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Vector Fields for Statistical Anisotropy and Slowly Rolling Inflaton

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University of Helsinki



Vector fields for statistical anisotropy



Vector fields for slowly rolling inflaton



Introduction Observational constraints Sources of Statistical Aniso

The Power Spectrum

- Inflation the dominant paradigm to explain the origin of primordial perturbation;
- Statistical properties of

 $\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle = (2\pi)^3 \, \delta \left(\boldsymbol{k} + \boldsymbol{k}' \right) \frac{2\pi}{L^3}$



Parametrized as power-law

$$\mathcal{P}_{\zeta}\left(k
ight) = \mathcal{P}_{\zeta}\left(k_{*}
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where $n_s = 0.9603 \pm 0.0073$ Planck 2013 Results. XXII

• Another observable: gravitational waves $r=rac{\mathcal{P}_h}{\mathcal{P}_c} < 0.11;$

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Simple Inflation Model

• \mathcal{P}_{ζ} and r gives information about the inflationary epoch

$$\begin{array}{ll} V' & \Rightarrow & \epsilon \equiv \frac{m_{\rm Pl}^2}{2} \left(\frac{V'}{V}\right)^2 \\ V'' & & \eta \equiv m_{\rm Pl}^2 \frac{V''}{V} \end{array}$$



- For example
 - The energy scale of the inflation and the slope of the potential: $r = 16\epsilon \text{ and } \mathcal{P}_{\zeta}(k_*) = rac{1}{24\pi^2 m_{
 m Pl}^4} rac{V}{\epsilon}$
 - The curvature of the potentia

$$\frac{\mathrm{d}\ln\mathcal{P}_{\zeta}}{\mathrm{d}\ln k} \equiv n_s - 1 = -6\epsilon + 2\eta$$

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Departures From Simple Models

- Departures from the simplest scenarios
 ⇒ more observables, e.g.:
 - features in the spectrum

$$\mathcal{P}_{\zeta} \neq \mathcal{P}_{\zeta}\left(k_{*}\right) \left(rac{k}{k_{*}}
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 $\langle \zeta_{k} \zeta_{k'} \zeta_{k''} \rangle = (2\pi)^{3} \, \delta \left(k + k' + k'' \right) B_{\zeta} \left(k, k', k'' \right)$

Reduced bispectrum

 $\frac{6}{5} f_{\rm NL} = \frac{B_{\zeta}\left(k, k', k''\right)}{P_{\zeta}\left(k\right) P_{\zeta}\left(k'\right) + 2 \text{ permutation}}$

where e.g. $f_{
m NL}^{
m local}=2.7\pm5.8$ Planck 2013 Results. XXIV

statistical anisotropy;



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Statistical Anisotropy

- In momentum space:
 - the spectrum

 $\mathcal{P}_{\zeta}\left(\boldsymbol{k}
ight)=\mathcal{P}_{\zeta}\left(\boldsymbol{k}
ight)\left[1+g_{\zeta}\left(\boldsymbol{k}\cdot\boldsymbol{n}
ight)
ight]$

- the reduced bispectrum
 - $f_{\rm NL} = f_{\rm NL}^{\rm iso} \left[1 + \mathcal{G} \left(\hat{k}, \hat{n} \right) \right]$



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Statistical Anisotropy

- In momentum space:
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$$\mathcal{P}_{\zeta}(\boldsymbol{k}) = \mathcal{P}_{\zeta}(\boldsymbol{k}) \left[1 + g_{\zeta} \left(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{n}} \right)^2 + \dots \right]$$

the reduced bispectrum

$$f_{
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Observational constraints

- Statistical anisotropy:
 - WMAP9 data Ramazanov & Rubtsov (1311.3272) :

 $-0.046 < g_{\zeta} < 0.048$ at 68% CL

Planck data Kim & Komatsu (2013) :

 $|g_{\zeta}| < 0.02$ at 68% CL

Two views:

- 1. Still some space available (1st part of the talk)
- Constraining models (2nd part of the talk);

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Sources of Statistical Anisotropy

Sources of statistical anisotropy

- Remnant of initial stages of inflation; Pereira et al. (2007)
- Three forms; Urban (2013)
- (pseudo) Conformal Universe; Libanov & Rubakov (2010)
- Vector fields; Soda & Yokoyama (2008)

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Vector Fields

- Ubiquitous in particle physics models ⇒ relevant during inflation?
- EMT of (homogeneous) massless $U\left(1
 ight)$ vector-field



A likely (but not necessary) signature: statistical anisotropy;

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Vector Fields

- Ubiquitous in particle physics models ⇒ relevant during inflation?
- EMT of (homogeneous) massless U(1) vector field

$$T^{\nu}_{\mu} = \left(\begin{array}{cc} \rho_{W} & & 0 \\ & -p_{W} & \\ & & -p_{W} \\ 0 & & +p_{W} \end{array} \right)$$

A likely (but not necessary) signature: statistical anisotropy;

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Conformal Invariance

• Massless $U\left(1
ight)$ vector field $\mathcal{L}=-rac{1}{4}F_{\mu
u}F^{\mu
u}$ is invariant under

$$g_{\mu\nu}(x) \to \Omega(x) g_{\mu\nu}(x)$$

- No classical perturbation;
- $\rho_W \propto a^{-4};$

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Breaking Conformal Invariance

- Breaking conformal invariance: many examples in the literature on PMFs.
 - 1. Modify potential term,
 - $\begin{array}{ll} 1.1 \quad \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} & \rightarrow \text{ghost?} \\ \text{Himmetogle of all (2000), but resolution} \\ 1.2 \quad \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 \\ \end{array}$
 - 2. Modify kinetic term, e.g.:

$$\mathcal{L} = -\frac{1}{4} f^2 \left(t \right) F_{\mu\nu} F^{\mu\nu}$$

Maxwell-type kinetic term: the only possibility without instabilities; *Carroll et al. (2009)*

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For concreteness I will consider

$$\mathcal{L} = -\frac{1}{4} f^{2}(t) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^{2}(t) A_{\mu} A^{\mu}$$

Can give statistically anisotropic as well as isotropic cases.

Physical, canonically normalized vector field

$$W_i = f \frac{A_i}{a}$$

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Generating Statistical Anisotropy

Make the background expansion anisotropic;
 Directly contribute or generate the total of *ζ*;
 Source inflaton perturbation;

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1. Make the background expansion anisotropic;

- 2. Directly contribute or generate the total of
- 3. Source inflaton perturbation;

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Anisotropic Background

• EMT $T^{\nu}_{\mu} = \operatorname{diag}\left[\rho_W, -p_W, -p_W, +p_W\right] \Rightarrow$

$$ds^{2} = dt^{2} - a^{2}(t) (dx^{2} + dy^{2}) - b^{2}(t) dz^{2}$$

- Single field inflation: ζ_k = ζ_k (t) at k = aH and ζ_k = const afterwards;
- Power spectrum

$$\mathcal{P}_{\zeta}\left(m{k}
ight) = rac{1}{24\pi^{2}m_{\mathrm{Pl}}^{4}} \left.rac{V}{\epsilon}
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- In this case $g_{\zeta} < 0;$ Gumrukcuoglu et al. (2010)
- Scalar-Tensor correlations Gumrukcuoglu et al. (2010):

$$\frac{\mathcal{P}_{\zeta h}}{\mathcal{P}_{hh}} \simeq \frac{|g_{\zeta}|}{2\sqrt{\epsilon}}.$$

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Direct Contribution to ζ

• $\delta W\left(oldsymbol{x}
ight) \Rightarrow \zeta_{W}\left(oldsymbol{x}
ight);$

