 + (A_1(\varphi)\Delta^3\varphi + A_2(\varphi)(\partial\varphi)^6 + \dots) G \sim \frac{1}{|\vec{k}|^6} \right. \\ \left. + (B_1(\varphi)\Delta^2\varphi + B_2(\varphi)(\partial\varphi)^4 + \dots) \right. \\ \left. + C^2(\varphi)\varphi\Delta\varphi - V_0(\varphi) \right]$$

Second order in time derivatives  no ghosts

$C^2(0) = c^2$   linear dispersion relation  
 $\omega^2 = c^2 p^2$  in IR

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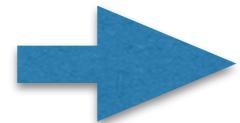
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linear dispersion relation  
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# Gravity with anisotropic scaling

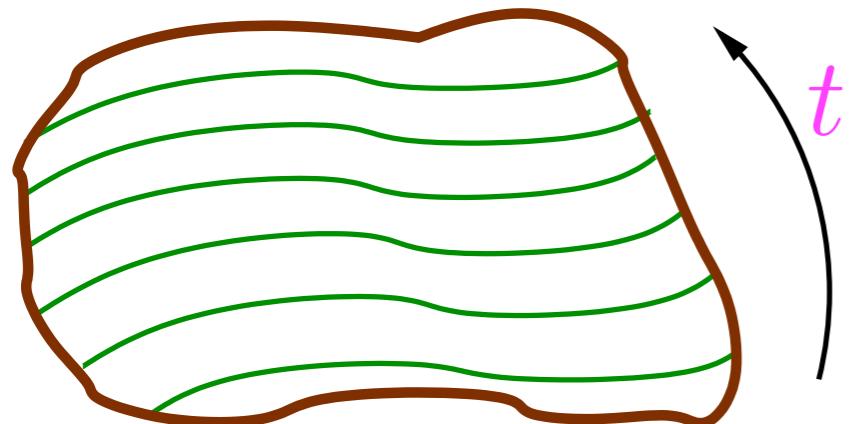
Let's do something similar for gravity

I) Lorentz group is a gauge group, thus its breaking gives



new degree of freedom

II) Geometrically, preferred time amounts to splitting coordinates in space and time, in other words, equipping space-time manifold with space-like foliations



Excitation of the foliation =  
new scalar dof.

ADM decomposition of the metric (in GR -- a gauge choice)

$$ds^2 = (N^2 - N_i N^i) dt^2 - 2N_i dt dx^i - \gamma_{ij} dx^i dx^j$$

# Horava gravity

+ Blas,Pujolas,Sibiryakov'09

$$S = \frac{M_P^2}{2} \int d^3x dt \sqrt{\gamma} N \left( K_{ij} K^{ij} - \zeta K^2 - \mathcal{V} \right)$$

$$K_{ij} = \frac{\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i}{2N}$$

$$\begin{aligned} \mathcal{V}_I = & -\xi R + M_*^{-2} (A_1 \Delta R + A_2 R_{ij} R^{ij} + \dots) \\ & + M_*^{-4} (B_1 \Delta^2 R + B_2 R_{ij} R^{jk} R_k^i + \dots) \end{aligned}$$

$$\begin{aligned} \mathcal{V}_{II} = & \mathcal{V}_I - \alpha a_i a^i \\ & + M_*^{-2} (C_1 a_i \Delta a^i + C_2 (a_i a^i)^2 + C_3 a_i a_j R^{ij} + \dots) \\ & + M_*^{-4} (D_1 a_i \Delta^2 a^1 + D_2 (a_i a^i)^3 + D_3 a_i a^i a_j a_k R^{jk} + \dots) \end{aligned}$$

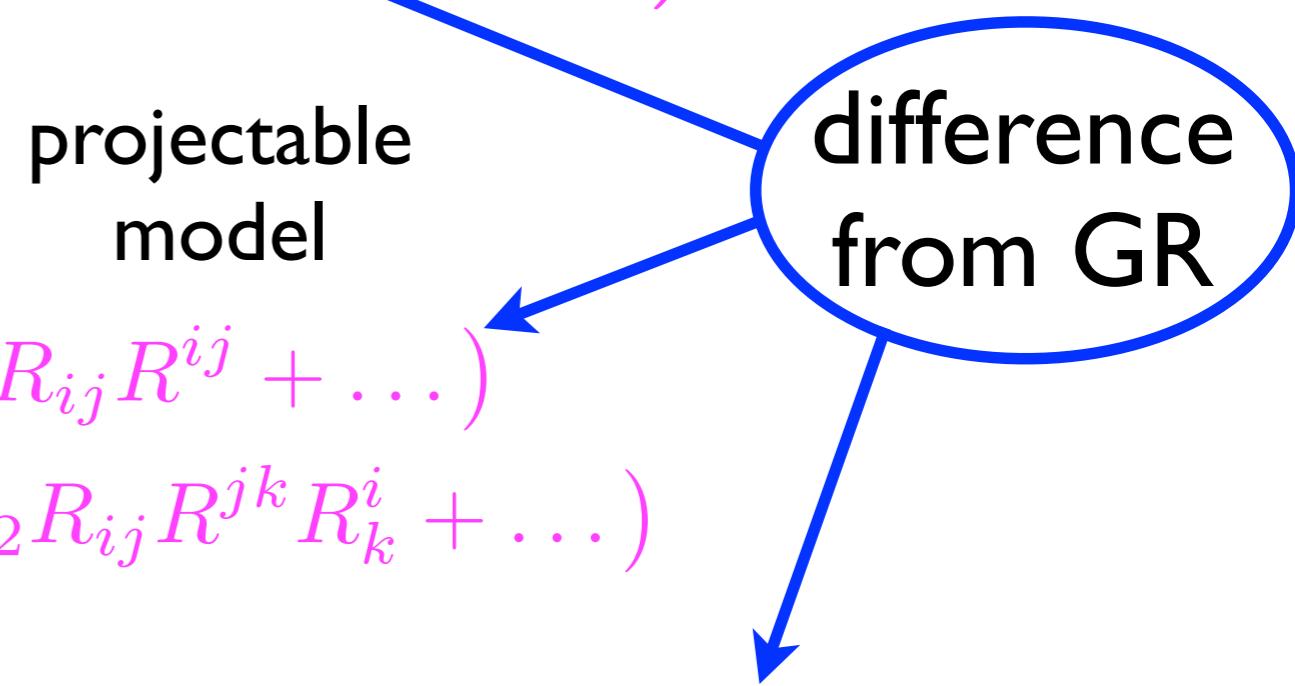
$R_{ij}$  -- 3d Ricci tensor

$$a_i \equiv N^{-1} \partial_i N$$

ADM decomposition

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Think of the splitting as physical



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**Gradient inst.  
Strong coupling**

$$\begin{aligned} \mathcal{V}_I &= -\frac{1}{M_*^4} (A_0 R + A_2 R_{ij} R^{ij} + \dots) \\ &\quad + M_*^{-4} (B_1 \Delta^2 R + B_2 R_{ij} R^{jk} R_k^i + \dots) \\ \mathcal{V}_{II} &= \mathcal{V}_I - \alpha a_i a^i \\ &\quad + M_*^{-2} (C_1 a_i \Delta a^i + C_2 (a_i a^i)^2 + C_3 a_i a_j R^{ij} + \dots) \\ &\quad + M_*^{-4} (D_1 a_i \Delta^2 a^i + D_2 (a_i a^i)^3 + D_3 a_i a^i a_j a_k R^{jk} + \dots) \end{aligned}$$

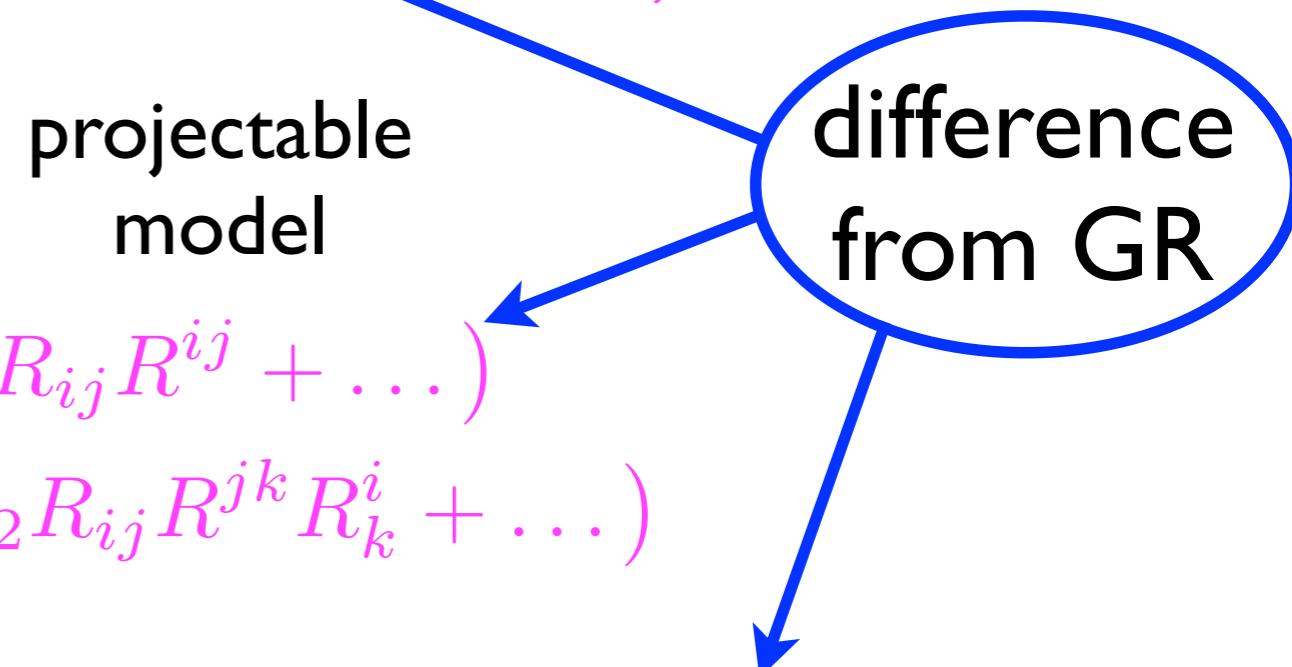
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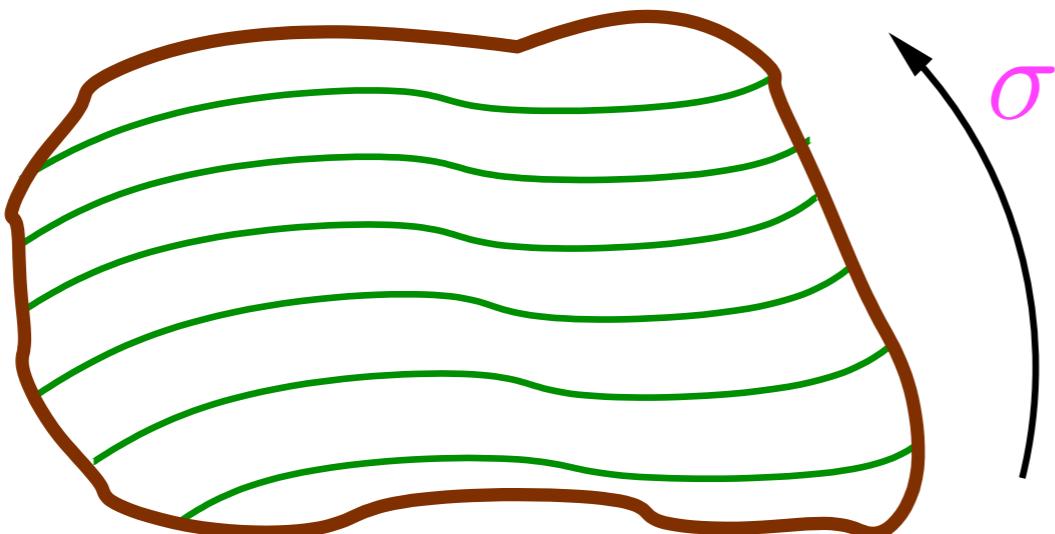
Think of the splitting as physical



# Stuckelberg description

**Convenient** to compare with GR at low energies (where deviations are weak)

**Inconvenient** to analyse the UV structure (where deviations from GR are strong)



introduce  
a field  $\sigma(\mathbf{x}, t)$   
to parametrize  
the foliation  
surfaces

ADM formulation = the gauge  $t = \sigma$

→  $\sigma$  sets global time

Khronon !

