Bimetric Massive Gravity

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Outline

- Introduction
- Bimetric gravity
- Cosmology
- Matter coupling
- Conclusion

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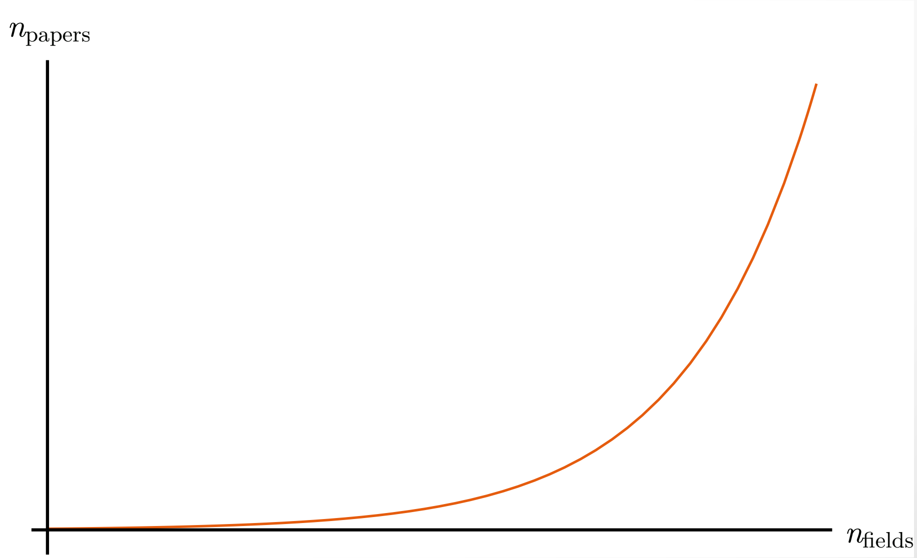
Motivations

- Why should the graviton be massless?

- Large distance modification:
  - dark energy?
  - dark matter?
  - degravitation?

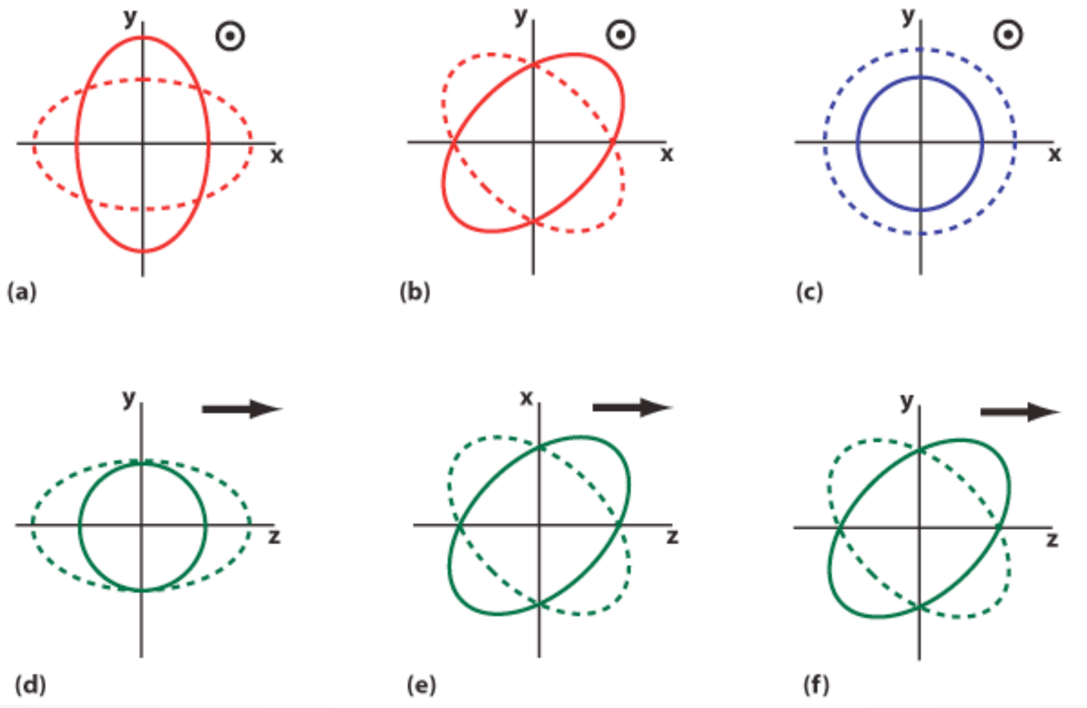


Polarizations

- GR: propagates 2 gravitational waves
- Metric has 10-4=6 degrees of freedom:

  + 2 vector modes

  + 2 scalar modes
- In massive theory these are set free



Fierz-Pauli theory

$$\int d^4x \underbrace{\sqrt{g}R_g} + m^2 \int d^4x h_{\mu\nu} h_{\alpha\beta} \left(\eta^{\mu\alpha}\eta^{\nu\beta} - \eta^{\mu\nu}\eta^{\alpha\beta}\right)$$

- Second order in $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$
- The only quadratic ghost-free theory for massive spin-2
- Breaks gauge invariance
- For reviews, see

Infrared-modified gravities and massive gravitons

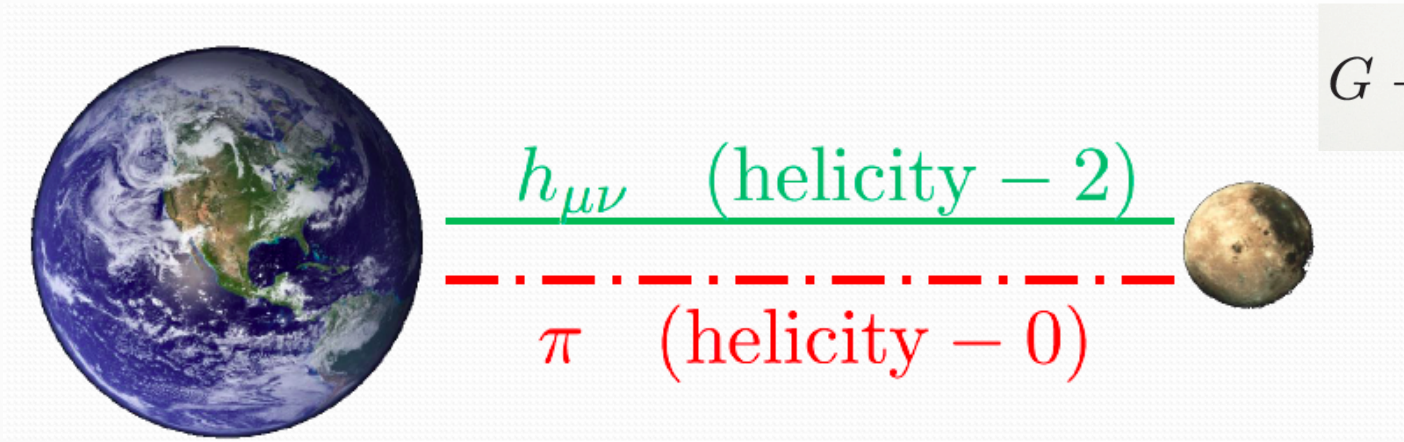
V.A. Rubakov[a] and P.G. Tinyakov[b,a]

Massive Gravity

Claudia de Rham

70's I: Newtonian limit

- The van Dam – Veltman – Zakharov discontinuity:



$$G \to \frac{4}{3} G$$

- The Vainshtein mechanism:

  nonlinear kinetic interactions
  screen the helicity-0 mode

$$F_{grav} = \begin{cases} \frac{GM}{R^2} & R < r* \\ \frac{4}{3}\frac{GM}{R^2} & R > r* \end{cases}$$

$$r_V = \left(\frac{M}{M_{Pl}^2 m_G^4} \right)^{1/5}$$

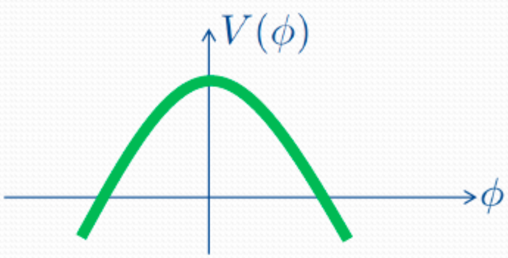
70's findings II: pathology

- <u>The Boulware-Deser ghost</u>:

  In curved backgrounds, the 6th polarisation is set to propagate. This is generically a ghost!