• $\delta W\left(oldsymbol{x}
ight)$ is statistically anisotropic

 $\left< \delta W_i\left(m{k}
ight) \delta W_j\left(m{k}'
ight)
ight> \propto T_{i,j}^{\rm L} \left(m{k} \left| \mathcal{P}_+\left(k
ight) + i
ight) + \left[m{k} \left| \mathcal{P}_-\left(m{k}
ight) + i
ight| \left(m{k}
ight) + \left[m{k} \left| m{k}
ight]
ight> \mathcal{P}_{||}\left(m{k}
ight)$

where $T_{ij}^{\Lambda}\left(\hat{k}
ight)=T_{ij}^{\Lambda}\left(e^{\mathrm{R}}\left(\hat{k}
ight),e^{\mathrm{L}}\left(\hat{k}
ight),e^{\mathrm{H}}\left(\hat{k}
ight)
ight)$

The anisotropy in $\mathcal{P}_{\zeta}\left(oldsymbol{k}
ight)$:

 $g_{\zeta}\left(k
ight) = N_{W}^{2} rac{\mathcal{P}_{||}\left(k
ight) - \mathcal{P}_{+}\left(k
ight)}{\mathcal{P}_{\zeta}^{\mathrm{iso}}\left(k
ight)}$

and preferred direction $\hat{m{n}}=\hat{m{W}}.$

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ight
angle \propto T_{ij}^{\mathrm{E}}\left(\hat{m{k}}
ight) \mathcal{P}_{+}\left(k
ight) + iT_{ij}^{\mathrm{O}}\left(\hat{m{k}}
ight) \mathcal{P}_{-}\left(k
ight) + T_{ij}^{||}\left(\hat{m{k}}
ight) \mathcal{P}_{||}\left(k
ight)$

where $T_{ij}^{\Lambda}\left(\hat{m{k}}\right) = T_{ij}^{\Lambda}\left(m{e}^{\mathrm{R}}\left(\hat{m{k}}\right),m{e}^{\mathrm{L}}\left(\hat{m{k}}\right),m{e}^{||}\left(\hat{m{k}}\right)
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and preferred direction $\hat{m{n}}=m{W}.$
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and preferred direction $\hat{n} = \hat{W}$.

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Avoiding Excessive Statistical Anisotropy

$$g_{\zeta}\left(k
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ight)-\mathcal{P}_{+}\left(k
ight)}{\mathcal{P}_{\zeta}^{\mathrm{iso}}\left(k
ight)}$$

1. Three orthogonal vector fields: $\sum_{i=3} \left(\hat{m{k}} \cdot \hat{m{W}}_i
ight)^2 = ext{const};$

2. Many randomly oriented vector fields: $(\mathbf{k} \cdot \mathbf{W}_i) \simeq \text{const};$ 3. A subdominant contribution: $\mathcal{P}_{\zeta_W} \ll \mathcal{P}_{\zeta};$ 4. $\mathcal{P}_0 = \mathcal{P}_{\perp};$

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1. Three orthogonal vector fields: $\sum_{i=3} \left(\hat{k} \cdot \hat{W}_i \right)^2 = \text{const};$ 2. Many randomly oriented vector fields: $\overline{\left(\hat{k} \cdot \hat{W}_i \right)^2} \simeq \text{const};$ 3. A subdominant contribution: $\mathcal{P}_{\zeta_W} \ll \mathcal{P}_{\zeta};$ 4. $\mathcal{P}_{||} = \mathcal{P}_+;$

Many Vector Fields

• Non-Abelian vector fields Karčiauskas (2012)

$$\mathcal{L} = -\frac{1}{4} f^2 \left(t \right) F^a_{\mu\nu} F^{\mu\nu}_a,$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_c f^{abc} A^b_\mu A^c_\nu$. Assuming $W_a = W \forall a$

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Subdominant Contribution: Vector Curvaton

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f = 1 & m = const < H(t)