# Low-energy action

Invariant object -- unit normal to the foliation surfaces:

$$u_\mu = \frac{\partial_\mu \sigma}{\sqrt{(\partial\sigma)^2}}$$

$$S_{kh-m} = -\frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[ {}^{(4)}R + \beta \nabla_\mu u_\nu \nabla^\nu u^\mu + \lambda (\nabla_\mu u^\mu)^2 + \alpha u^\mu u^\nu \nabla_\mu u_\rho \nabla_\nu u^\rho \right]$$

cf. with Einstein-aether theory (Jacobson and Mattingly'01): an EFT for unit time-like LV vector

$$+ c_1 \sqrt{-g} \nabla_\mu u^\nu \nabla^\mu u_\nu + l(u_\mu^2 - 1)$$

Both theories have the same scalar and tensor sectors!  
(completely characterised by  $\alpha, \beta, \lambda$ )

# Degrees of freedom

$$S_{kh-m} = -\frac{M_P^2}{2} \int d^4x \sqrt{-g} \left[ {}^{(4)}R + \beta \nabla_\mu u_\nu \nabla^\nu u^\mu + \lambda (\nabla_\mu u^\mu)^2 + \alpha u^\mu u^\nu \nabla_\mu u_\rho \nabla_\nu u^\rho \right]$$

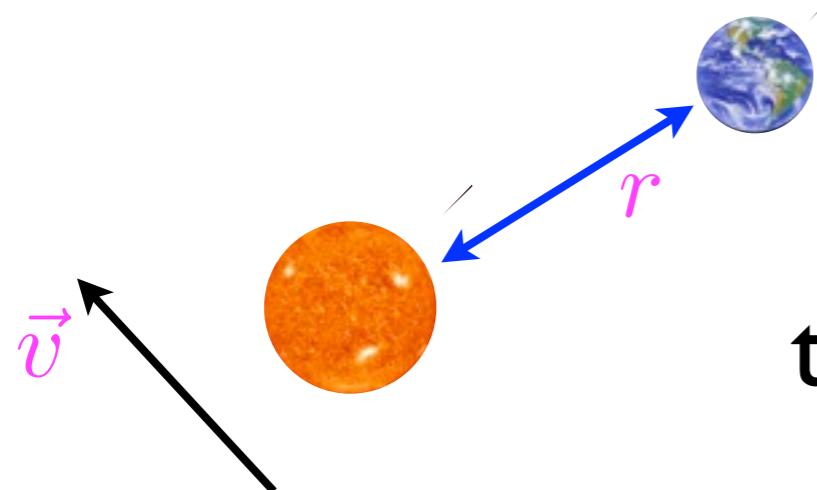
- ★ Massless Spin 2 graviton  $\omega^2 = c_t^2 k^2$
- ★  $\varphi = t + \chi$  massless scalar (new force!)  $\omega^2 = c_\chi^2 k^2$
- ★ Khronon

$$c_t^2 = \frac{1}{1 - \beta}$$
$$c_\chi^2 = \frac{\beta + \lambda}{\alpha}$$

EA-case: extra vector polarisations

not relevant for CMB-TT and LSS

## Constraints from the visible sector



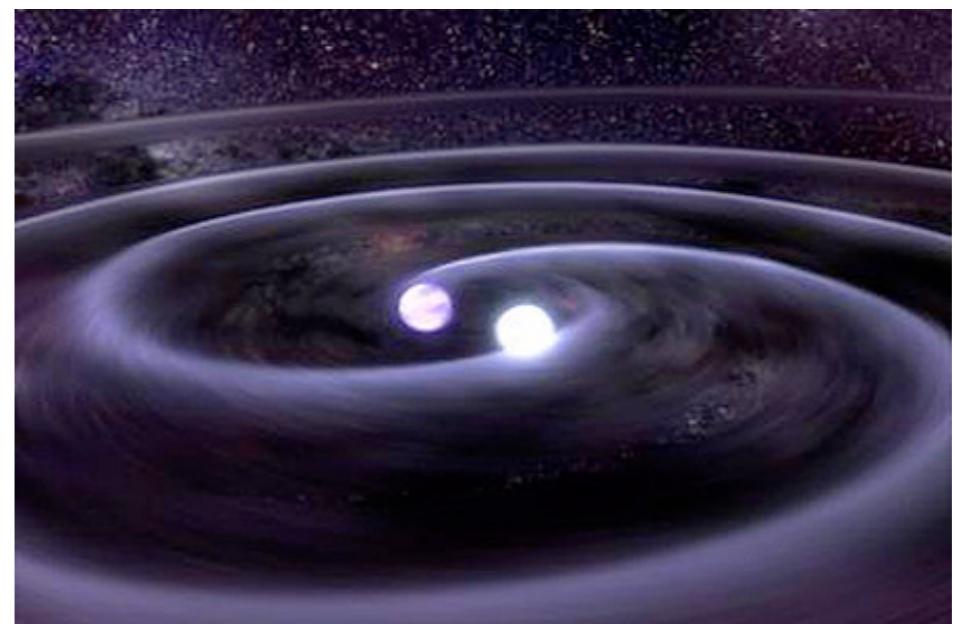
PPN bounds  $|\alpha, \beta, \lambda| < 10^{-7}$   
can be avoided for  
the special choice of parameters:  
**Khronometric:**  $\alpha = 2\beta$   
**Einstein-aether:**  $\alpha = -(\beta + 3\lambda)$

Constraints from GW emission in  
binary systems, **cosmology**  
(Einstein -aether only)

$$|\alpha, \beta, \lambda| \lesssim 0.01$$

Yagi, Blas, Yunes, Barouse'14

Zuntz, Ferreira ,Zlosnik'08



# Dark matter

Is non-relativistic ([small velocities](#)).

Is it possible to test its Lorentz invariance?

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DM-aether interaction:  
(assume DM - perfect  
pressureless fluid)

D.Blas, MI, S.Sibiryakov' 12

$$S_{[DMu]} = -m \int d^4x \sqrt{-g} n F(u_\mu v^\mu)$$

number density

aether

DM 4-velocity

LV effects related to DM are summarised in

$$Y \equiv F'(1)$$

# Relativistic cosmology - $\Lambda$ LVDM model

$$G_{\mu\nu} = \frac{1}{M_P^2} T_{\mu\nu}^{SM} + \frac{1}{M_P^2} T_{\mu\nu}^{dm, LV} + \frac{1}{M_P^2} T_{\mu\nu}^{aether} + \Lambda g_{\mu\nu}$$

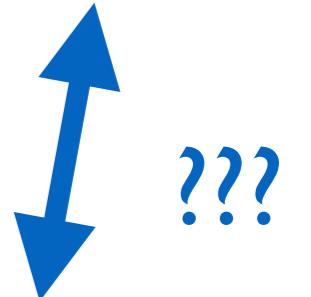
$$T_{\mu\nu}^{dm, LV} = T_{\mu\nu}^{dm} + \textcolor{red}{Y} \cdot \rho_{[dm]} O(u_\mu v_\nu^{[dm]})$$

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???

D.Blas, MI, S.Sibiryakov' 12

Blas, Sibiryakov' 11

Audren, Blas,  
Lesgourges, Sibiryakov' 13

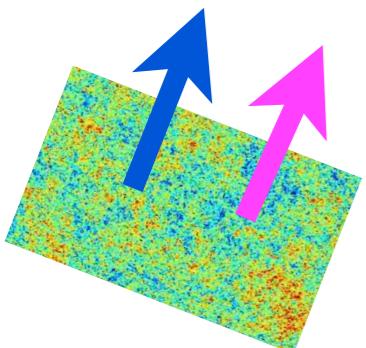
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D.Blas, MI, S.Sibiryakov' 12

**Background:** Homogeneous and isotropic  
(preferred foliation aligned with CMB frame)



$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a(t)^2 dx^i dx^i \\ u_\mu &= (u_0(t), 0, 0, 0) = v_\mu \quad , \quad \rho(t) \end{aligned}$$

Friedmann equations almost not modified!

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_c}{3} \rho_m$$

$$G_c = \frac{1}{8\pi M_P^2 [1 + 3\lambda/2 + \beta/2]}$$

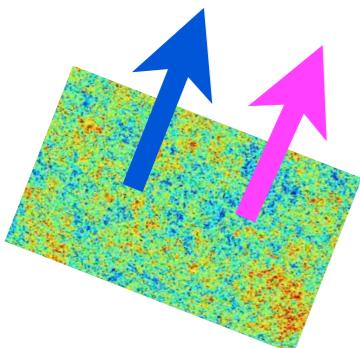
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**Background:** Homogeneous and isotropic  
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$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$
$$u_\mu = (u_0(t),$$

differs from Newtonian limit

$$G_N = \frac{1}{8\pi M_P^2(1 - \alpha/2)}$$

Friedmann eq.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_c}{3} \rho_m$$

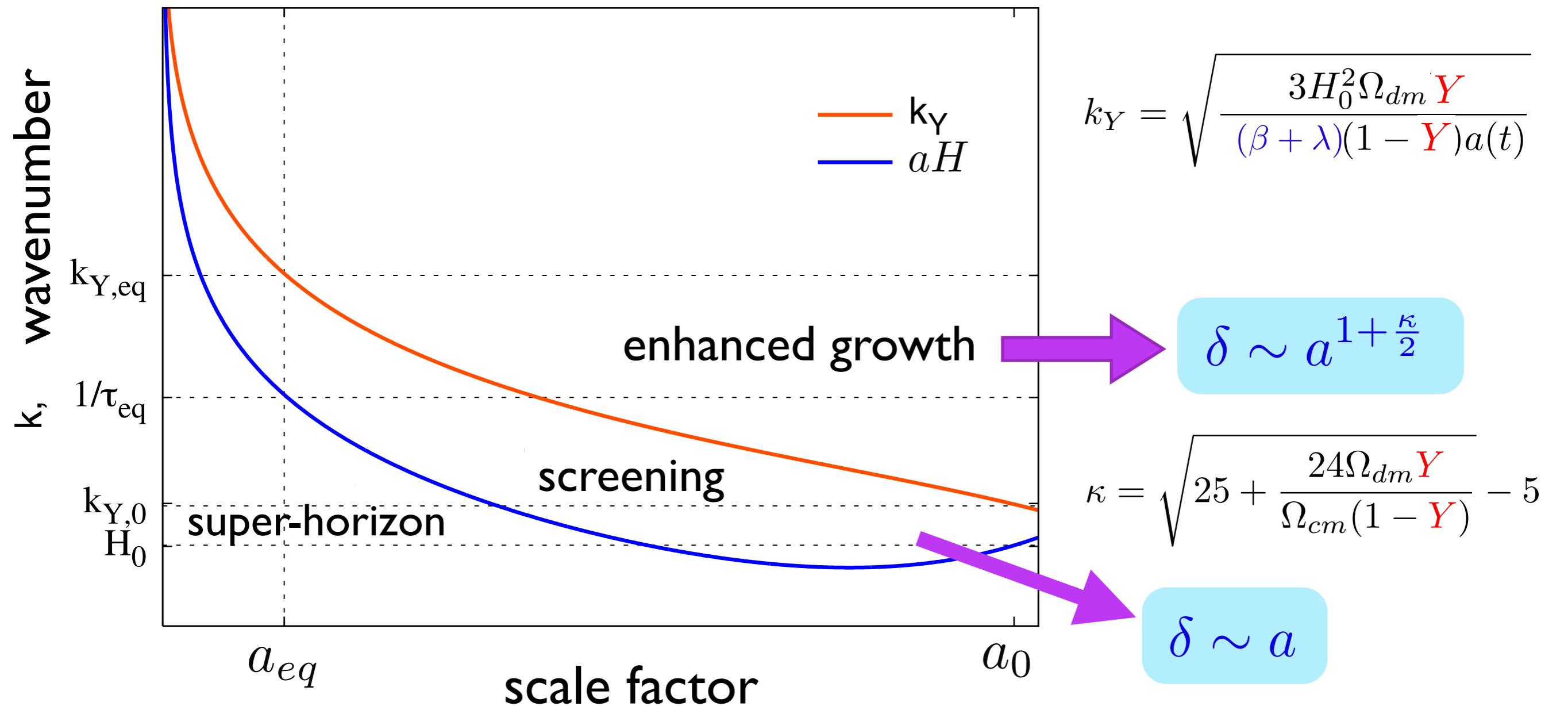


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# Effects on perturbations: LV in DM

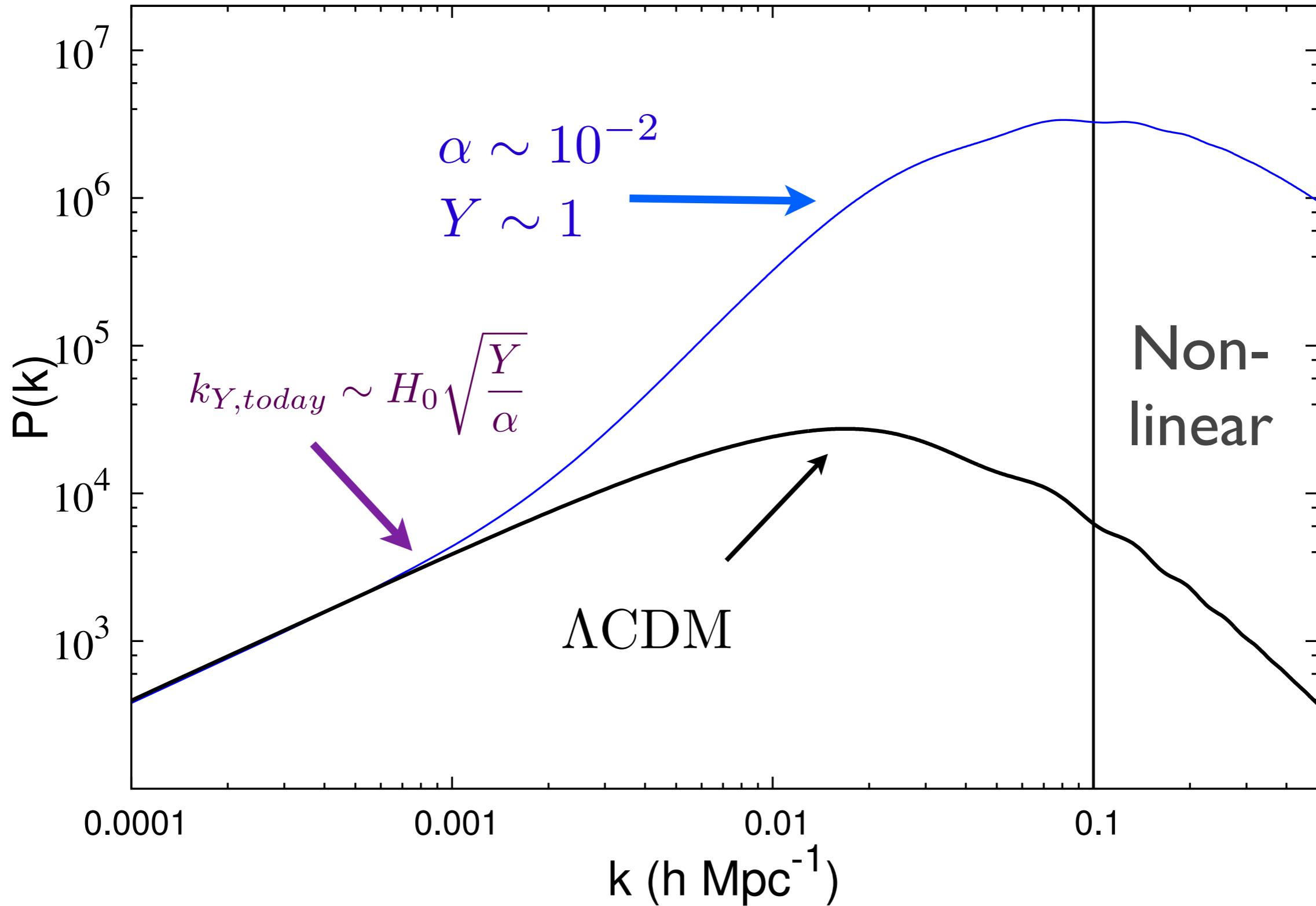
$$\rho(x, t) \equiv \rho(t)(1 + \delta(x, t))$$