- A ***ghost*** is a field with the wrong sign kinetic term $S_{\phi,\chi} = \int d^4x \left(-\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 - V(\phi,\chi) \right)$

- Different from a ***Tachyon***, which has an instability in the potential

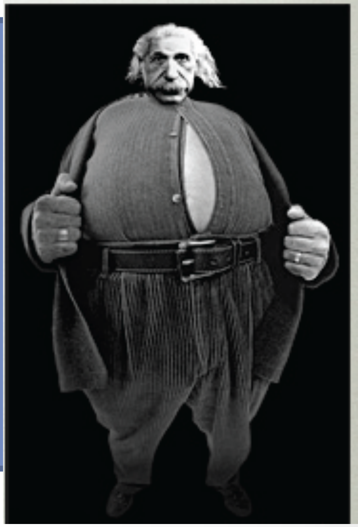


$$S = \int d^4x \left(-\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 \right)$$

Scale of instability: m

dRGT massive gravity

$$S = -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R$$
$$+ m^2 M_g^2 \int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$



- The unique non-linear action for a single massive spin-2
- The f is a "reference metric"
- e_n are elementary symmetric polynomials given by…

$$S=-\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R+m^2M_g^2\int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$

For a matrix X, the elementary symmetric polynomials are ([] = trace)

$$e_0(X)\equiv 1,$$

$$e_1(X)\equiv [X],$$

$$e_2(X)\equiv \frac{1}{2}\left([X]^2-[X^2]\right),$$

$$e_3(X)\equiv \frac{1}{6}\left([X]^3-3[X][X^2]+2[X^3]\right),$$

$$e_4(X)\equiv \det(X)$$

ADM analysis

$3+1$ dimensional parametrization of the gravitational field $g_{\mu\nu}$:

$$\gamma_{ij} = g_{ij}, \quad N = \sqrt{-g^{00}}, \quad N_i = g_{0i}$$

Arnowitt, Deser, Misner (1959)

allows to rewrite the action of general relativity in the first order form

$$\mathcal{L}_E = M_{Pl}^2 \sqrt{-g} R = M_{Pl}^2 \left[-\gamma_{ij} \partial_t \pi^{ij} - N R^0 - N_i R^i \right]$$

$\Rightarrow$ the Einstein action is linear in the lapse N and shift N_i $\Rightarrow$ 2 degrees of freedom!

Meanwhile, the interaction term of massive gravity in the unitary gauge can be written in a closed form as

$$\mathcal{L}_{FP} = 2 m_g^2 \sqrt{\gamma} N \left[\mathrm{tr} \sqrt{g^{-1}\eta} - 3 \right]$$

Hassan, Rosen (2011)

$\Rightarrow$ the massive gravity action is non-linear in N, N_i $\Rightarrow$ 6 degrees of freedom!

CLAIM: After an appropriate field redefinition $N^i = \left(\delta^i_j + N D^i_j \right) n^j$
the Lagrangian is linear in N and gives rise to a secondary constraint.

The reference metric

- …is required for local, Lorenz-invariant massive gravity

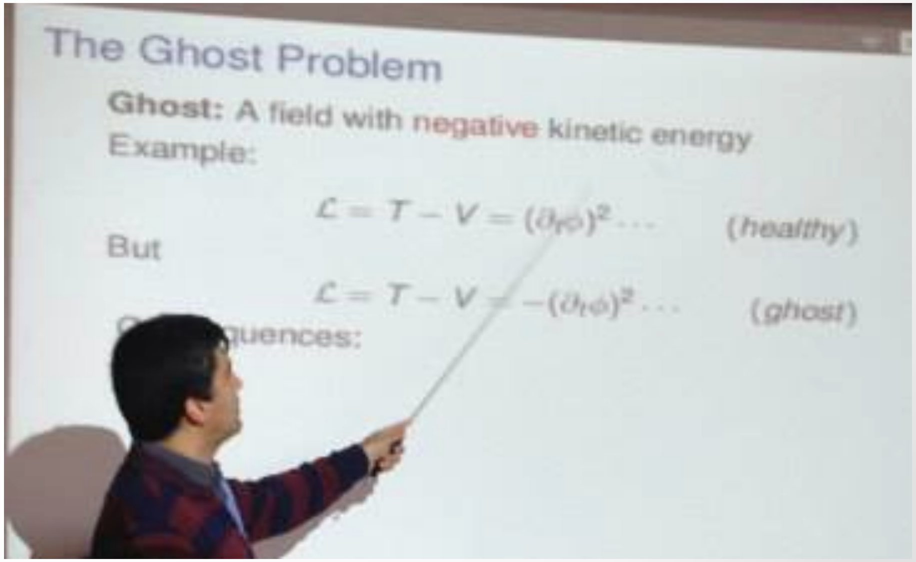
- *But*
  - There are infinite number of (arbitrary) dRGT's
  - Phenomenologically, it doesn't work too well!
  - Claims of extra theoretical problems too

- Resolution is simple…

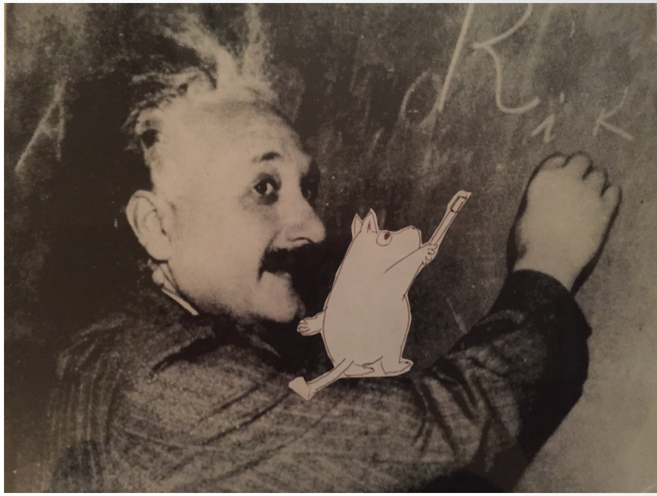
Hassan-Rosen theory

$$S = -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R(g) - \frac{M_f^2}{2}\int d^4x\sqrt{-\det f}R(f)$$
$$+ m^2 M_g^2 \int d^4x\sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{-1}f}\right)$$

- Now 5+2 d.o.f.'s propagate
- The theory of spin-2 fields
-



The bigravity field equations



$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \frac{m^2}{2}\sum_{n=0}^{3}(-1)^n\beta_n\left[g_{\mu\lambda}Y^{\lambda}_{(n)\nu}\left(\sqrt{g^{-1}f}\right) + g_{\nu\lambda}Y^{\lambda}_{(n)\mu}\left(\sqrt{g^{-1}f}\right)\right] = \frac{1}{M_g^2}T_{\mu\nu}, \qquad (2.3)$$

$$\bar{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\bar{R} + \frac{m^2}{2M_\star^2}\sum_{n=0}^{3}(-1)^n\beta_{4-n}\left[f_{\mu\lambda}Y^{\lambda}_{(n)\nu}\left(\sqrt{g^{-1}f}\right) + f_{\nu\lambda}Y^{\lambda}_{(n)\mu}\left(\sqrt{g^{-1}f}\right)\right] = 0, \qquad (2.4)$$

$$M_\star^2 \equiv M_f^2/M_g^2$$

$$Y_{(0)}(\mathbb{X}) = \mathbb{1}, \quad Y_{(1)}(\mathbb{X}) = \mathbb{X} - \mathbb{1}[\mathbb{X}], \quad Y_{(2)}(\mathbb{X}) = \mathbb{X}^2 - \mathbb{X}[\mathbb{X}] + \frac{1}{2}\mathbb{1}\left([\mathbb{X}]^2 - [\mathbb{X}^2]\right),$$

$$Y_{(3)}(\mathbb{X}) = \mathbb{X}^3 - \mathbb{X}^2[\mathbb{X}] + \frac{1}{2}\mathbb{X}\left([\mathbb{X}]^2 - [\mathbb{X}^2]\right) - \frac{1}{6}\mathbb{1}\left([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3]\right)$$