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• $\frac{6}{5} f_{\rm NL}^{\rm equil} \simeq \frac{2g^2}{\Omega_W} \left(\frac{m_{\rm end}}{3H}\right)^4 \left[1 + \frac{1}{8} \left(\frac{3H}{m_{\rm end}}\right)^4 \hat{W}_{\perp}^2\right]$



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Subdominant Contribution: End-of-Inflation

- End-of-Inflation Scenario Yokoyama & Soda (2008)
- 1. Hybrid inflation

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi + \frac{1}{2} \partial_{\mu} \chi \partial^{\mu} \chi + V(\phi, \chi)$$

with

$$V(\phi, \chi) = \frac{1}{4}\lambda \left(\chi^{2} - M^{2}\right)^{2} + \frac{1}{2}\kappa^{2}\phi^{2}\chi^{2} + V(\phi)$$

2. End-of-Inflation with a vector field $\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{4} f^{2} (t) F^{a}_{\mu\nu} F^{\mu\nu}_{a} + \frac{1}{2} (D_{\mu} \Phi)^{\dagger} D^{\mu} \Phi + V (\phi)^{\dagger} D^{\mu} \Phi + V (\phi)^{\mu} \Phi + V$



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Subdominant Contribution: End-of-Inflation

$$g_{\zeta} \simeq rac{\epsilon_*}{\epsilon_{
m end}} \left(rac{\kappa^2}{e^2} rac{W}{\phi_{
m end}}
ight)^2$$

$$\frac{6}{5} f_{\rm NL}^{\rm local} \simeq \eta g_{\zeta}^2 \left[\left(C^2 - 1 \right) \left(1 - W_{\perp}^2 \right) - \frac{1}{4} \left(\sin \varphi \right)^2 W_{\perp}^4 \right] \\ \frac{6}{5} f_{\rm NL}^{\rm local} \simeq \eta g_{\zeta}^2 \left[\left(C^2 - 1 \right) - \left(\frac{7}{8} C^2 - 1 \right) W_{\perp}^2 - \frac{3}{16} W_{\perp}^4 \right]$$

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 $\mathcal{P}_+ = \mathcal{P}_|$

Consider

$$\mathcal{L} = -\frac{1}{4} f^{2}(t) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^{2}(t) A_{\mu} A^{\mu}$$

with $f \propto a^{-2}$ and $m \propto a$.

1. $m_{
m end} < 3H$ - statistical anisotropy

$$\mathcal{P}_{+} = \left(\frac{H}{2\pi}\right)^{2} \ll \mathcal{P}_{||} = \left(\frac{3H}{m_{\text{end}}}\right)^{2} \left(\frac{H}{2\pi}\right)$$

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Generating Statistical Anisotropy

Make the background expansion anisotropic;
 Directly contribute or generate the total of *ζ*;
 Source inflaton perturbation;

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Generating Statistical Anisotropy

- 1. Make the background expansion anisotropic;
- 2. Directly contribute or generate the total of ζ
- 3. Source inflaton perturbation;

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Source Inflaton Perturbation

• If inflaton is coupled to the vector field

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + V''\left(\phi\right)\delta\phi = -\left[V''\left(\phi,W\right)\delta\phi + \frac{\partial V'\left(\phi,W\right)}{\partial W_{i}}\delta W_{i}\right]$$

• Since δW_i statistically anisotropic $\Rightarrow \delta \phi$ also statistically anisotropic;

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Source Inflaton Perturbation: Example

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi + V(\phi) - \frac{1}{4} f^{2}(\phi) F_{\mu\nu} F^{\mu\nu}$$

Slowly rolling vector field

Negligible backreaction

We live in a typical location

 $W \gg H$

$$\delta\phi_{k} = \delta\phi_{k}^{0} + \frac{6N\left(k\right)}{\sqrt{2\epsilon}m_{\mathrm{Pl}}}\delta\left(W^{2}\right)_{k}$$

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$$f\left(\phi\right) \propto a^{-2}$$