Screening horizon

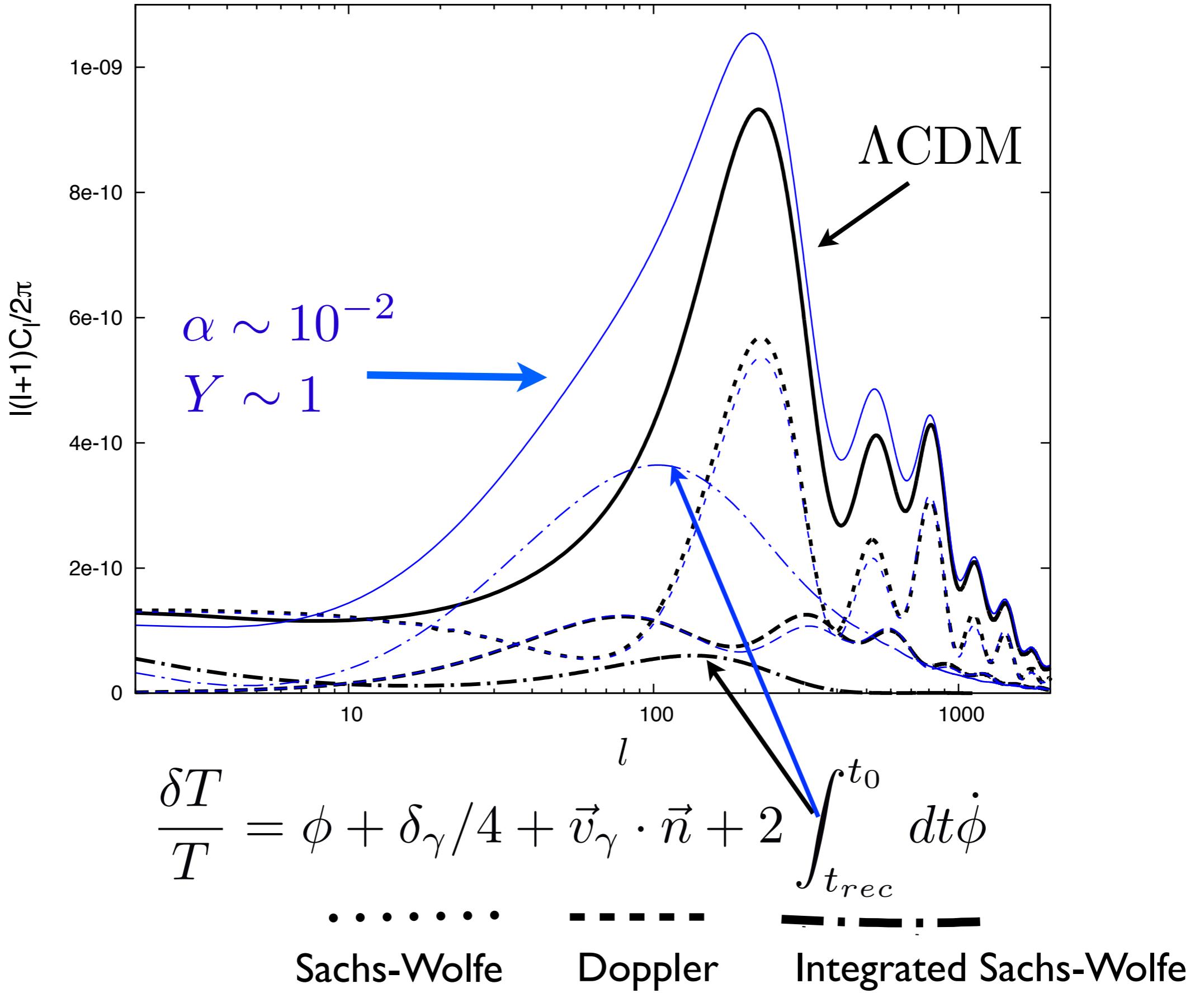


# LV in DM: Matter power spectrum

$$\langle \delta(k)\delta(k') \rangle \equiv \delta^{(3)}(k+k')P(k)k^3$$



# LV in DM: Cosmic microwave background



# LV in gravity: effects on perturbations

I) Modified Poisson equation:

$$k^2 \phi = -4\pi G_N a^2 [\Sigma \rho_i \delta_i + \delta \rho_{aether}]$$

$$H^2 = 8\pi G_c \rho / 3$$

$$G_c = \frac{G_0}{1 + \beta/2 + 3\lambda/2}$$

$$G_N = G_0 / (1 - \alpha/2)$$

$$\frac{G_N}{G_c} - 1 = \frac{\alpha + \beta + 3\lambda}{2} + O(2)$$

DM, baryons  
matter domination

$$\delta \sim \tau^{(-1 + \sqrt{1 + 24 \frac{G_N}{G_c}})/2}$$

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DM, baryons  
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+ Solar system constraints

**Khronometric** ( $\alpha = 2\beta$ )

$$\frac{G_N}{G_c} - 1 = \frac{3(\beta + \lambda)}{2} + O(2) > 0$$

**Einstein-Aether:**

$$\frac{G_N}{G_c} - 1 = O(2)$$

# LV in gravity: effects on perturbations

II) Anisotropic stress: aether  fluid

$$ds^2 = a^2(t)[(1 + 2\psi)dt^2 - (1 - 2\phi)d\mathbf{x}^2]$$

$$\phi - \psi = O(\beta)$$

Free-streaming below the scale imposed by

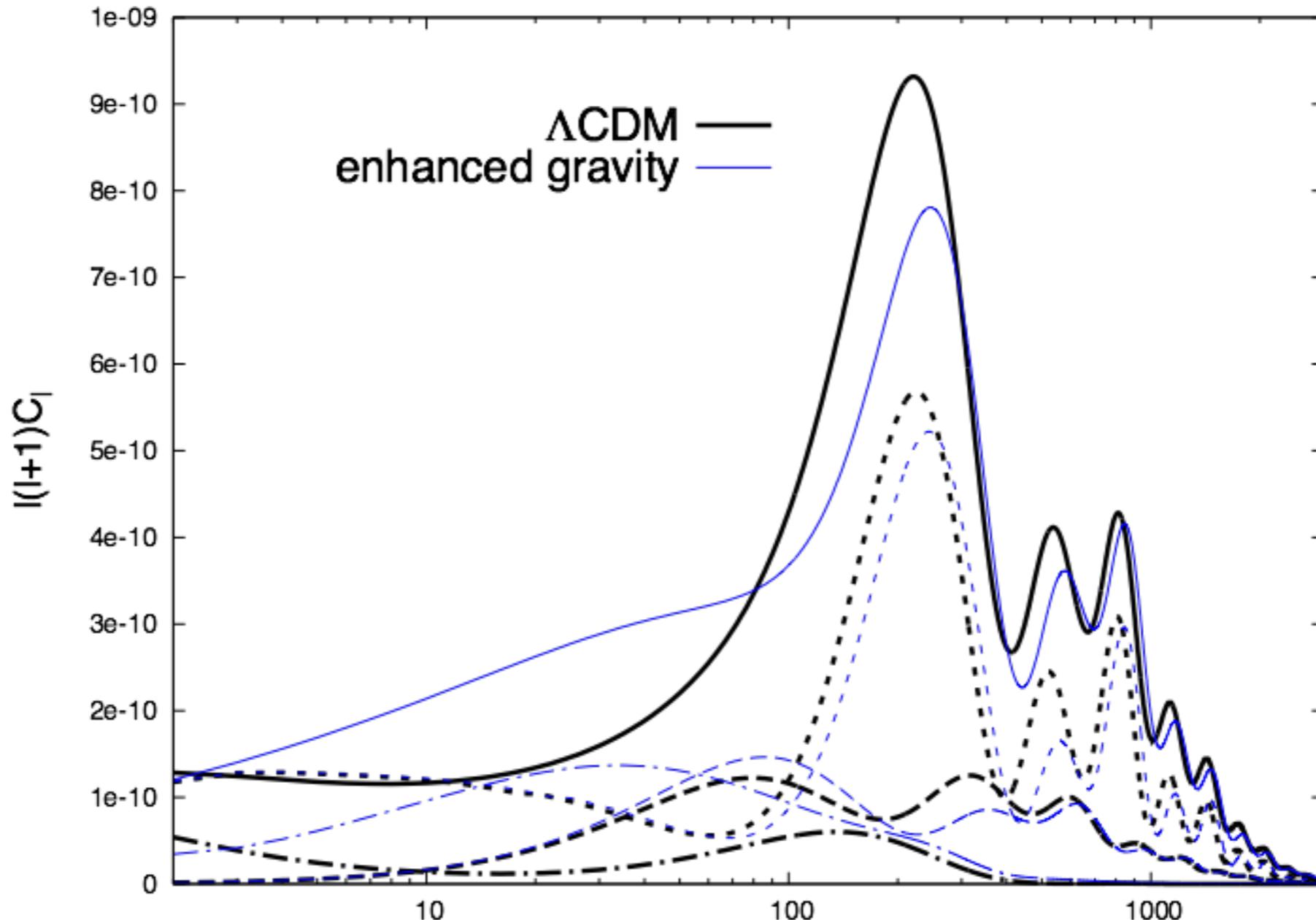
$c_\chi$  - aether speed of sound

# Enhanced gravity: effects on CMB

$$\frac{G_N}{G_c} - 1 \sim 1$$

$$\beta = 0$$

$$Y = 0$$



$$\frac{\delta T}{T} = \phi + \delta_\gamma/4 + \vec{v}_\gamma \cdot \vec{n} + \int_{\tau_{rec}}^{\tau_0} dt (\dot{\phi} + \dot{\psi})$$

• • • • •

- - - - -

- - - - -

Sachs-Wolfe

Doppler

Integrated Sachs-Wolfe

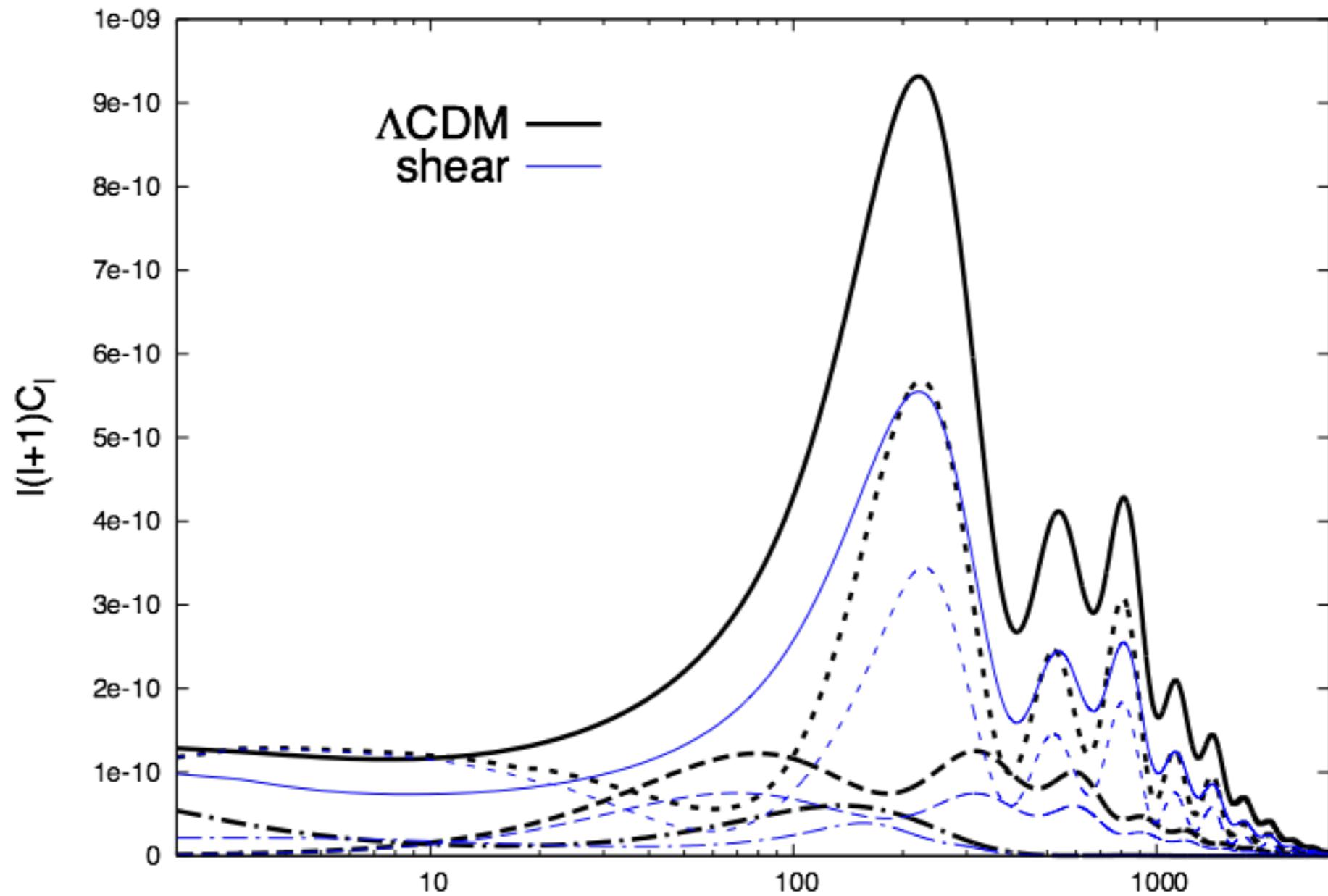
B.Audren, D.Blas,  
J.Lesgourgues,  
S.Sibiryakov 13'

# Shear: effects on CMB

$$\beta \sim 1$$

$$\frac{G_N}{G_c} - 1 = 0$$

$$Y = 0$$



$$\frac{\delta T}{T} = \phi + \delta_\gamma/4 + \vec{v}_\gamma \cdot \vec{n} + \int_{\tau_{rec}}^{\tau_0} dt (\dot{\phi} + \dot{\psi})$$