Massive bigravity has self-accelerating cosmologies

- Homogeneous and isotropic solution:

$$ds_g^2 = a^2\left(-d\tau^2 + d\vec{x}^2\right),$$
$$ds_f^2 = -X^2 d\tau^2 + Y^2 d\vec{x}^2$$

the background dynamics are determined by

$$3\mathcal{H}^2 = \frac{a^2\rho}{M_g^2} + m^2 a^2\left(\beta_0 + 3\beta_1 y + 3\beta_2 y^2 + \beta_3 y^3\right)$$

$$y \equiv \frac{Y}{a}$$

$$\beta_3 y^4 + (3\beta_2 - \beta_4)\, y^3 + 3\,(\beta_1 - \beta_3)\, y^2 + \left(\frac{\rho}{M_g^2 m^2} + \beta_0 - 3\beta_2\right) y - \beta_1 = 0$$

As ρ -> 0, y -> constant, so the mass term approaches a (positive) constant ➔ late-time acceleration

Confronting with the data

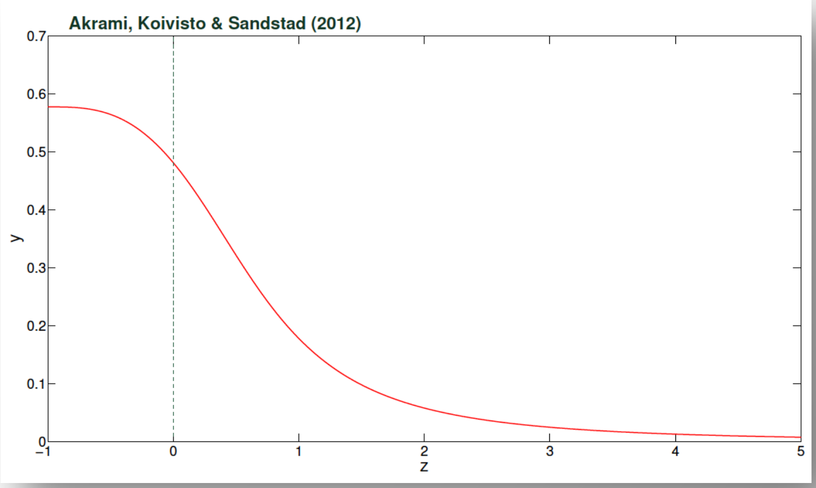
- A comprehensive comparison to background data was undertaken by Akrami, Koivisto, & Sandstad [arXiv:1209.0457]
- Data sets:
  - Luminosity distances from Type Ia supernovae (Union 2.1)
  - Position of the first CMB peak – angular scale of sound horizon at recombination (WMAP7)
  - Baryon-acoustic oscillations (2dFGRS, 6dFGS, SDSS and WiggleZ)

Take-home points:

No exact ΛCDM without explicit Λ

Phantom behavior (w < -1) is common

✓Viable alternative to ΛCDM

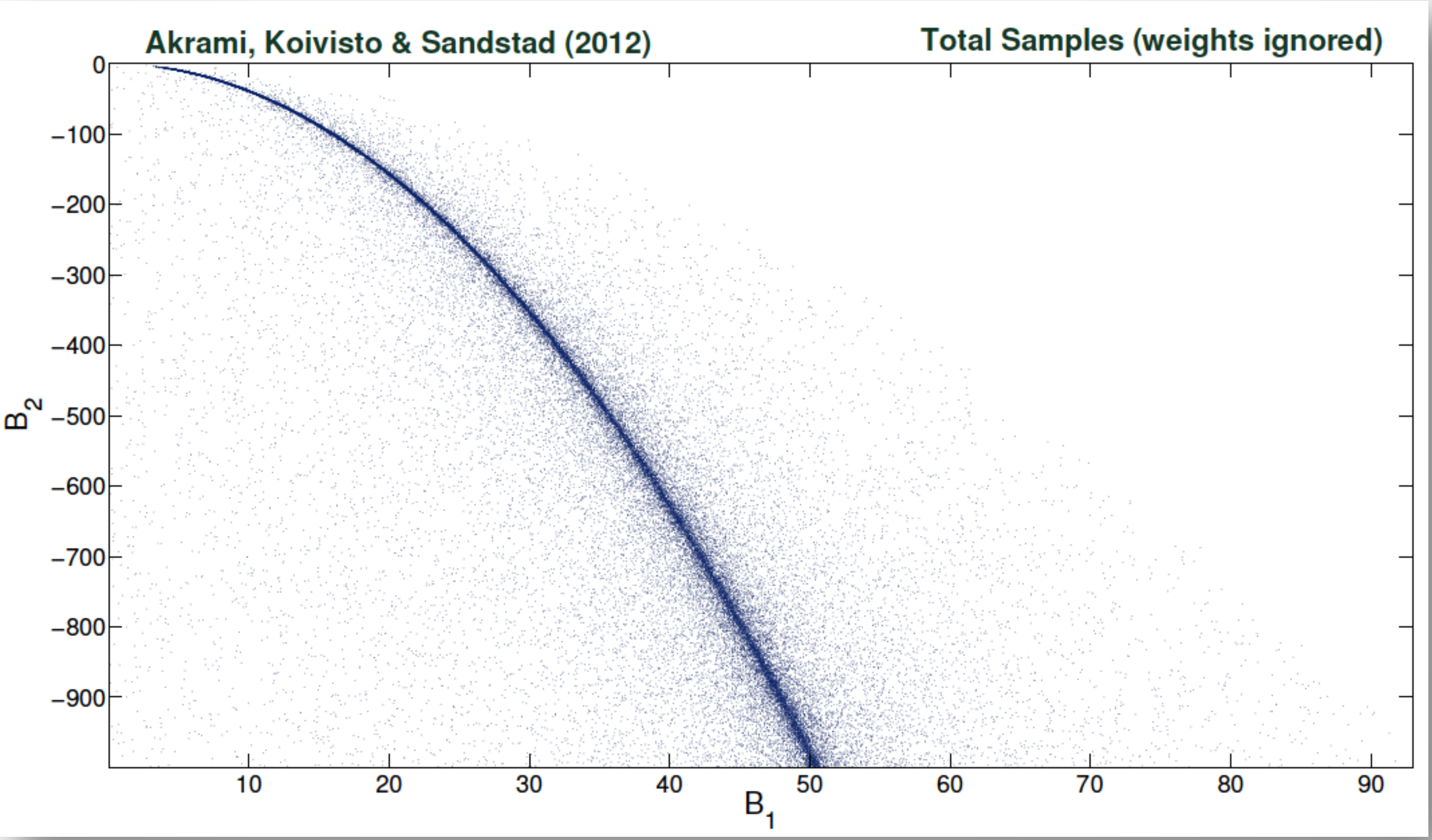


Massive bigravity effectively competes with ΛCDM

Y. Akrami, T. Koivisto, and M. Sandstad [arXiv:1209.0457]
See also F. Könnig, A. Patil, and L. Amendola [arXiv:1312.3208]; A. Solomon, Y. Akrami, and T. Koivisto [arXiv:1404.4061]

Model	B_0	B_1	B_2	B_3	B_4	Ω_m	χ^2_{min}	p-value	log-evidence
ΛCDM	free	0	0	0	0	free	546.54	0.8709	-278.50
(B_1, Ω_m^0)	0	free	0	0	0	free	551.60	0.8355	-281.73
~~(B_2, Ω_m^0)~~	~~0~~	~~0~~	~~free~~	~~0~~	~~0~~	~~free~~	~~894.00~~	~~< 0.0001~~	~~-450.25~~
~~(B_3, Ω_m^0)~~	~~0~~	~~0~~	~~0~~	~~free~~	~~0~~	~~free~~	~~1700.50~~	~~< 0.0001~~	~~-850.26~~
(B_1, B_2, Ω_m^0)	0	free	free	0	0	free	546.52	0.8646	-279.77
(B_1, B_3, Ω_m^0)	0	free	0	free	0	free	542.82	0.8878	-280.10
~~(B_2, B_3, Ω_m^0)~~	~~0~~	~~0~~	~~free~~	~~free~~	~~0~~	~~free~~	~~548.04~~	~~0.8543~~	~~-280.91~~
(B_1, B_4, Ω_m^0)	0	free	0	0	free	free	548.86	0.8485	-281.42
~~(B_2, B_4, Ω_m^0)~~	~~0~~	~~0~~	~~free~~	~~0~~	~~free~~	~~free~~	~~806.82~~	~~< 0.0001~~	~~-420.87~~
~~(B_3, B_4, Ω_m^0)~~	~~0~~	~~0~~	~~0~~	~~free~~	~~free~~	~~free~~	~~685.30~~	~~0.0023~~	~~-351.14~~
$(B_1, B_2, B_3, \Omega_m^0)$	0	free	free	free	0	free	546.50	0.8582	-279.61
$(B_1, B_2, B_4, \Omega_m^0)$	0	free	free	0	free	free	546.52	0.8581	-279.56
$(B_1, B_3, B_4, \Omega_m^0)$	0	free	0	free	free	free	546.78	0.8563	-280.00
~~$(B_2, B_3, B_4, \Omega_m^0)$~~	~~0~~	~~0~~	~~free~~	~~free~~	~~free~~	~~free~~	~~549.68~~	~~0.8353~~	~~-282.89~~
$(B_1, B_2, B_3, B_4, \Omega_m^0)$	0	free	free	free	free	free	546.50	0.8515	-279.60
full bigravity model	free	free	free	free	free	free	546.50	0.8445	-279.82

Degeneracies



Scalar perturbations in massive bigravity

-These models provide a good fit to the background data, but look similar to ΛCDM and can be degenerate with each other.
-Can we tease these models apart by looking beyond the background to structure formation? (Spoiler alert: yes.)