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Negligible backreaction

$$\frac{\rho_W}{\epsilon\rho} \ll 1$$

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Source Inflaton Perturbation: Example

Anisotropy in the spectrum

$$g_{\zeta}\left(k_{0}
ight)\simeq-1.3 imes10^{-3}\left(rac{N\left(k
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- Observations $|g_{\zeta}| < 0.02$: if $\mathcal{P}_{\zeta_{\phi}} = \mathcal{P}_{\zeta}$ the condition $W \gg H$ is only marginally allowed.
- non-Gaussianity

$$f_{\mathrm{NL}}^{\mathrm{iso}} = -10N\left(k_{0}
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Vector fields for statistical anisotropy



Vector fields for slowly rolling inflaton


Equations of Motion I

- The work of Wagstaff, J. M. and Dimopoulos, K. (2011)
- The action

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[-\frac{1}{2} m_{\mathrm{Pl}}^2 R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V\left(\phi\right) - \frac{1}{4} f^2\left(\phi\right) F_{\mu\nu} F^{\mu\nu} \right],$$

where

$$ds^{2} = dt^{2} - a^{2}(t) \left[e^{2\sigma(t)} \left(dx^{2} + dy^{2} \right) + e^{-4\sigma(t)} dz^{2} \right]$$

Equations of Motion I

• This gives

$$H^{2} (1 - \Sigma^{2}) = \frac{\frac{1}{2}\dot{\phi}^{2} + V + \rho_{\rm kin}}{3m_{\rm Pl}^{2}}$$
$$\dot{H} + 3H^{2} = \frac{3V + \rho_{\rm kin}}{3m_{\rm Pl}^{2}},$$
$$H\dot{\Sigma} + \Sigma\dot{H} + 3H^{2}\Sigma = \frac{2\rho_{\rm kin}}{3m_{\rm Pl}^{2}},$$
$$\ddot{\phi} + 3H\dot{\phi} + V' - \rho'_{\rm kin} = 0,$$
$$\ddot{A}_{i} + H\left(1 + 4\Sigma + 2\frac{\dot{f}}{Hf}\right)\dot{A}_{i} = 0,$$

where

$$H \equiv \frac{\dot{a}}{a}, \quad \Sigma \equiv \frac{\dot{\sigma}}{H}, \quad \rho_{\rm kin} \equiv \frac{1}{2} \left(f \frac{\dot{A}_i}{a} \right)^2$$

Equations of Motion II

• It is useful to introduce

$$x \equiv \frac{\dot{\phi}}{\sqrt{6}m_{\rm Pl}H}, \quad y \equiv \frac{\sqrt{V}}{\sqrt{3}m_{\rm Pl}H}, \quad z \equiv \frac{\sqrt{\rho_{\rm kin}}}{\sqrt{3}m_{\rm Pl}H}$$

Then Equations of motion become
$$1 = x^2 + y^2 + z^2 + \Sigma^2$$
$$\frac{d\ln H}{dN} = 3(x^2 + \Sigma^2) + 2z^2$$
$$\Sigma \frac{d\ln \Sigma}{dN} = \Sigma \left[3(1 - x^2 - \Sigma^2) - 2z^2\right] - 2z^2$$
$$\frac{dx}{dN} = x \left[3(1 - x^2 - \Sigma^2) - 2z^2\right] + (1 - x^2 - z^2 - \Sigma^2) \lambda - z^2 \Gamma$$
$$\frac{dz}{dN} = x \left[3(1 - x^2 - \Sigma^2) - 2z^2\right] + (4\Sigma + x\Gamma - 1)z$$