• • • • •

- - - - -

- . - . -

Sachs-Wolfe

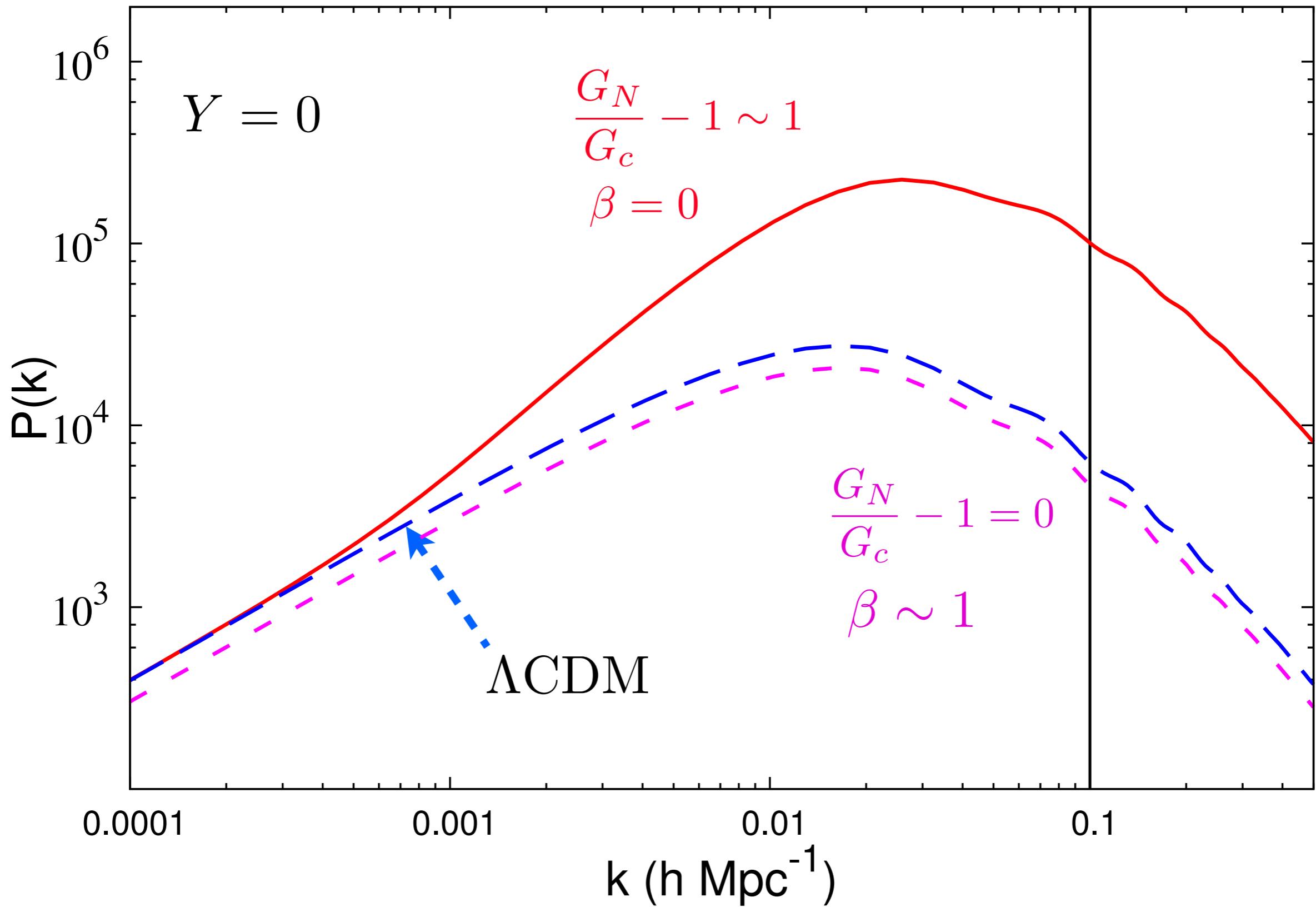
Doppler

Integrated Sachs-Wolfe

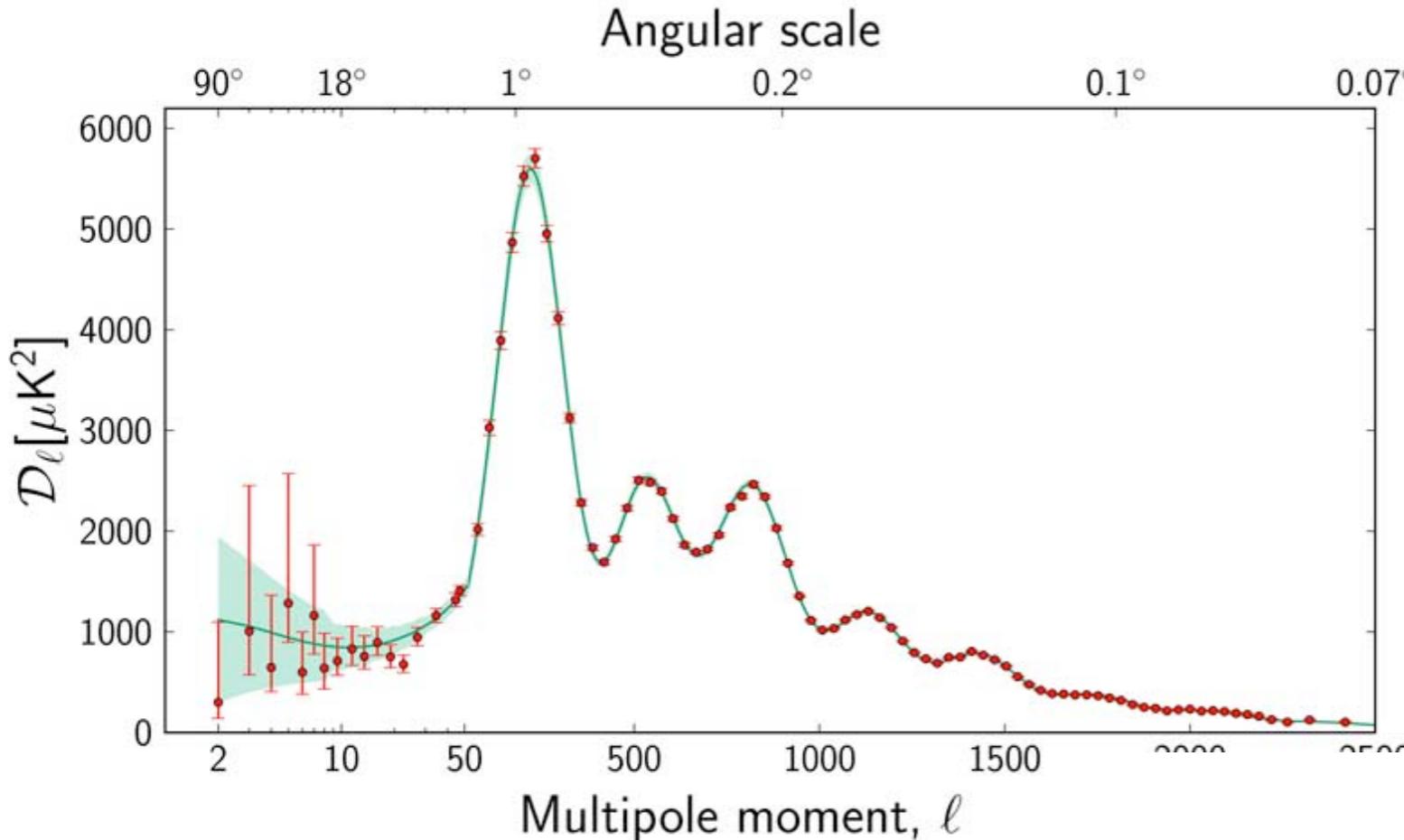
B.Audren, D.Blas,  
J.Lesgourgues,  
S.Sibiryakov 13'

# LV in gravity: effects on MPS

$$\langle \delta(k)\delta(k') \rangle \equiv \delta^{(3)}(k+k')P(k)k^3$$



# Observational data



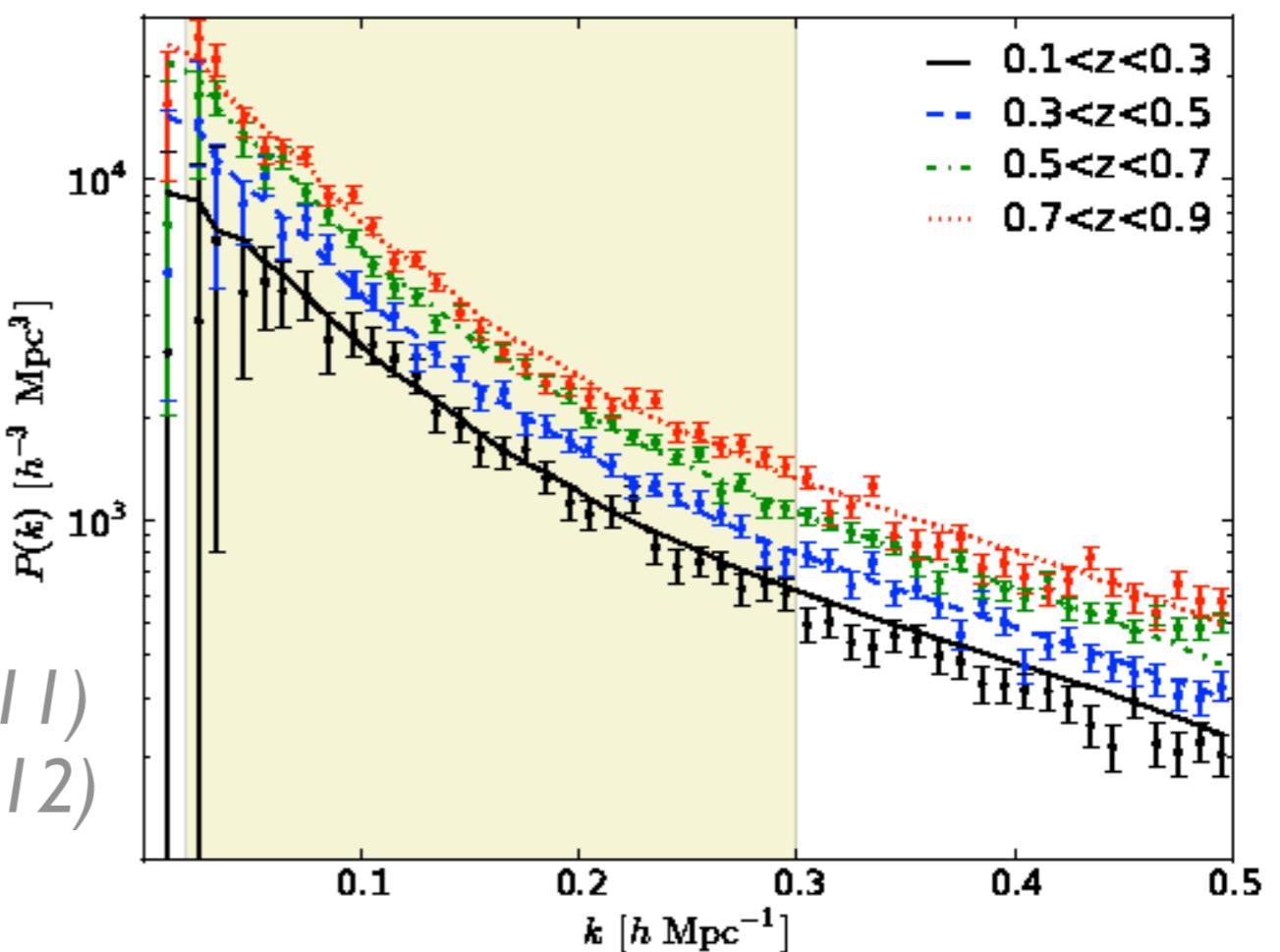
Planck



WiggleZ survey



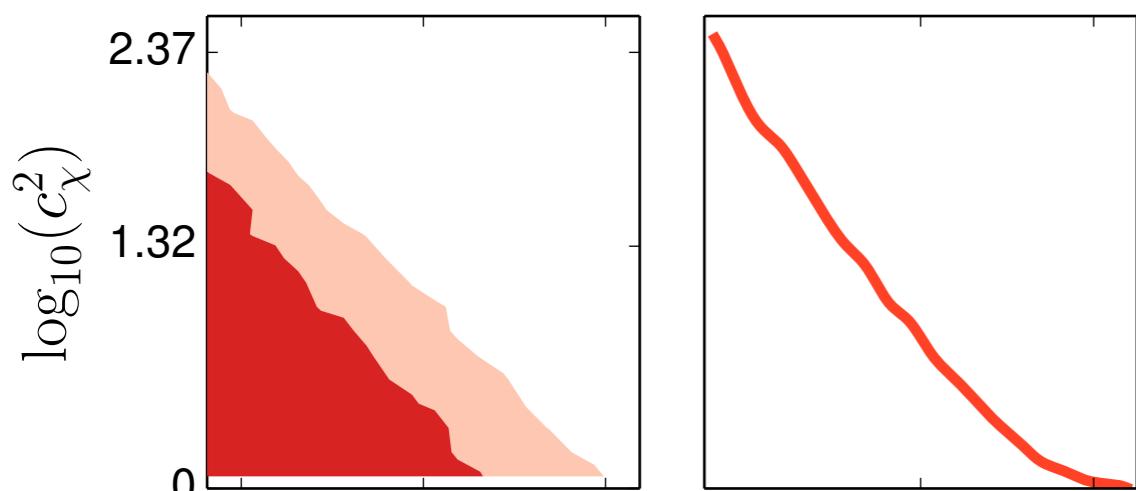
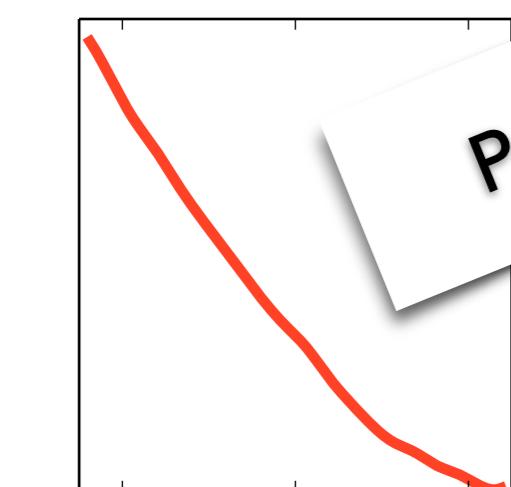
use CLASS Blas, Lesgourgues, Tram (2011)  
and MONTE PYTHON Audren et. al. (2012)