- Extensive analysis of perturbations undertaken by A. Solomon, Y. Akrami, and T. Koivisto, arXiv:1404.4061
- Linearize metrics around FRW backgrounds, restrict to scalar perturbations $\{E_{g,f}, A_{g,f}, F_{g,f},$ and $B_{g,f}\}$:

$$ds_g^2 = a^2 \left\{-(1+E_g)d\tau^2 + 2\partial_i F_g d\tau dx^i + \left[(1+A_g)\delta_{ij} + \partial_i\partial_j B_g\right] dx^i dx^j\right\}$$
$$ds_f^2 = -X^2(1+E_f)d\tau^2 + 2XY\partial_i F_f d\tau dx^i + Y^2\left[(1+A_f)\delta_{ij} + \partial_i\partial_j B_f\right] dx^i dx^j$$

Linearized Einstein Equations for g

0-0:

$$\frac{3H}{N^2}\left(HE_g-\dot{A}_g\right)+\nabla^2\left[\frac{A_g}{a^2}+H\left(\frac{2F_g}{Na}-\frac{\dot{B}_g}{N^2}\right)\right]+\frac{m^2}{2}yP\left(3\Delta A+\nabla^2\Delta B\right)=\frac{1}{M_g^2}\delta T^0{}_0,$$

0-i:

$$\frac{1}{N^2}\partial_i\left(\dot{A}_g-HE_g\right)+m^2\frac{P}{x+y}\frac{Y}{N}\partial_i\left(xF_f-yF_g\right)=\frac{1}{M_g^2}\delta T^0{}_i,$$

i-i:

$$\frac{1}{N^2}\left[\left(2\dot{H}+3H^2-2\frac{\dot{N}}{N}H\right)E_g+H\dot{E}_g-\ddot{A}_g-3H\dot{A}_g+\frac{\dot{N}}{N}\dot{A}_g\right]+\frac{1}{2}\left(\partial_j^2+\partial_k^2\right)D_g$$

$$+m^2\left[\frac{1}{2}xP\Delta E+yQ\left(\Delta A+\frac{1}{2}\left(\partial_j^2+\partial_k^2\right)\Delta B\right)\right]=\frac{1}{M_g^2}\delta T^i{}_i,$$

$$D_g\equiv\frac{A_g+E_g}{a^2}+\frac{H}{N}\left(\frac{4F_g}{a}-\frac{3\dot{B}_g}{N}\right)+\frac{2\dot{F}_g}{Na}-\frac{1}{N^2}\left(\ddot{B}_g-\frac{\dot{N}}{N}\dot{B}_g\right),$$

Off-diagonal i-j:

$$-\frac{1}{2}\partial^i\partial_jD_g-\frac{m^2}{2}yQ\partial^i\partial_j\Delta B=\frac{1}{M_g^2}\delta T^i{}_j,$$

$$P\equiv\beta_1+2\beta_2y+\beta_3y^2$$

$$Q\equiv\beta_1+(x+y)\,\beta_2+xy\beta_3$$

$$x\equiv X/N \qquad y\equiv Y/a$$

$$\Delta A\equiv A_f-A_g$$

$$\Delta B\equiv B_f-B_g$$

$$\Delta E\equiv E_f-E_g$$

Linearized Einstein Equations for f

0-0:

$$\frac{3K}{X^2}\left(KE_f-\dot{A}_f\right)+\nabla^2\left[\frac{A_f}{Y^2}+K\left(\frac{2F_f}{XY}-\frac{\dot{B}_f}{X^2}\right)\right]-\frac{m^2}{2M_\star^2}\frac{P}{y^3}\left(3\Delta A+\nabla^2\Delta B\right)=0,$$

0-i:

$$\frac{1}{X^2}\partial_i\left(\dot{A}_f-KE_f\right)+\frac{m^2}{M_\star^2}\frac{P}{y^2}\frac{1}{x+y}\frac{a}{X}\partial_i\left(yF_g-xF_f\right)=0,$$

i-i:

$$\frac{1}{X^2}\left[\left(2\dot{K}+3K^2-2\frac{\dot{X}}{X}K\right)E_f+K\dot{E}_f-\ddot{A}_f-3K\dot{A}_f+\frac{\dot{X}}{X}\dot{A}_f\right]+\frac{1}{2}\left(\partial_j^2+\partial_k^2\right)D_f$$
$$-\frac{m^2}{M_\star^2}\frac{1}{xy^2}\left[\frac{1}{2}P\Delta E+Q\left(\Delta A+\frac{1}{2}\left(\partial_j^2+\partial_k^2\right)\Delta B\right)\right]=0,$$
$$D_f\equiv\frac{A_f+E_f}{Y^2}+\frac{K}{X}\left(\frac{4F_f}{Y}-\frac{3\dot{B}_f}{X}\right)+\frac{2\dot{F}_f}{XY}-\frac{1}{X^2}\left(\ddot{B}_f-\frac{\dot{X}}{X}\dot{B}_f\right),$$

Off-diagonal i-j:

$$-\frac{1}{2}\partial^i\partial_jD_f+\frac{m^2}{2M_\star^2}\frac{Q}{xy^2}\partial^i\partial_j\Delta B=0,$$

Perturbations in massive bigravity: subhorizon

A. Solomon, Y. Akrami, and T. Koivisto, arXiv:1404.4061 (gory details)

- Most observations of cosmic structure are taken in the subhorizon limit: $k^2\Phi \gg H^2\Phi \sim H\dot{\Phi} \sim \ddot{\Phi}$
- Specializing to this limit, and assuming only matter is dust (P=0)…
  - Five perturbations ($E_{g,f}$, $A_{g,f}$, and $B_f - B_g$) are determined algebraically in terms of the density perturbation δ
  - Meanwhile, δ is determined by the same evolution equation as in GR:

$$\delta'' + \mathcal{H}\delta' + \frac{1}{2}k^2 E_g(\delta) = 0$$

$$\delta'' + \mathcal{H}\delta' + \frac{1}{2}k^2 E_g(\delta) = 0$$

(GR *and* massive bigravity)

- In GR, there is no anisotropic stress so E_g (time-time perturbation) is related to δ through Poisson's equation,

$$k^2 E_g = -(a^2 \bar{\rho}/M_g^2)\delta$$

- In bigravity, the relation beteen E_g and δ is significantly more complicated
➔ modified structure growth

The "observables"

$$ds_g^2 = a^2\left[-(1+E_g)d\tau^2 + (1+A_g)\delta_{ij}dx^i dx^j\right]$$

We calculate three parameters which are commonly used to distinguish modified gravity from GR:

- **Growth rate/index** (f/γ): measures growth of structures

$$f(a,k) \equiv \frac{d\log\delta}{d\log a} \approx \Omega_m^\gamma$$

- **Modification of Newton's constant** in Poisson eq. (Q):

$$\frac{k^2}{a^2}A_g \equiv \frac{Q(a,k)\bar{\rho}}{M_g^2}\delta$$

- **Anisotropic stress** (η):

$$\eta(a,k) \equiv -\frac{A_g}{E_g}$$

GR:

$$\gamma \approx 0.545$$
$$Q = \eta = 1$$

The "observables":

- We have analytic solutions (messy) for A_g and E_g as (stuff) x δ, so
  - Can immediately read off analytic expressions for Q and η:

$$Q = h_1 \left(\frac{1 + k^2 h_4}{1 + k^2 h_3} \right), \qquad \eta = h_2 \left(\frac{1 + k^2 h_4}{1 + k^2 h_5} \right)$$