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$$1 = x^{2} + y^{2} + z^{2} + \Sigma^{2}$$

$$\frac{d \ln H}{dN} = 3 (x^{2} + \Sigma^{2}) + 2z^{2}$$

$$\Sigma \frac{d \ln \Sigma}{dN} = \Sigma [3 (1 - x^{2} - \Sigma^{2}) - 2z^{2}] - 2z^{2}$$

$$\frac{dx}{dN} = x [3 (1 - x^{2} - \Sigma^{2}) - 2z^{2}] + (1 - x^{2} - z^{2} - \Sigma^{2}) \lambda - z^{2}\Gamma$$

$$\frac{dz}{dN} = z [3 (1 - x^{2} - \Sigma^{2}) - 2z^{2}] + (4\Sigma + x\Gamma - 1) z$$

where

$$\lambda \equiv \sqrt{\frac{3}{2}} m_{\rm Pl} \frac{V'}{V} = \sqrt{3\epsilon} \quad \Gamma \equiv \sqrt{6} m_{\rm Pl} \frac{f'}{f}$$

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Equations of motion become

$$\begin{array}{rcl} 1 & = & x_c^2 + y_c^2 + z_c^2 + \Sigma_c^2 \\ z_H & = & 3 \left(x_c^2 + \Sigma_c^2 \right) + 2 z_c^2 \\ 0 & = & \Sigma_c \left[3 \left(1 - x_c^2 - \Sigma_c^2 \right) - 2 z_c^2 \right] - 2 z_c^2 \\ 0 & = & x_c \left[3 \left(1 - x_c^2 - \Sigma_c^2 \right) - 2 z_c^2 \right] + \left(1 - x_c^2 - z_c^2 - \Sigma_c^2 \right) \lambda - z_c^2 \Gamma \\ 0 & = & z_c \left[3 \left(1 - x_c^2 - \Sigma_c^2 \right) - 2 z_c^2 \right] + \left(4 \Sigma_c + x_c \Gamma - 1 \right) z_c \end{array}$$

• Take $\lambda \simeq \text{constant}$, $\Gamma \simeq \text{constant}$;

1. Standard slow-roll solution

$$x_{c} = -\frac{\lambda}{3}, \quad y_{c} = \sqrt{1 - \frac{\lambda^{2}}{9}}, \quad z_{c} = 0, \quad \Sigma_{c} = 0$$

stable fixed point for $\lambda < 3$ and $\lambda (\lambda + \Gamma) < 6$ Anisotropic kination solution

$$x_c = x, \quad y_c = 0, \quad z_c = 0, \quad \Sigma_c = \sqrt{1 - x^2}$$

3. Vector scaling solution

$$x_c \simeq -\frac{2}{\Gamma}, \quad y_c \simeq 1, \quad z_c \simeq \frac{\sqrt{\lambda\Gamma - 6}}{\Gamma}, \quad \Sigma_c \simeq \frac{2}{3} \frac{\lambda\Gamma - 6}{\Gamma^2}$$

stable fixed point for $\lambda \Gamma > 6$, $\Gamma \gg \lambda$ and $\Gamma \gg 1$.

Check the rate of change of λ and Γ posteriori.

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Statistical Anisotropy Slow-Roll Attractor

Scale Invariant Spectrum

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$$\mathcal{P}_+ \simeq \left(\frac{H}{2\pi}\right)^2$$

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No Backreaction Problem

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- The background anisotropy

 $\Sigma \simeq \text{const} \ll 1$

The vector field energy density

 $\frac{\rho_{\rm kin}}{\rho_{\phi}} \simeq {\rm const} \ll 1$

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The η problem

• A generic SUGRA inflation model gives $|\eta| \gtrsim 1$ $V(\phi) = V(0) \left[1 + \frac{|\phi|^2}{m_{\rm Pl}^2} + \frac{1}{2} \frac{|\phi|^4}{m_{\rm Pl}^4} + \dots\right]$

This gives $V''(\phi) \simeq V''(0) + 3H^2$

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Solution of the η problem

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In the attractor

$$V_{
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Conclusions

- Vector fields can play a role in inflationary dynamics;
- If so, statistical anisotropy is likely;
- Can help solving the η problem;