# Cosmological bounds

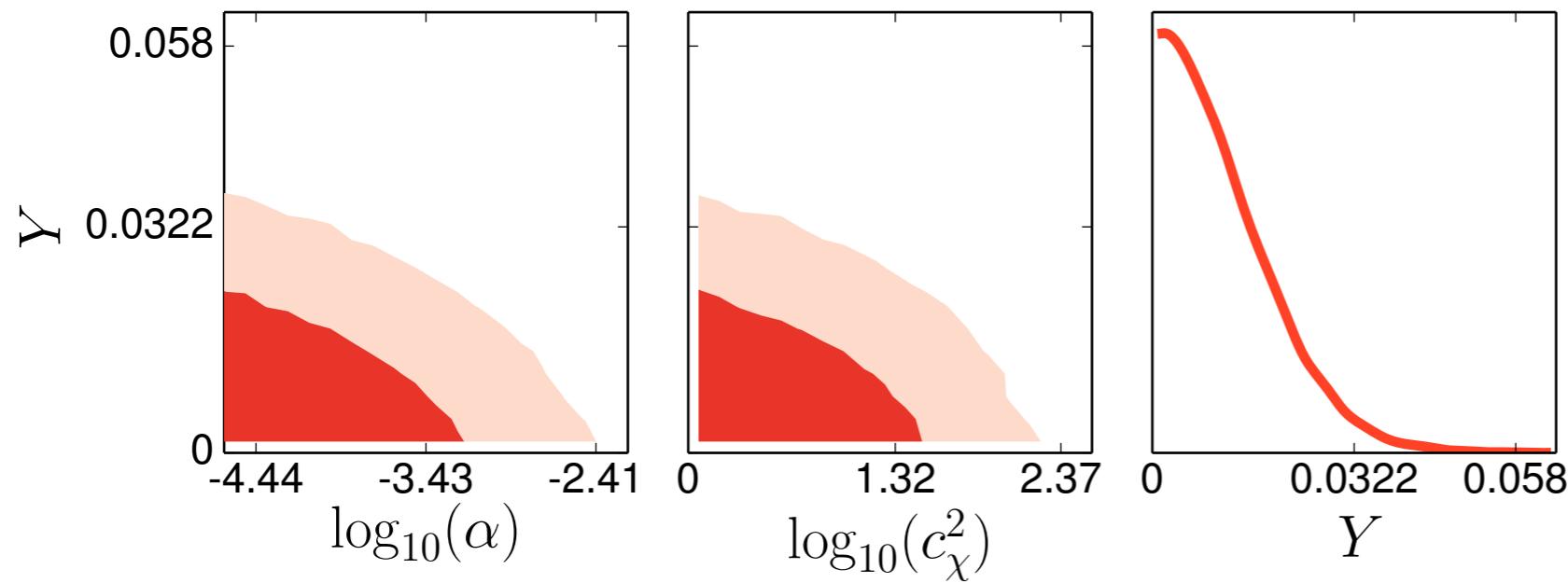
95% CL upper limits

PPN Einstein-aether  $3\lambda = -(\alpha + \beta)$



$$\begin{aligned}\alpha &< 5.0 \cdot 10^{-3} \\ c_\chi^2 &< 240 \\ Y &< 0.028\end{aligned}$$

PPN Khronometric  $\alpha = 2\beta$



$$\begin{aligned}\alpha &< 1.1 \cdot 10^{-3} \\ c_\chi^2 &< 54 \\ Y &< 0.029\end{aligned}$$

Preliminary

## Conclusions:



Lorentz violation is a consistent framework to test deviations from  $\Lambda$ CDM motivated by quantum gravity



Consequences of LV in cosmology:  
enhanced growth of structures at small scales (i)  
accelerated growth of structures (ii)  
additional cosmic shear (iii)



Bounds at the level  $\text{few} \times 10^{-3}$  on LV in gravity  
and 0.03 on LV in Dark Matter

## Outlook:



\* Nonlinear structure formation, ‘DM problems’

## Other directions:



Technically natural dark energy with LV  
- resolves Cosm. Const. Problem

[Blas, Sibiryakov' 11](#)

[Audren, Blas,](#)

[Lesgourges, Sibiryakov' 13](#)



Inflation with LV  
- curious phenomenology for NG

[MI, Sibiryakov' 13](#)

Rigorous treatment of renormalisability  
in extended Horava's gravity



.....

**Thank you for your attention!**

## Conclusions:



Lorentz violation is a consistent framework to test deviations from  $\Lambda$ CDM



Consequences of LV in cosmology:  
accelerated growth of structures



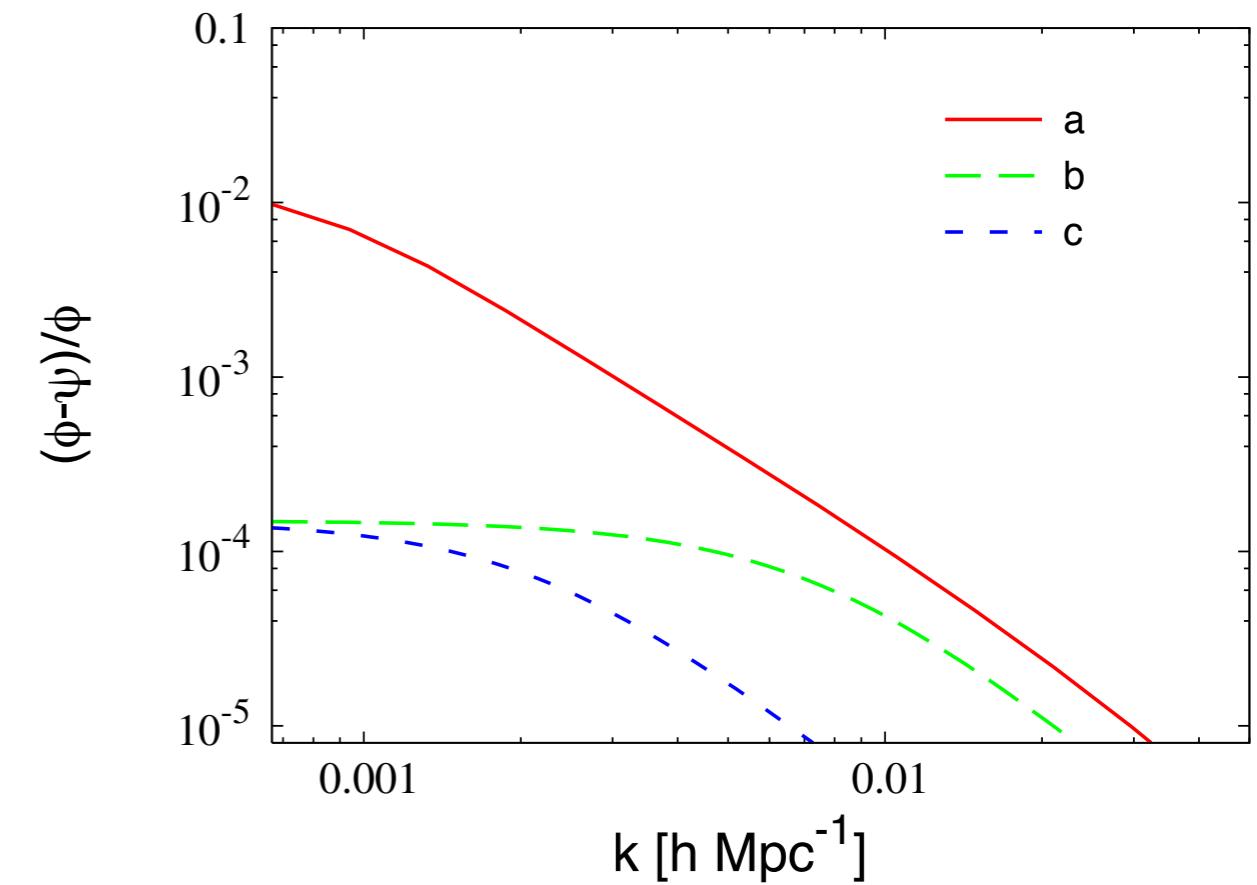
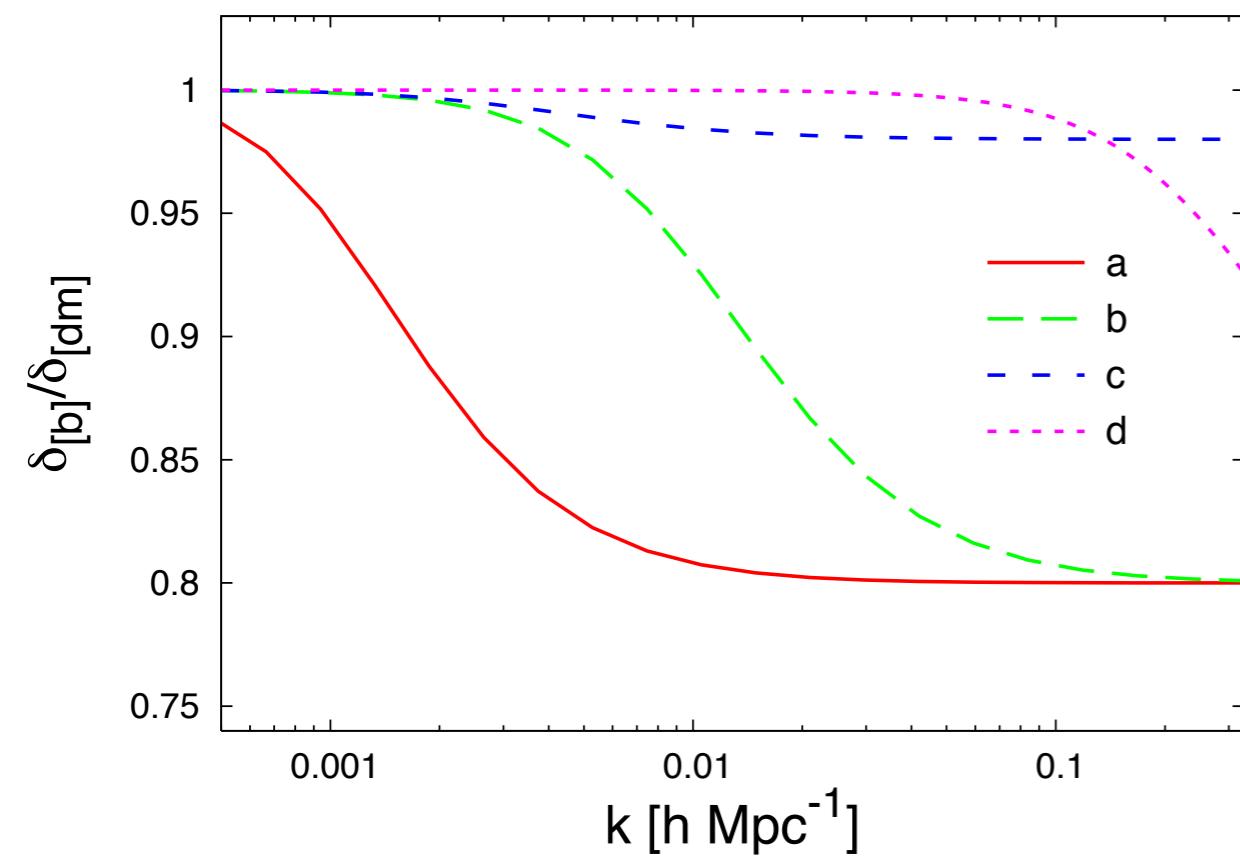
Thank you for your attention!

(depending on LV in gravity)

## Outlook:

- \* Nonlinear structure formation, ‘DM problems’

# Baryonic bias and anisotropic stress:

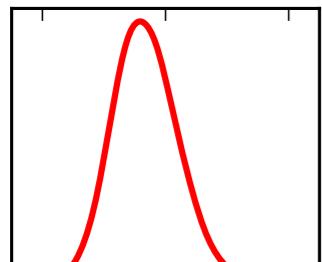


	$\alpha$	$\beta$	$\lambda$	$Y$	$k_{Y,0}$ (h Mpc $^{-1}$ )	$k_{Y,eq}$ (h Mpc $^{-1}$ )
a	$2 \cdot 10^{-2}$	$10^{-2}$	$10^{-2}$	0.2	$9.2 \cdot 10^{-4}$	$6.5 \cdot 10^{-2}$
b	$2 \cdot 10^{-4}$	$10^{-4}$	$10^{-4}$	0.2	$9.1 \cdot 10^{-3}$	0.65
c	$2 \cdot 10^{-4}$	$10^{-4}$	$10^{-4}$	0.02	$2.6 \cdot 10^{-3}$	0.18
d	$10^{-7}$	0	$10^{-7}$	0.2	0.41	29

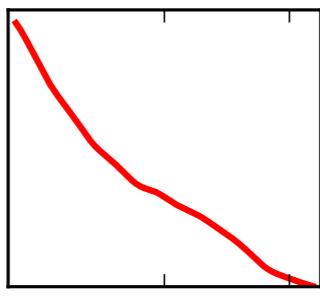
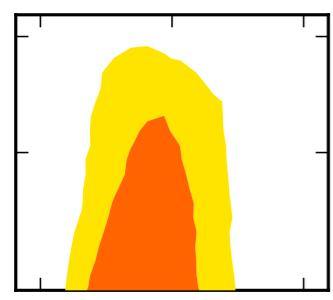
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95% CL upper limits

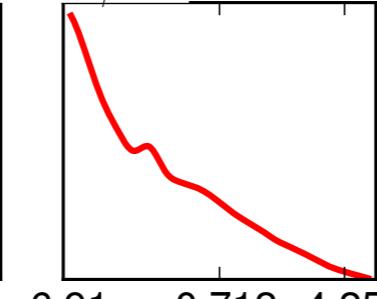
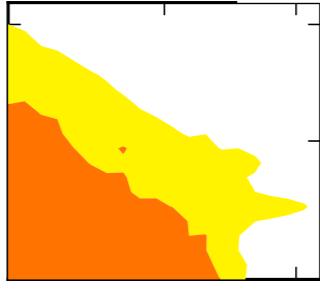
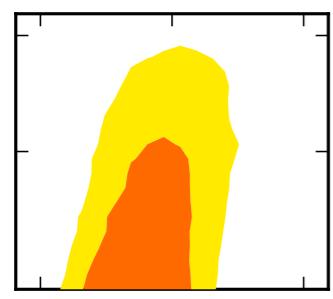
$$H_0 = 67.9^{+0.961}_{-1.15}$$