    (h_i are non-trivial functions of time…)
  - Can solve numerically for δ using Q and η:

$$\delta'' + \mathcal{H}\delta' - \frac{1}{2} \frac{Q}{\eta} \frac{a^2 \bar{\rho}}{M_g^2} \delta = 0$$

The h-functions 1

$$h_1 = \frac{1}{1+y^2},$$

$$h_2 = -\frac{\left(1+y^2\right)\left(\beta_1 + 3\beta_3 y^4 + (6\beta_2 - 2\beta_4)\, y^3 + 3\,(\beta_1 - \beta_3)\, y^2\right)}{-\beta_1 + (3\beta_2 - \beta_4)\, y^5 + (6\beta_1 - 9\beta_3)\, y^4 + (3\beta_0 - 15\beta_2 + 2\beta_4)\, y^3 + (3\beta_3 - 7\beta_1)\, y^2},$$

$$\begin{aligned} h_3 = -\frac{y^2}{h_6}\frac{3}{1+y^2}\Big[&\beta_3^2 y^7 + (4\beta_2\beta_3 - 2\beta_3\beta_4)\, y^6 + 3\,(2\beta_1 - 3\beta_3)\,\beta_3 y^5 + (4\beta_0\beta_3 - 19\beta_2\beta_3 - \beta_4\beta_3 + 2\beta_1\beta_4)\, y^4 \\ &+ \left(-3\beta_1^2 - 18\beta_2^2 - 6\beta_3^2 + 4\beta_0\beta_2 + 2\beta_2\beta_4\right) y^3 + 3\left((\beta_0 - 3\beta_2)\,\beta_3 + \beta_1\,(\beta_4 - 5\beta_2)\right) y^2 \\ &+ \left(-7\beta_1^2 + 2\beta_3\beta_1 + 2\,(\beta_0 - 3\beta_2)\,\beta_2\right) y - \beta_1\,(\beta_0 + \beta_2)\Big], \end{aligned}$$

$$\begin{aligned} h_4 = \frac{y^2}{h_6}\Big[&2\beta_3\beta_4 y^6 + 2\left(3\beta_2^2 - \beta_4\beta_2 - 3\,(\beta_1 - 2\beta_3)\,\beta_3\right) y^5 + \left(\beta_1\,(6\beta_2 - 4\beta_4) + 3\beta_3\,(-2\beta_0 + 9\beta_2 + \beta_4)\right) y^4 \\ &+ 2\left(3\beta_1^2 - 2\beta_3\beta_1 + 18\beta_2^2 + 9\beta_3^2 - 3\beta_0\beta_2 - 3\beta_2\beta_4\right) y^3 + (37\beta_1\beta_2 + 27\beta_3\beta_2 - 9\beta_0\beta_3 - 9\beta_1\beta_4)\, y^2 \\ &+ 2\left(10\beta_1^2 - 3\beta_3\beta_1 - 3\,(\beta_0 - 3\beta_2)\,\beta_2\right) y + 3\beta_1\,(\beta_0 + \beta_2)\Big], \end{aligned}$$

$$\begin{aligned} h_6 = 3m^2a^2\left(1+y^2\right)\left(\beta_1 + \beta_3 y^2 + 2\beta_2 y\right)\Big(&\beta_1^2 + \beta_3\beta_4 y^5 + \left(3\beta_2^2 - \beta_4\beta_2 - 3\,(\beta_1 - 2\beta_3)\,\beta_3\right) y^4 \\ &+ (3\beta_1\beta_2 + 12\beta_3\beta_2 - 3\beta_0\beta_3 - 2\beta_1\beta_4)\, y^3 + \left(3\beta_1^2 + \beta_3\beta_1 - 3\,(\beta_0 - 3\beta_2)\,\beta_2\right) y^2 + 5\beta_1\beta_2 y\Big), \end{aligned}$$

$$h_7 = \frac{\left(\beta_1 + y\,(2\beta_2 + \beta_3 y)\right)\left(3\beta_0 y^3 + y^2\left(3\beta_2 y\left(y^2 - 5\right) + \beta_3\left(3 - 9y^2\right) - \beta_4 y\left(y^2 - 2\right)\right) + \beta_1\left(6y^4 - 7y^2 - 1\right)\right)}{1+y^2}$$

The h-functions 2

$$
\begin{aligned}
h_5 = -\frac{y^2}{h_6}\frac{1}{h_7}\Big[& 4\beta_3^2\beta_4^2 y^{11} + \beta_3\left(24\beta_4\beta_2^2 + \left(9\beta_3^2 - 8\beta_4^2\right)\beta_2 + 3\beta_3\left(19\beta_3 - 8\beta_1\right)\beta_4\right) y^{10} \\
& + 2\left(18\beta_2^4 - 12\beta_4\beta_2^3 + \left(117\beta_3^2 - 36\beta_1\beta_3 + 2\beta_4^2\right)\beta_2^2 + 6\beta_3\left(4\beta_1 + 5\beta_3\right)\beta_4\beta_2\right. \\
& \quad \left. + \beta_3\left(99\beta_3^3 - 81\beta_1\beta_3^2 + 18\beta_1^2\beta_3 - 3\beta_4^2\beta_3 - 12\beta_0\beta_4\beta_3 - 8\beta_1\beta_4^2\right)\right) y^9 \\
& - \left(72\beta_3\left(\beta_2 - \beta_4\right)\beta_1^2 + \left(-72\beta_2^3 + 72\beta_4\beta_2^2 + \left(117\beta_3^2 - 16\beta_4^2\right)\beta_2 + \beta_3^2\left(85\beta_4 - 72\beta_0\right)\right)\beta_1\right. \\
& \quad \left. + 9\beta_3\left(-60\beta_2^3 + 8\beta_0\beta_2^2 + 12\beta_4\beta_2^2 - 96\beta_3^2\beta_2 + 19\beta_0\beta_3^2 + 5\beta_3^2\beta_4\right)\right) y^8 \\
& - 2\left(36\beta_3\beta_1^3 - \left(54\beta_2^2 - 36\beta_4\beta_2 + 69\beta_3^2 + 8\beta_4^2\right)\beta_1^2\right. \\
& \quad - 2\beta_3\left(123\beta_2^2 - 76\beta_4\beta_2 + 3\left(13\beta_3^2 + \beta_4\left(4\beta_0 + \beta_4\right)\right)\right)\beta_1 \\
& \quad - 3\left(72\beta_2^4 - 36\beta_4\beta_2^3 + \left(255\beta_3^2 + 4\beta_4^2\right)\beta_2^2 - 21\beta_3^2\beta_4\beta_2 - 9\beta_3^4 + 6\beta_0^2\beta_3^2\right. \\
& \qquad \left.\left. + \beta_0\left(-12\beta_2^3 + 4\beta_4\beta_2^2 - 93\beta_3^2\beta_2 + 3\beta_3^2\beta_4\right)\right)\right) y^7 \\
& + \left(24\left(3\beta_2 - 2\beta_4\right)\beta_1^3 + \beta_3\left(-72\beta_0 + 507\beta_2 - 77\beta_4\right)\beta_1^2\right. \\
& \quad + \left(876\beta_2^3 - 508\beta_4\beta_2^2 + 600\beta_3^2\beta_2 + 48\beta_4^2\beta_2 - 3\beta_3^2\beta_4 + \beta_0\left(-72\beta_2^2 + 48\beta_4\beta_2 - 69\beta_3^2\right)\right)\beta_1 \\
& \quad \left. + 3\beta_3\left(24\beta_2\beta_0^2 + \left(-228\beta_2^2 + 16\beta_4\beta_2 + 9\beta_3^2\right)\beta_0 + 9\beta_2\left(48\beta_2^2 - 4\beta_4\beta_2 - 7\beta_3^2\right)\right)\right) y^6 \\
& + 2\left(18\beta_1^4 + 45\beta_3\beta_1^3 + \left(477\beta_2^2 - 36\beta_0\beta_2 - 170\beta_4\beta_2 + 14\beta_3^2 + 9\beta_4^2\right)\beta_1^2\right. \\
& \quad + 3\beta_3\left(126\beta_2^2 - 42\beta_0\beta_2 + 6\beta_4\beta_2 - 3\beta_3^2 + 2\beta_0\beta_4\right)\beta_1 \\
& \quad \left. + 6\left(\beta_0 - 3\beta_2\right)\beta_2\left(-15\beta_2^2 + 3\beta_0\beta_2 + 2\beta_4\beta_2 + 6\beta_3^2\right)\right) y^5 \\
& + \left(\left(441\beta_2 - 79\beta_4\right)\beta_1^3 + \beta_3\left(-33\beta_0 - 8\beta_2 + 33\beta_4\right)\beta_1^2\right. \\
& \quad \left. + \left(648\beta_2^3 - 156\beta_0\beta_2^2 - 60\beta_4\beta_2^2 + 9\beta_3^2\beta_2 + 9\beta_0\beta_3^2\right)\beta_1 + 36\left(\beta_0 - 3\beta_2\right)\beta_2^2\beta_3\right) y^4 \\
& + 2\beta_1\left(39\beta_1^3 - 26\beta_3\beta_1^2 + \left(167\beta_2^2 - 15\beta_4\beta_2 + 15\beta_3^2 - 3\beta_0\left(\beta_2 + \beta_4\right)\right)\beta_1 - 12\beta_0\beta_2\beta_3\right) y^3 \\
& + \beta_1\left(36\beta_2^3 + 9\beta_1\beta_3\beta_2 + 3\beta_0\left(3\beta_1^2 - 5\beta_3\beta_1 - 4\beta_2^2\right) + \beta_1^2\left(112\beta_2 - 9\beta_4\right)\right) y^2 \\
& + 2\beta_1^2\left(11\beta_1^2 - 3\beta_3\beta_1 + 12\beta_2^2\right) y + 3\beta_1^3\left(\beta_0 + \beta_2\right)\Big],
\end{aligned}
\tag{B.8}
$$