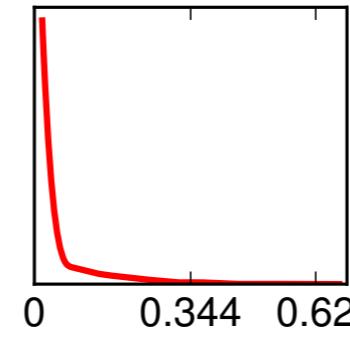
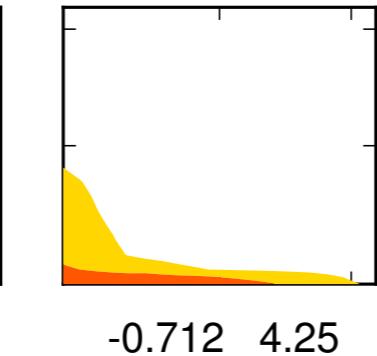
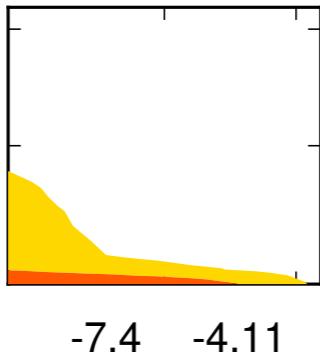
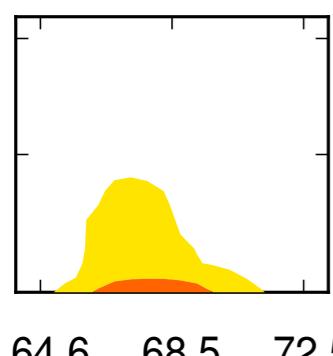
$$\ln \beta = -9.14^{+0.73}_{-2.38}$$



$$\ln \frac{\beta + \lambda}{2\beta} = -3.49^{+0.000878}_{-3.42}$$



$$Y = 0.045^{+0.00815}_{-0.045}$$



PPN Khronometric  $\alpha = 2\beta$

$$\beta < 0.004$$

$$\beta + \lambda < 2 \cdot 10^{-4}$$

$$Y < 0.2$$

PPN Einstein-aether

$$3\lambda = -(\alpha + \beta)$$

$$\alpha < 0.004$$

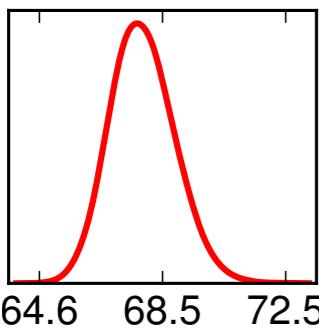
$$\beta < 0.003$$

$$Y < 0.6$$

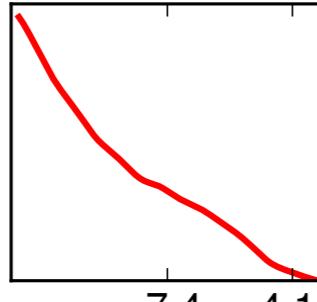
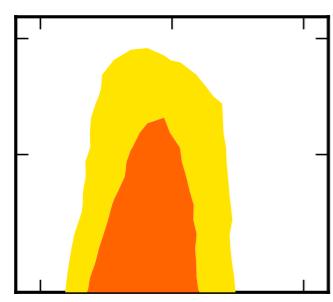
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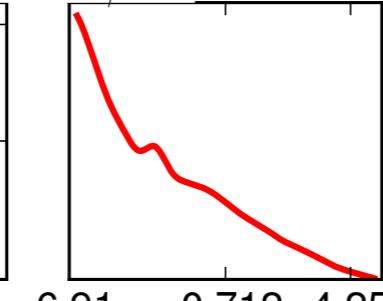
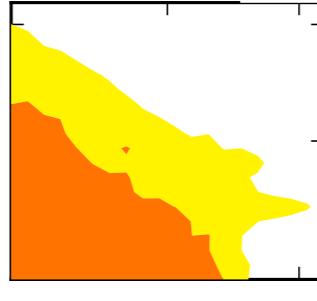
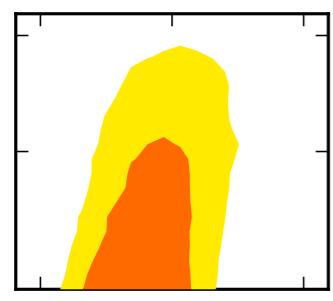
$$H_0 = 67.9^{+0.961}_{-1.15}$$



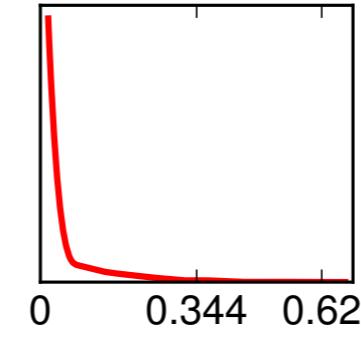
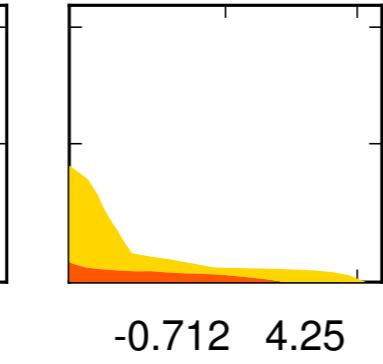
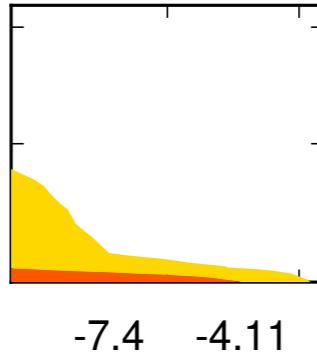
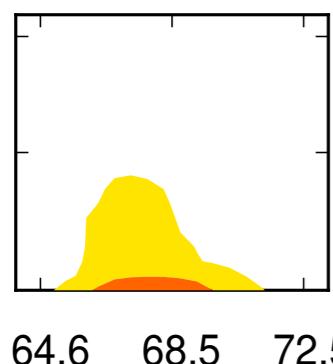
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$$Y = 0.045^{+0.00815}_{-0.045}$$



Correlation with  $H_0$   
Khronon resembles dark radiation for  
some LV parameters !

PPN Einstein-aether

$$3\lambda = -(\alpha + \beta)$$

$$\alpha < 0.004$$

$$\beta < 0.003$$

$$Y < 0.6$$

# LV in gravity: effects on perturbations

I) Modified Poisson equation:

$$k^2 \phi = -4\pi G_N a^2 [\Sigma \rho_i \delta_i + \delta \rho_{aether}] \quad H^2 = 8\pi G_c \rho / 3$$

different from  $G_c$ ,  $\frac{G_N}{G_c} - 1 = \frac{\alpha + \beta + 3\lambda}{2} + O(2)$

DM, baryons

matter domination

$$\delta \sim \tau^{(-1 + \sqrt{1 + 24 \frac{G_N}{G_c}})/2}$$

+ Solar system constraints

**Khronometric** ( $\alpha = 2\beta$ )

$$\frac{G_N}{G_c} - 1 = \frac{3(\beta + \lambda)}{2} + O(2) > 0$$

**Einstein-Aether:**

$$\frac{G_N}{G_c} - 1 = O(2)$$

## Gravity action: $g_{\mu\nu}, u^\mu$

Einstein-aether:

$$S_{\text{ae}} \equiv -\frac{M_0^2}{2} \int d^4x \sqrt{-g} \left[ R + K^{\mu\nu}{}_{\sigma\rho} \nabla_\mu u^\sigma \nabla_\nu u^\rho + l(u_\mu u^\mu - 1) \right]$$

$$K^{\mu\nu}{}_{\sigma\rho} \equiv c_1 g^{\mu\nu} g_{\sigma\rho} + c_2 \delta_\sigma^\mu \delta_\rho^\nu + c_3 \delta_\rho^\mu \delta_\sigma^\nu + c_4 u^\mu u^\nu g_{\sigma\rho}$$

Khronometric:  $u_\mu = \frac{\partial_\mu \varphi}{\sqrt{(\partial\varphi)^2}}$  

$c_i$  are not independent!

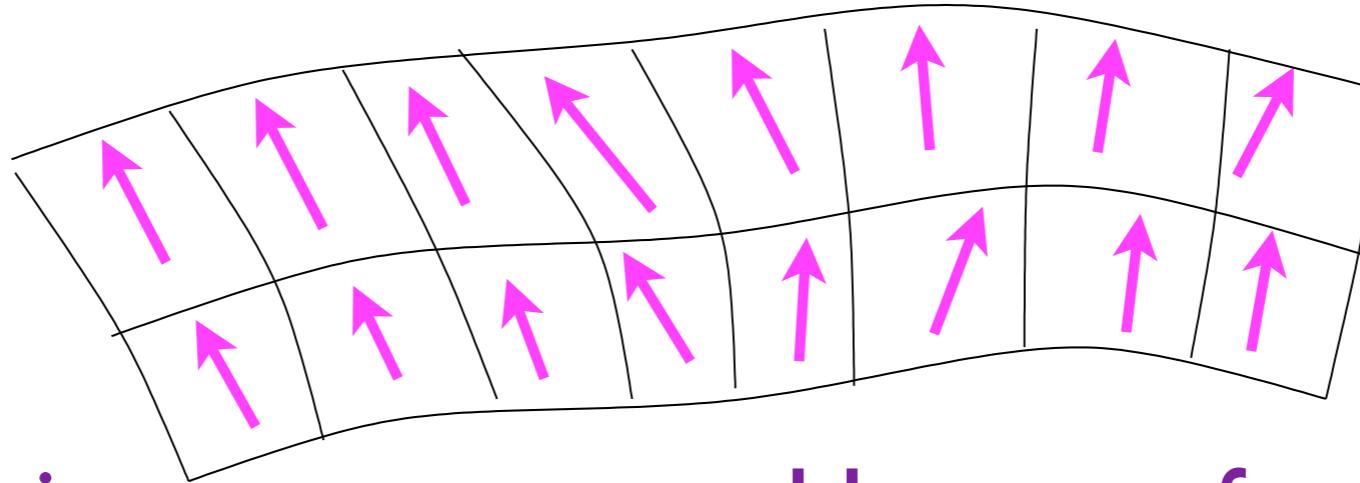
$$\begin{aligned}\lambda &\equiv c_2 \\ \beta &\equiv c_1 + c_3 \\ \alpha &\equiv c_1 + c_4\end{aligned}$$

Both theories have the same scalar and tensor sectors!  
(completely characterized by  $\alpha, \beta, \lambda$ )

Vectors not relevant for CMB-TT and LSS

# Breaking Lorentz Invariance

Space-time filled by a preferred **time** direction  
Associated to a time-like unit vector  $u_\mu$



Generic:  
Einstein-aether theory

$$u^\mu u_\mu = 1$$

Jacobson,  
Mattingly' 01

Hypersurface orthogonal:  
Khronon

$$u_\mu = \frac{\partial_\mu \sigma}{\sqrt{(\partial\sigma)^2}}$$



# Gravity Lagrangian:

Einstein-aether:  $\mathcal{L}_{GR} + \mathcal{L}_u$

$$\mathcal{L}_u \sim c_1, c_2, c_3, c_4 M_P^2 (\nabla u_\mu)^2$$

Khronometric:

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Both

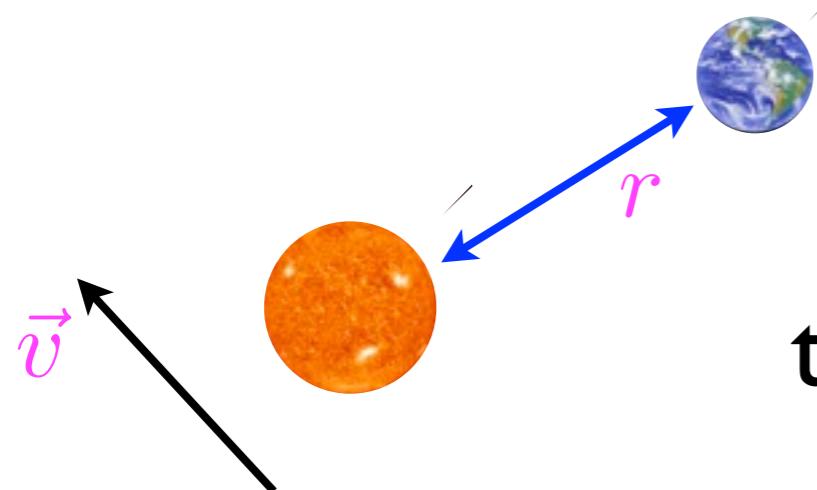
Low-energy limit of Horava-Lifshitz gravity

$$\Lambda_{IR} \sim \sqrt{\alpha} M_P$$

sectors!

SS

# Constraints from the visible sector



PPN bounds  $|\alpha, \beta, \lambda| < 10^{-7}$

can be avoided for  
the special choice of parameters:

Khron

Einstein

Constrains from GW emission  
binary systems, cosmology  
(Einstein -aether only)

$$|\alpha, \beta, \lambda| \lesssim 0.01$$

Zuntz, Ferreira ,Zlosnik'08

Can be improved with new  
cosmological data !



All current data are compatible with  
the  $\Lambda CDM$  model  
(assumes Lorentz Invariance as a  
fundamental property of Nature)



Reasons to question this:

Recent successes of Lorentz-violating  
theory of quantum gravity (Horava' 09)

Lorentz invariance has been tightly  
constrained only in the sector of Standard  
Model particles

$$< 10^{-20}$$

What about other sectors?



For other sectors  
bounds are milder or even don't exist!

### Gravity

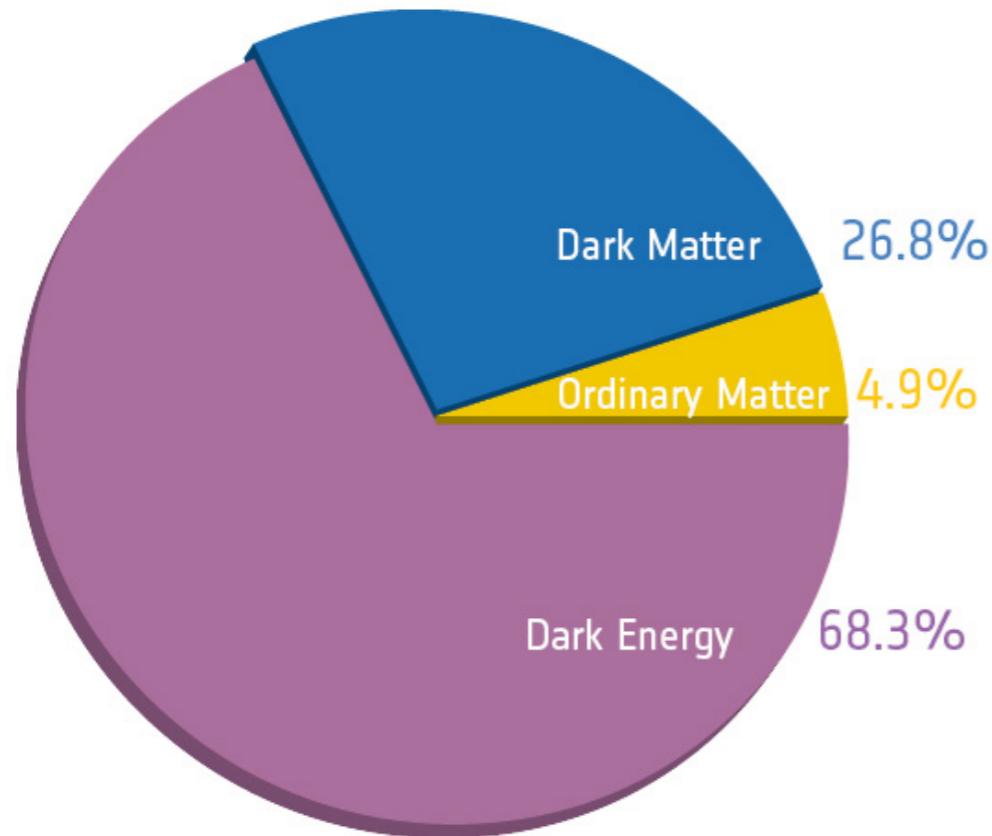
$< 10^{-7}$

### Dark Matter

???

### Dark Energy

$< 10^{-2}?$



Given the key role played by LI in modeling Nature, it is essential to test it to the best possible accuracy in all the sectors

# Dark matter

Is non-relativistic ([small velocities](#)).

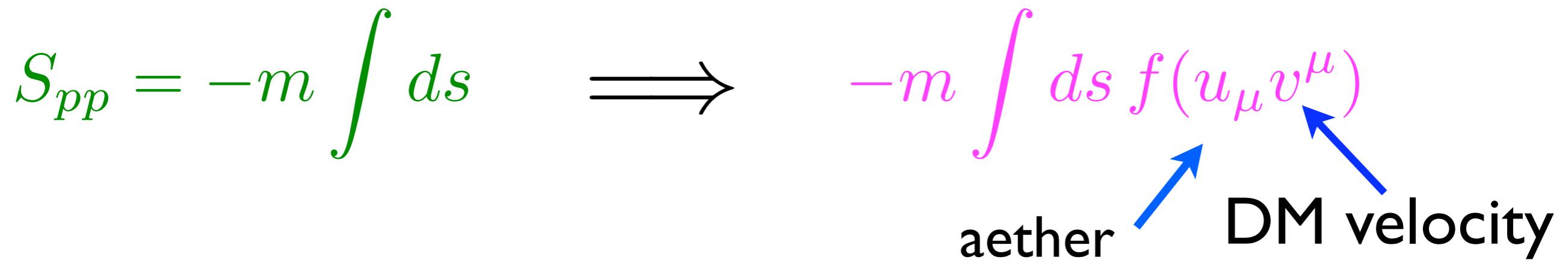
Is it possible to test its Lorentz invariance?

[Yes!](#)

# Generalized point particle action

$$S_{pp} = -m \int ds \quad \Longrightarrow \quad -m \int ds f(u_\mu v^\mu)$$

aether      DM velocity



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$$S_{pp} = -m \int ds \quad \Longrightarrow \quad -m \int ds f(u_\mu v^\mu)$$

aether      DM velocity

Newtonian limit:  $u^i, v^i$  - small

$$S = \int d^4x \left[ M_P^2 \phi \Delta \phi + \frac{M_P^2 \alpha}{2} u^i \Delta u^i \right] + \int d^4x \rho \left[ \frac{(v^i)^2}{2} - \phi - Y \frac{(u^i - v^i)^2}{2} \right]$$

DM density       $f'(1)$

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- modified inertial mass (coefficient in front of  $(v^i)^2$ )

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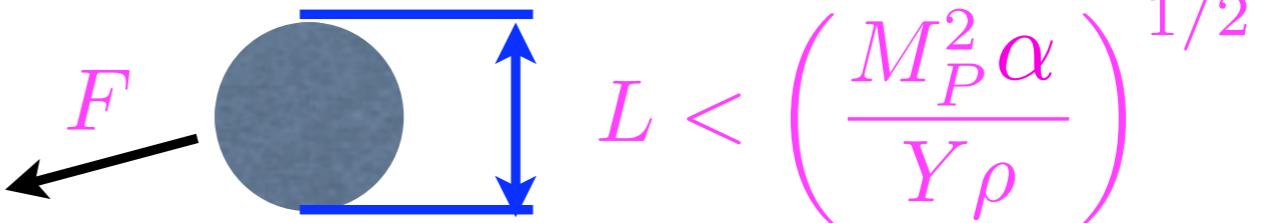
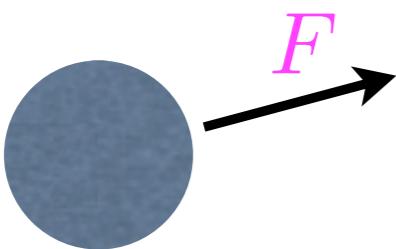
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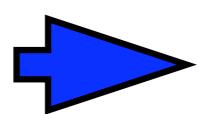
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DM density       $f'(1)$

- all effects are encoded in one parameter  $Y$
- modified inertial mass (coefficient in front of  $(v^i)^2$ )
- effective potential for aether in matter  $m_{eff} = \frac{Y\rho}{\alpha M_P^2}$



$$L < \left( \frac{M_P^2 \alpha}{Y \rho} \right)^{1/2}$$



$$F = \frac{F_N}{(1 - Y)}$$

$$m_{\text{inert}} = m_{\text{grav}} (1 - Y)$$

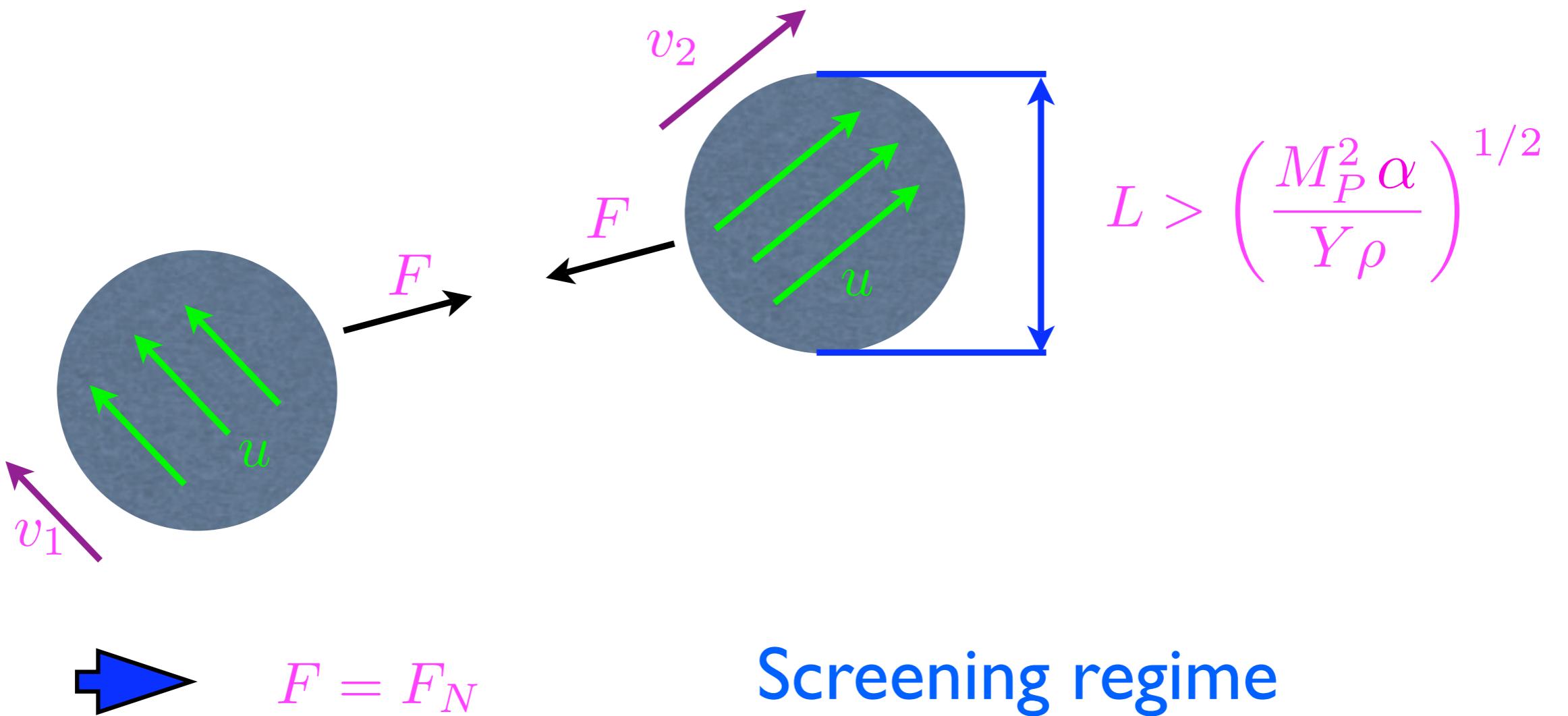
Accelerated Jeans instability !

$$\frac{\delta \rho}{\rho}$$

$$\delta \propto \tau^\gamma,$$

$$\gamma = \frac{1}{6} \left[ -1 + \sqrt{\frac{25 - Y}{1 - Y}} \right]$$

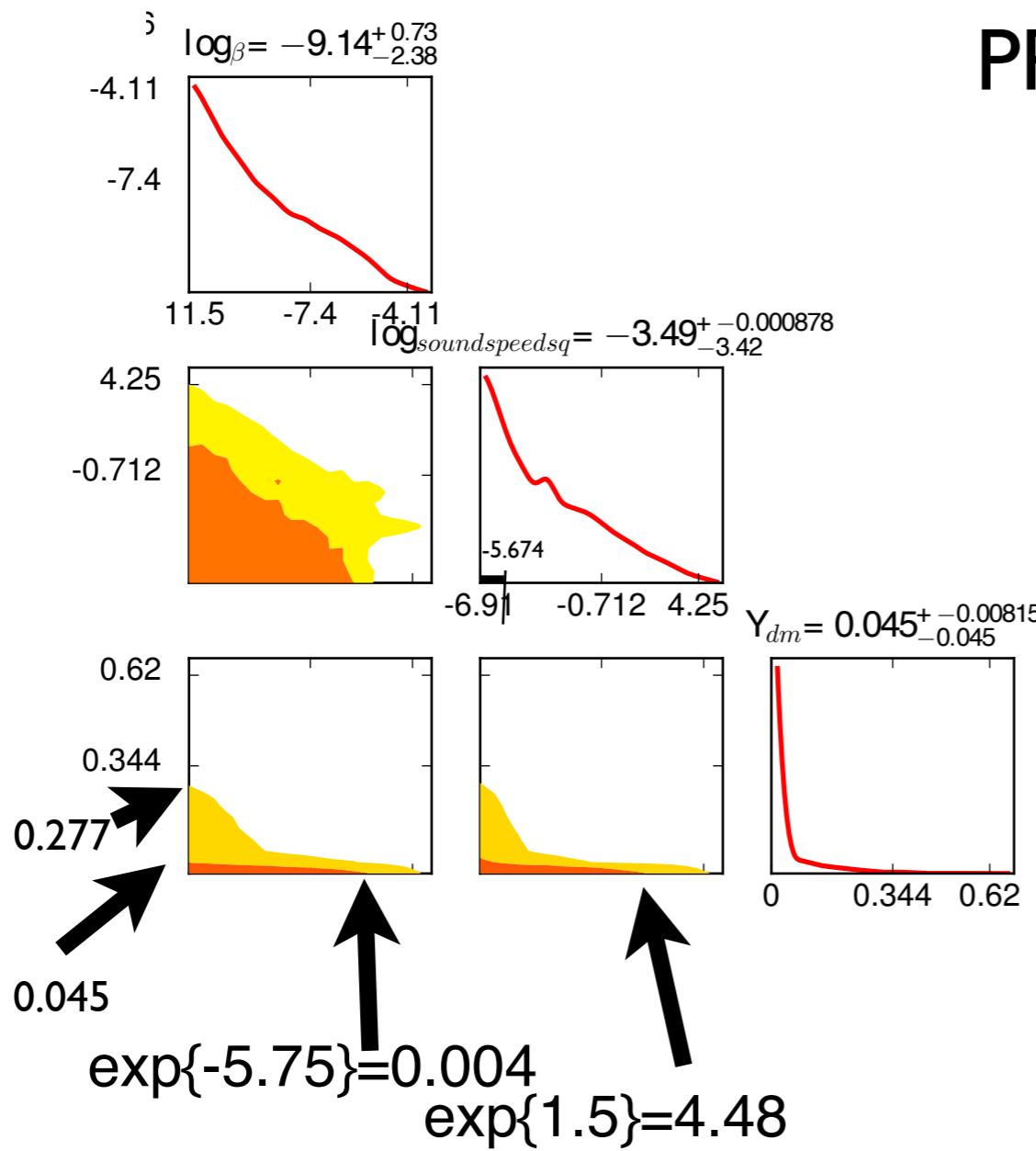
density contrast



**Standard growth of structures  
chameleon-like mechanism**

$$\delta \propto \tau^{2/3}$$

## PPN Khronometric $\alpha = 2\beta$



$$\begin{aligned}\beta &< 0.004 \\ \beta + \lambda &< 2 \cdot 10^{-4} \\ Y &< 0.2\end{aligned}$$

## PPN Einstein-aether

$$3\lambda = -(\alpha + \beta)$$

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# LV in gravity: effects on perturbations

I) Modified Poisson equation:

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different from  $G_c$ ,  $\frac{G_N}{G_c} - 1 = \frac{\alpha + \beta + 3\lambda}{2} + O(2)$