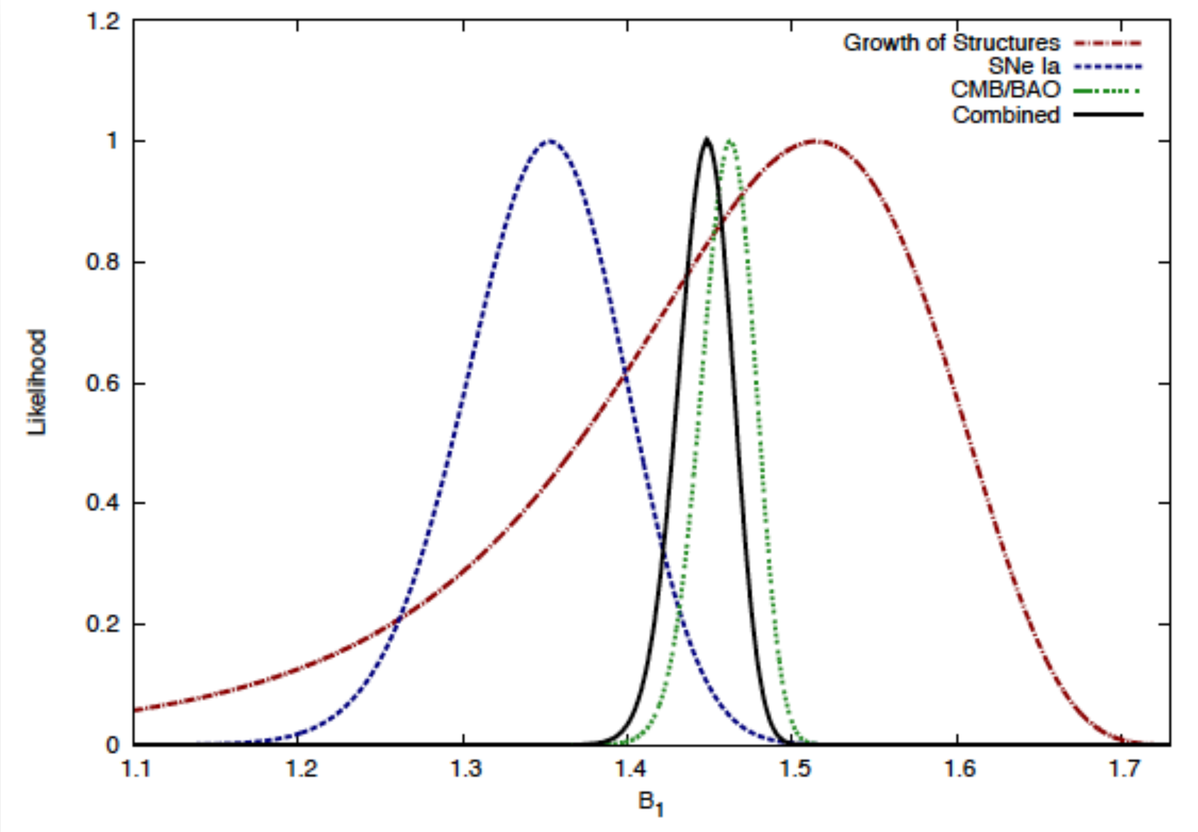
The Minimal Model (B_1)

SNe only:

$B_1 = 1.3527 \pm 0.0497$

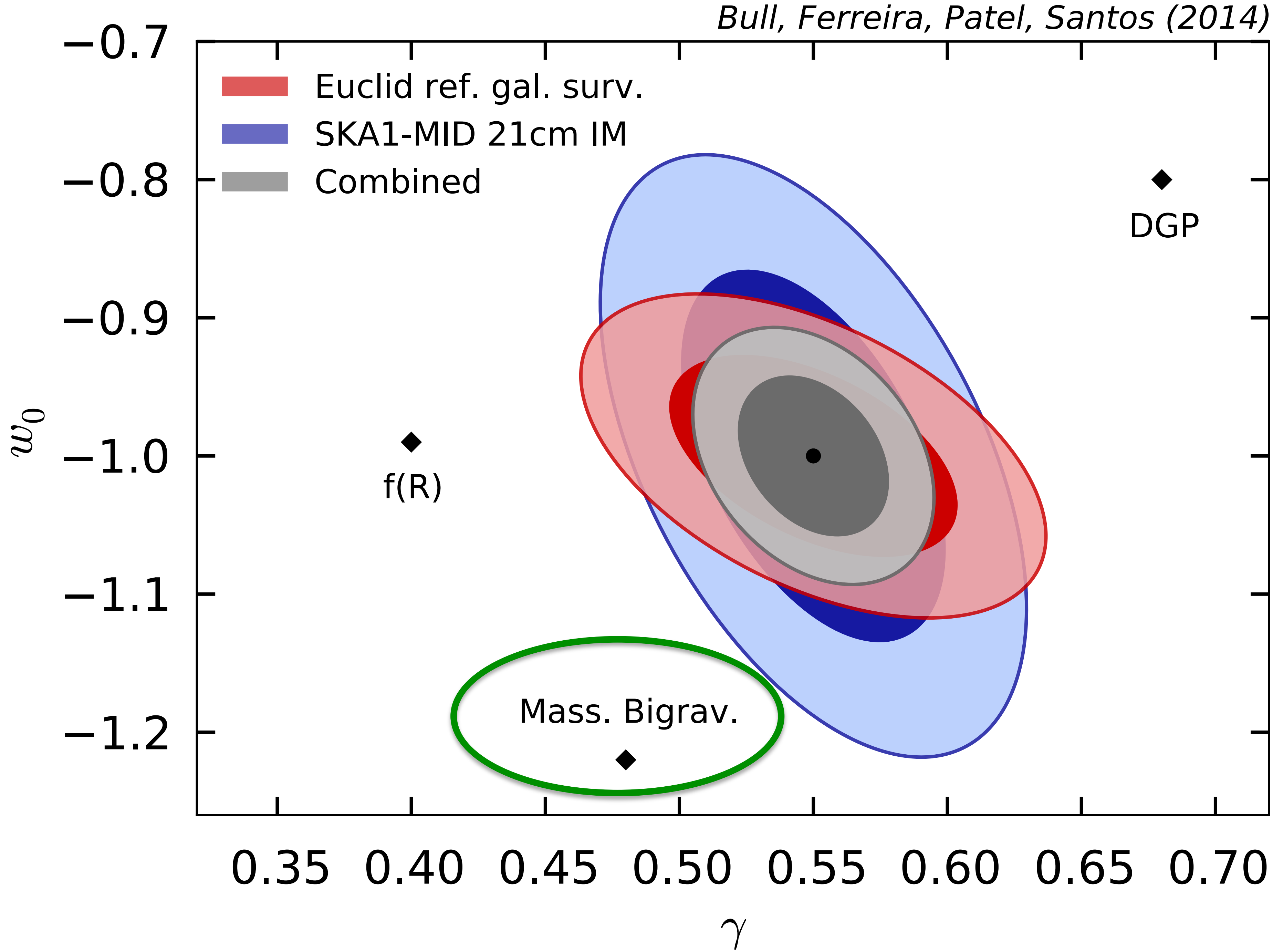
SNe + CMB + BAO:

$B_1 = 1.448 \pm 0.0168$



A. Solomon, Y. Akrami, T. Koivisto [arXiv:1404.4061] F. Könnig, L. Amendola [arXiv:1402.1988]

$$w(z) \approx -1.22^{+0.02}_{-0.02} - 0.64^{+0.05}_{-0.04} z/(1+z)$$


Bull, Ferreira, Patel, Santos (2014)
Euclid ref. gal. surv.
SKA1-MID 21cm IM
Combined
DGP
f(R)
Mass. Bigrav.
w_0
γ
−0.7
−0.8
−0.9
−1.0
−1.1
−1.2
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70

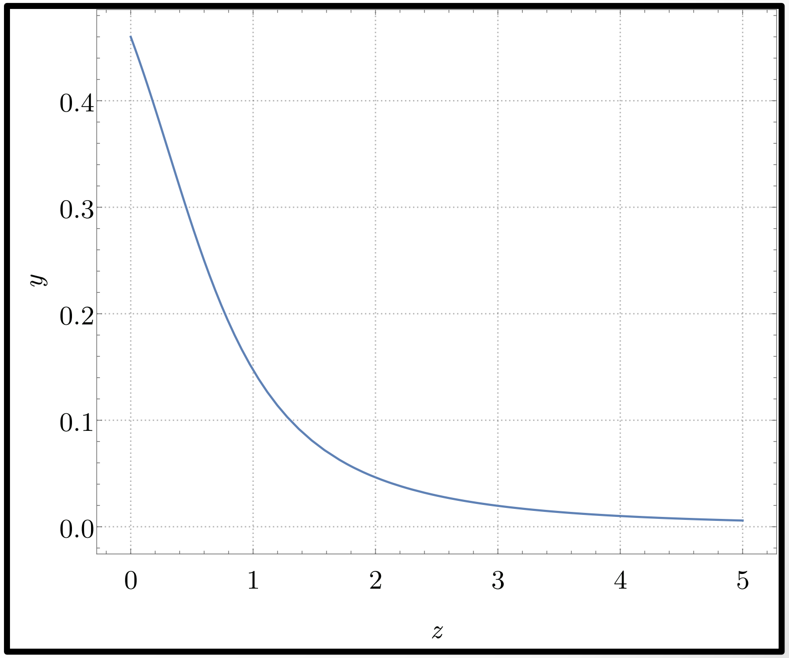
Scalar fluctuations can suffer from instabilities

- B_1-only model – simplest allowed by background

$$\omega_{B_1} = \pm \frac{k}{H} \frac{\sqrt{-1 + 12y^2 + 9y^4}}{1 + 3y^2}$$

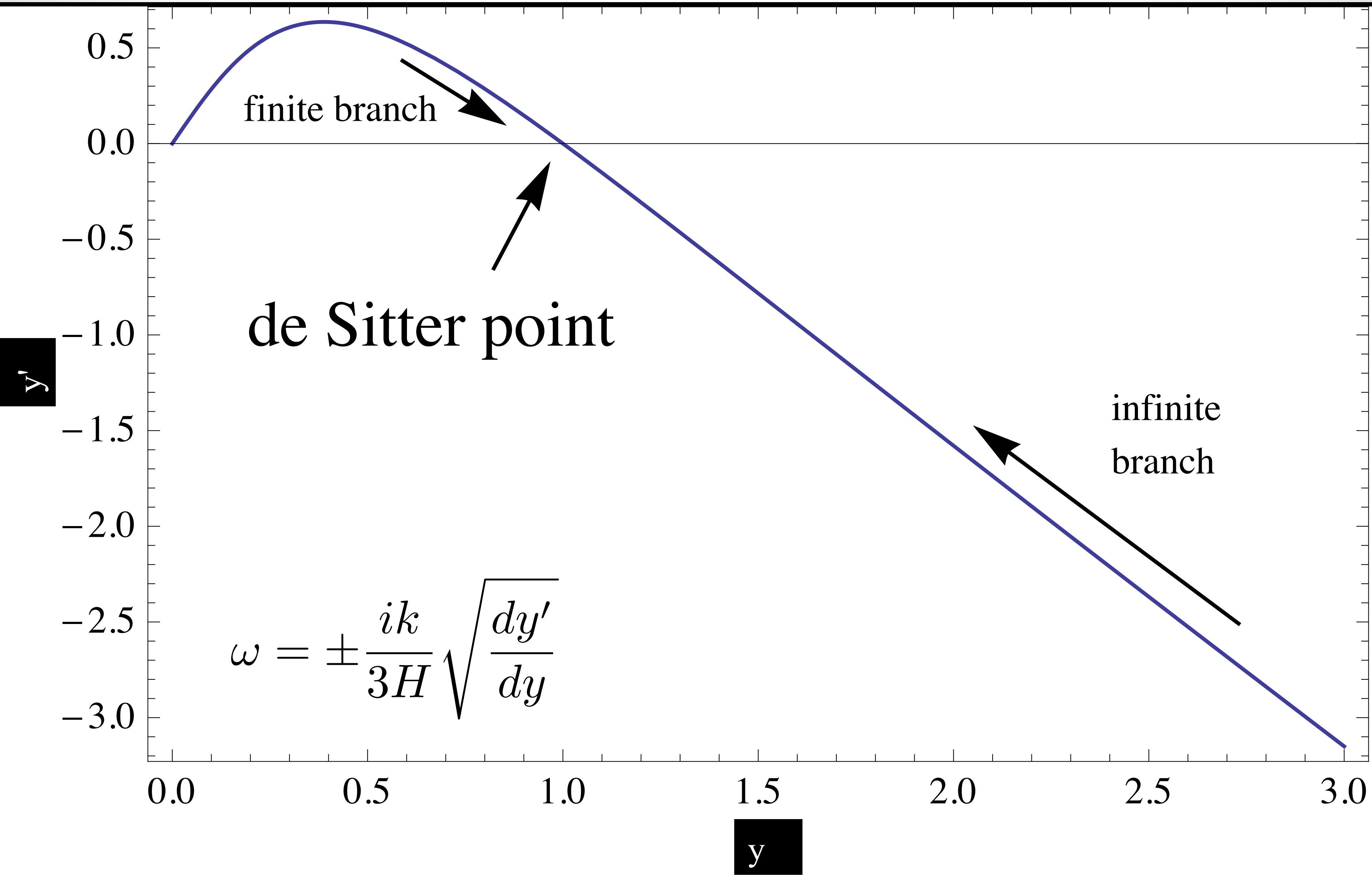
- **Unstable** for small y (early times)

For realistic parameters, model is only (linearly) stable for z <~ 0.5



- The instability is avoided by **infinite-branch** solutions, where y starts off at infinity
- Viability requires $B_1 > 0$
- Existence of infinite branch requires $0 < B_4 < 2B_1$
  - i.e., turn on the f-metric cosmological constant

B_1-B_4 model: background dynamics



$$\omega = \pm \frac{ik}{3H}\sqrt{\frac{dy'}{dy}}$$

Nature of Spacetime: 2 Metrics or None?

Y. Akrami, T. Koivisto, A. Solomon [arXiv:1404.0006]

$$S = -\int d^4x\sqrt{-g}\left[\frac{M_g^2}{2}R(g) - \alpha_g \mathcal{L}_m(g,\Psi)\right] - \int d^4x\sqrt{-f}\left[\frac{M_f^2}{2}R(f) - \alpha_f \mathcal{L}_m(f,\Psi)\right]$$
$$+ m^2 M_g^2 \int d^4x\sqrt{-g}\sum_{n=0}^{4}\beta_n e_n\left(\sqrt{g^{\mu\alpha}f_{\alpha\nu}}\right) .$$

No physical Riemannian metric exists to which matter minimally couples.