DM, L

matter c

Kin

$$\frac{G_N}{G_c} - 1 - \frac{1}{2} + O(2) > 0$$

$$\frac{G_N}{G_c} - 1 = O(2)$$

Aether - effective relativistic dof,  
undergoes free-streaming

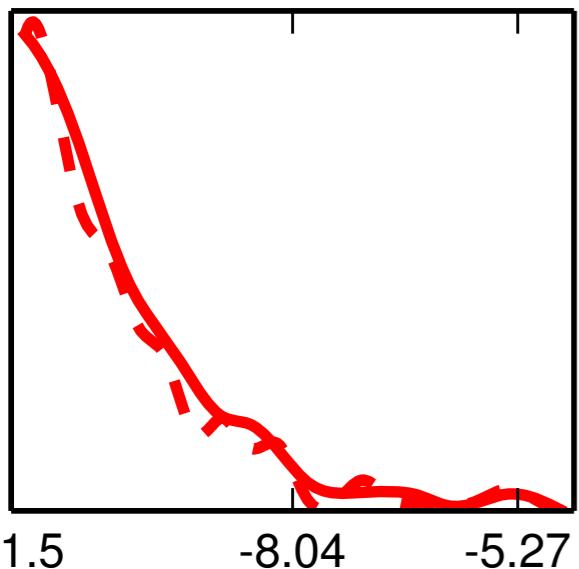
II) Anisotropic stress: aether

$$ds^2 = a(t)^2 [(1 + 2\phi)dt^2 - \delta_{ij}(1 - 2\psi)dx^i dx^j]$$



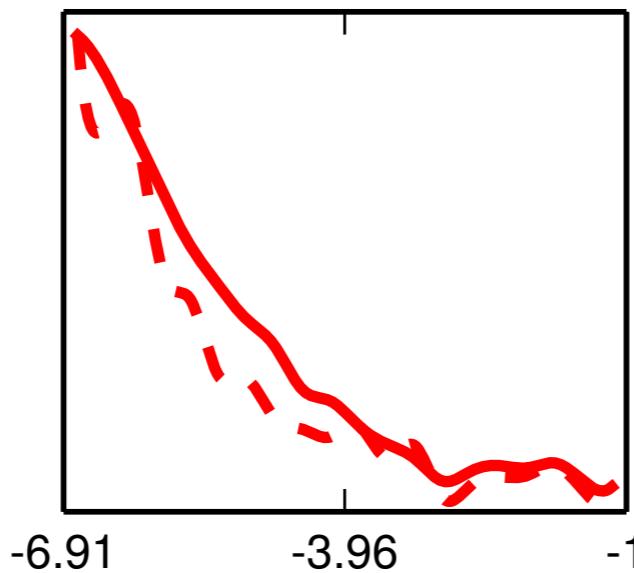
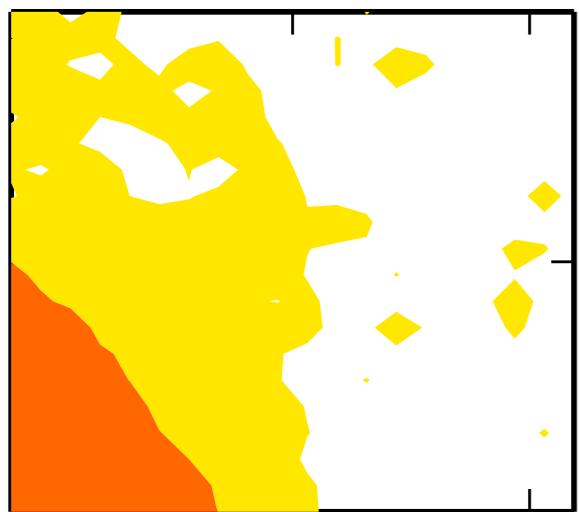
$$\phi - \psi = O(\beta)$$

$$\log_{\alpha} = -10.2^{+0.243}_{-1.34}$$

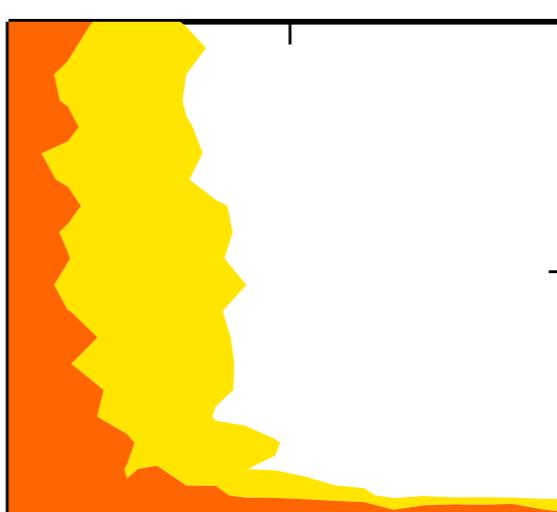
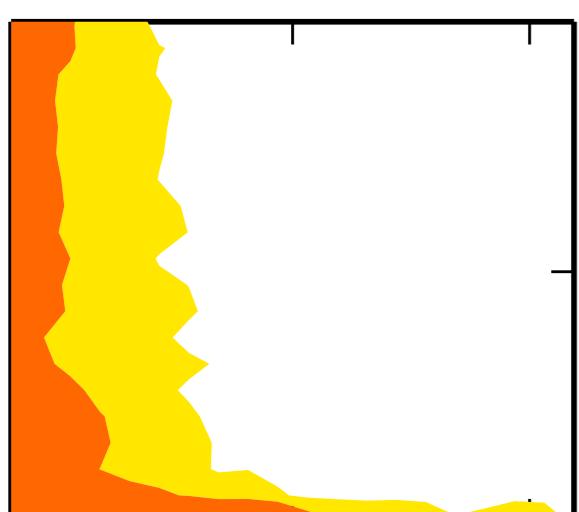


$$\ln c_{\chi}^2$$

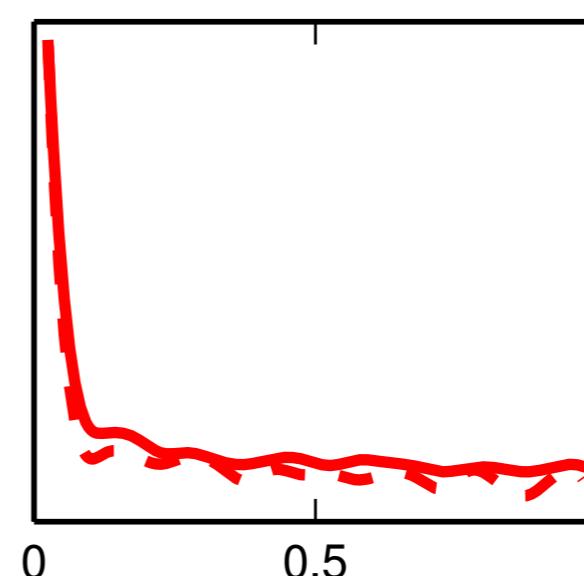
$$\log_{sound speed sq} = -5.36^{+0.309}_{-1.55}$$



$$\begin{aligned}\lambda &\equiv c_2 \\ \beta &\equiv c_1 + c_3 \\ \alpha &\equiv c_1 + c_4\end{aligned}$$

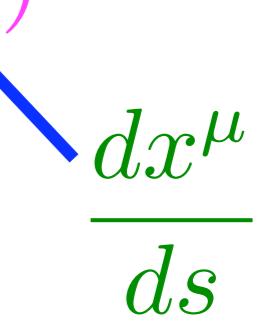


$$Y_{dm} = 0.343^{+0.324}_{-0.343}$$



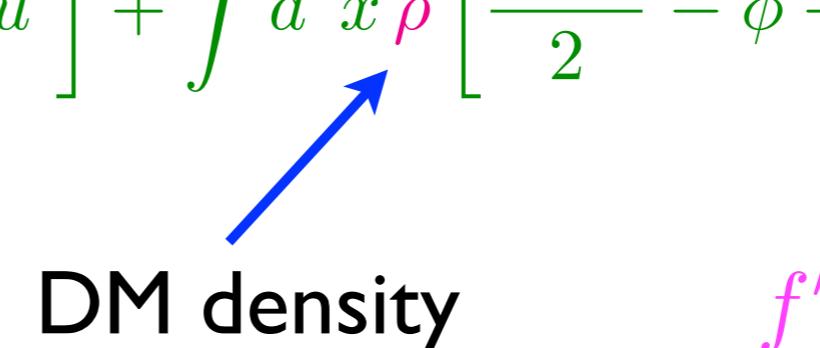
# Generalized point particle action

$$S_{pp} = -m \int ds \quad \Longrightarrow \quad -m \int ds f(u_\mu v^\mu)$$


 $\frac{dx^\mu}{ds}$

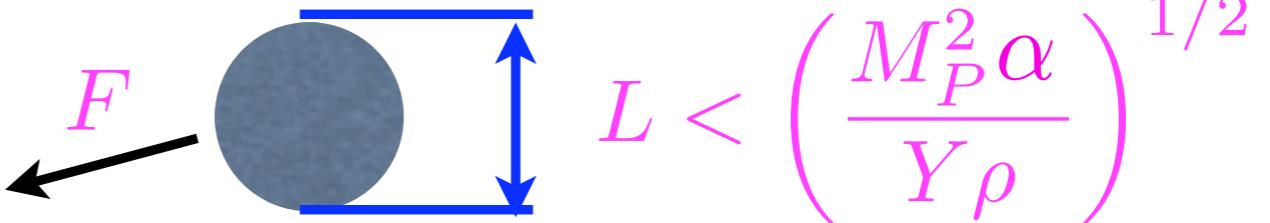
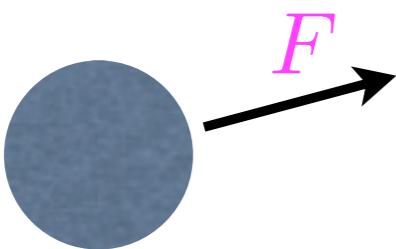
**Newtonian limit:**  $u^i, v^i$  - small

$$S = \int d^4x \left[ M_P^2 \phi \Delta \phi + \frac{M_P^2 \alpha}{2} u^i \Delta u^i \right] + \int d^4x \rho \left[ \frac{(v^i)^2}{2} - \phi - Y \frac{(u^i - v^i)^2}{2} \right]$$

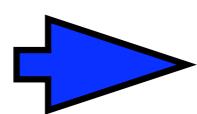

  
**DM density**       $f'(1)$

**Big halos:**  $\dot{v}_i = -\partial_i \phi$

**Small halos:**  $\dot{v}_i = -\frac{\partial_i \phi}{1 - Y}$



$$L < \left( \frac{M_P^2 \alpha}{Y \rho} \right)^{1/2}$$



$$F = \frac{F_N}{(1 - Y)}$$

$$m_{\text{inert}} = m_{\text{grav}} (1 - Y)$$

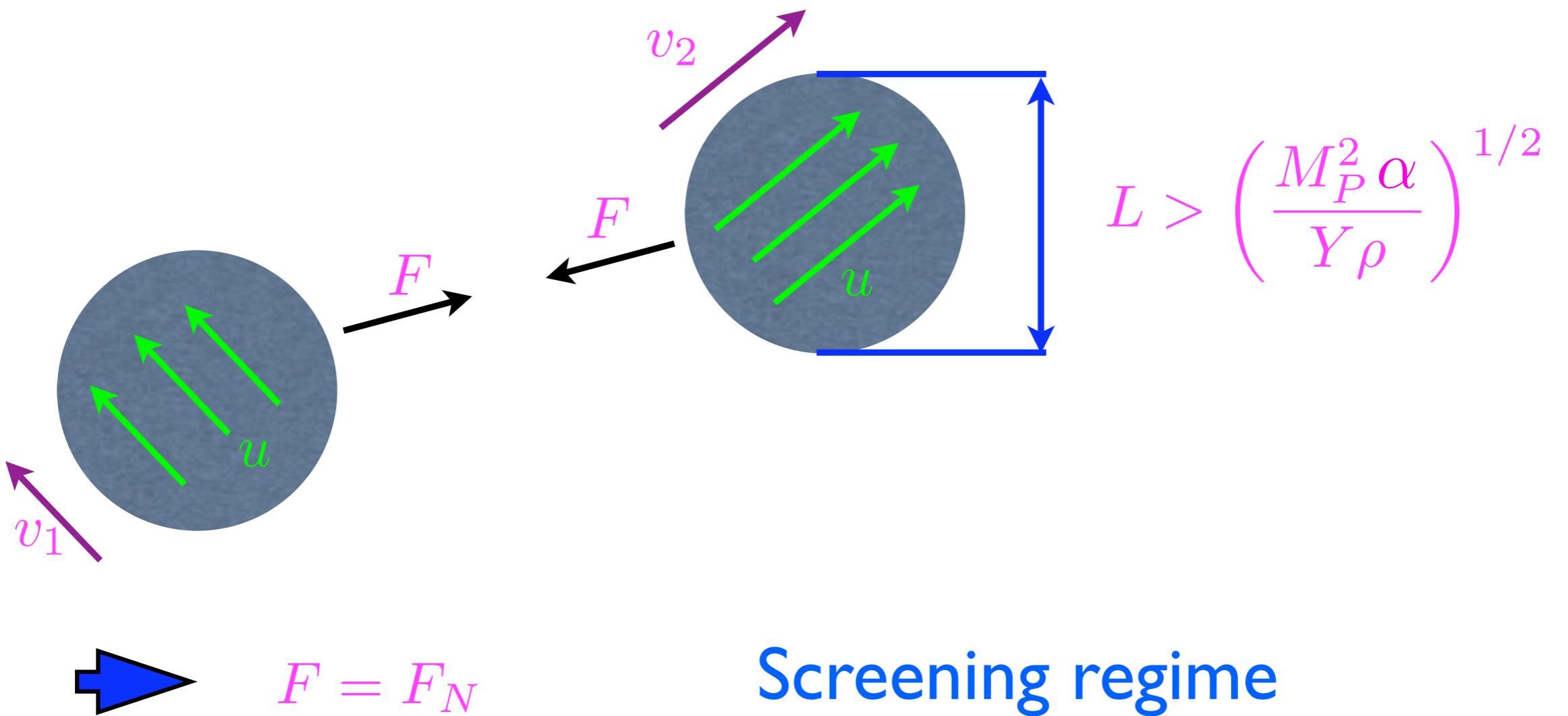
Accelerated Jeans instability !

$$\frac{\delta \rho}{\rho}$$

density contrast

$$\delta \propto \tau^\gamma,$$

$$\gamma = \frac{1}{6} \left[ -1 + \sqrt{\frac{25 - Y}{1 - Y}} \right]$$



**Standard growth of structures  
chameleon-like mechanism**

$$\delta \propto \tau^{2/3}$$