For photons (we make observations by tracking photons):

$$S_A = -\frac{1}{4}\alpha_g \int d^4x\sqrt{-g}g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} - \frac{1}{4}\alpha_f \int d^4x\sqrt{-f}f^{\mu\alpha}f^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$$

Minimally coupled to an effective metric h, if:

$$S_A = -\frac{1}{4}\int d^4x\sqrt{-h}h^{\mu\alpha}h^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$$

$$\alpha_g\sqrt{-g}g^{\mu\nu}g^{\alpha\beta} + \alpha_f\sqrt{-f}f^{\mu\nu}f^{\alpha\beta} = \sqrt{-h}h^{\mu\nu}h^{\alpha\beta}$$

This overconstrains h (in general, cannot simultaneously satisfy 00-00, 00-ii and ii-ii components).

Similar for other fields, such as a massive scalar; massless scalar does have an effective metric:

$$\sqrt{-h}h^{\mu\nu} = \alpha_g\sqrt{-g}g^{\mu\nu} + \alpha_f\sqrt{-f}f^{\mu\nu}$$

Nature of Spacetime: 2 Metrics or None?

- Possesses mathematically two metrics, but physically none.
- We need to step beyond the confines of metric geometry.

Point-particle of mass m (simplest possible type of matter):

$$S_{\rm pp} = -m\alpha_g \int dt \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} - m\alpha_f \int dt \sqrt{f_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}$$
$$= -m\alpha_g \int ds_g - m\alpha_g \int ds_f,$$

Geodesic equation:

$$\alpha_g g_{\alpha\beta}\left(\frac{du_g^\alpha}{ds_g} + \overset{g}{\Gamma}{}^\alpha_{\mu\nu} u_g^\mu u_g^\nu\right) + \alpha_f f_{\alpha\beta}\frac{ds_f}{ds_g}\left(\frac{du_f^\alpha}{ds_f} + \overset{f}{\Gamma}{}^\alpha_{\mu\nu} u_f^\mu u_f^\nu\right) = 0$$

$$u_g^\mu \equiv dx^\mu / ds_g$$

Not the geodesic equation for any Riemannian metric:

$$S_{\rm pp} = -m \int ds$$

$$ds^2 = \left(\alpha_g^2 g_{\mu\nu} + \alpha_f^2 f_{\mu\nu}\right) dx^\mu dx^\nu + 2\alpha_g \alpha_f \sqrt{g_{\mu\nu} f_{\alpha\beta} dx^\mu dx^\nu dx^\alpha dx^\beta}$$

Line element of a Finsler geometry!

Nature of Spacetime: 2 Metrics or None?

Finslerian geometry:

The line element is the most general one that is homogeneous of degree 2 in coordinate intervals:

$$ds^2 = f(x^\mu, dx^\nu); \qquad f(x^\mu, \lambda dx^\nu) = \lambda^2 f(x^\mu, dx^\nu).$$

Quasimetric:

$$f = ds^2 = \mathcal{G}_{\mu\nu} dx^\mu dx^\nu \,, \quad \mathcal{G}_{\mu\nu} = \frac{1}{2} \frac{\partial^2 f}{\partial dx^\mu \partial dx^\nu} \,.$$

$$\mathcal{G}_{\mu\nu} = \alpha_g^2 g_{\alpha\beta} + \alpha_f^2 f_{\alpha\beta} + \alpha_g \alpha_f \left[\frac{ds_f}{ds_g} \left(g_{\mu\nu} - u^g_\mu u^g_\nu \right) + \frac{ds_g}{ds_f} \left(f_{\alpha\beta} - u^f_\mu u^f_\nu \right) + 2 u^g_{(\mu} u^f_{\nu)} \right]$$

disformally related to original metrics.

Define proper time:

$$d\tau^2 = -ds^2$$

Massive (massless) point particles travel on unit-norm timelike (null) geodesics with respect to the quasimetric.

The geometry that emerges for an observer in a bimetric spacetime depends quite nontrivially upon, in addition to the two metric structures, the observer's four-velocity. This means she is disformally coupled to her own four-velocity, and thus effectively lives in a Finslerian spacetime.

Double-coupling revives B-D ghost!

$$\mathcal{L}_{\rm matter} = \lambda_g \mathcal{L}_g[g_{\mu\nu},\chi] + \lambda_f \mathcal{L}_f[f_{\mu\nu},\chi]$$
$$= -\frac{\lambda_g}{2}\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + M^2\chi^2\right) - \frac{\lambda_f}{2}\sqrt{-f}\left(f^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + M^2\chi^2\right)$$

$$ds_g^2 = g_{\mu\nu}dx^\mu dx^\nu = -N^2(t)dt^2 + a^2(t)dx^2$$
$$ds_f^2 = f_{\mu\nu}dx^\mu dx^\nu = -\mathcal{N}^2(t)dt^2 + b^2(t)dx^2\,,$$

$\longrightarrow$
$$\mathcal{L}_{\rm matter} = \frac{1}{2}\left(\frac{\lambda_g a^3}{N} + \frac{\lambda_f b^3}{\mathcal{N}}\right)\dot{\chi}^2 - \frac{1}{2}M^2\left(\lambda_g a^3 N + \lambda_f b^3\mathcal{N}\right)\chi^2 \qquad p_\chi = \left(\frac{\lambda_g a^3}{N} + \frac{\lambda_f b^3}{\mathcal{N}}\right)\dot{\chi}$$

$\longrightarrow$
$$\mathcal{H}_{\rm matter} = \frac{1}{2}\frac{N\mathcal{N}}{\lambda_g a^3\mathcal{N} + \lambda_f b^3 N}p_\chi^2 + \frac{1}{2}M^2\left(\lambda_g a^3 N + \lambda_f b^3\mathcal{N}\right)\chi^2$$

- Yamashita, de Felice, Tanaka [arXiv:1408.0487]
- De Rham, Heisenberg, Ribeiro [arXiv: 1408.1678]
- Noller, Melville [arXiv:1408.5131]
- Hassan, Kocic, Schmidt-May [1409.3146]

New coupling

- One loop requirement for an effective metric:

$$\mathcal{L}_{\rm matter} = -\frac{1}{2}\sqrt{-g_{\rm eff}}\left(g^{\mu\nu}_{\rm eff}\partial_\mu\chi\partial_\nu\chi + M^2\chi^2\right)$$

- $\mathcal{L}_{1-\rm loop} = M^4\sqrt{-g_{\rm eff}} + \text{curvature corrections}\,.$ looks as $\frac{m^2}{4}\sum_{n=0}^{4}\alpha_n\mathcal{U}_n[\mathcal{K}]$
- A candidate metric: $g^{\rm eff}_{\mu\nu} = \alpha^2 g_{\mu\nu} + 2\alpha\beta\; g_{\mu\alpha}\, X^\alpha_{\ \nu} + \beta^2 f_{\mu\nu}$
- Shown to cure the instability in cosmology
- But: $\mathcal{L}^{(\rm matter\ \ loops)}_{1,\log} = M^4\frac{\det f}{\sqrt{\det g}}\log(M/\mu)$ -> ghost, $m^2_{\rm ghost} = \frac{\Lambda^6}{M^4}$,

$$\Lambda = (m^2 M_{\rm Pl})^{1/3}$$

Classes of new couplings

- The criterion $\sqrt{\det \hat{g}_{\text{eff}}} = \sqrt{\det \hat{g}}\, \det(\alpha \mathbb{1} + \beta \hat{X})$
- Is satisfied with any unit-determinant *M* such that

$$\hat{g}_{\text{eff}} = \hat{g}(\alpha + \beta \hat{X})^2 \hat{\mathcal{M}}$$

- For example

$$\hat{\mathcal{M}} = \mathbb{1}, \qquad \hat{\mathcal{M}} = \frac{\sqrt{\hat{g}^{-1} f}}{\det\left(\sqrt{\hat{g}^{-1} f}\right)^{1/4}}, \qquad \hat{\mathcal{M}} = \frac{\sqrt{\hat{f}^{-1} g}}{\det\left(\sqrt{\hat{f}^{-1} g}\right)^{1/4}}$$

- Ghosts or not? Remains to be seen…
- And what about e.g. the criterion

$$\sqrt{\det \hat{g}_{\text{eff}}} = \sqrt{\det \hat{g}} + \sqrt{\det \hat{f}} \qquad ?$$

Conclusions

- **The graviton could have a mass**
  - need to introduce a new metric
  - 4 free interaction parameters
- **Viable cosmology possible**
  - only specific class of models is stable
  - self-acceleration, falsifiable
- **Issue of matter coupling**
  - symmetric coupling with ghost $m > \Lambda$
  - ghost-free couplings? Open question