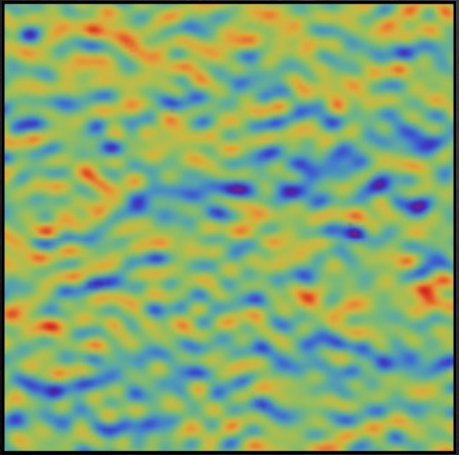
Vector Fields for Statistical Anisotropy and Slowly Rolling Inflaton

Mindaugas Karčiauskas

University of Helsinki

Vector fields for statistical anisotropy



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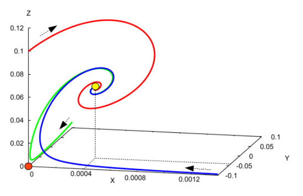
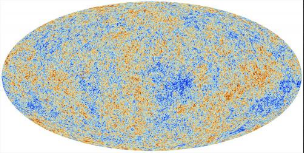


The Power Spectrum

- Inflation - the dominant paradigm to explain the origin of primordial perturbation;



- Statistical properties of ζ

$$\langle \zeta_{\boldsymbol{k}} \zeta_{\boldsymbol{k}'} \rangle = (2\pi)^3 \delta\left(\boldsymbol{k} + \boldsymbol{k}'\right) \frac{2\pi^2}{k^3} \mathcal{P}_\zeta(k)$$

  - Parametrized as power-law

$$\mathcal{P}_\zeta(k) = \mathcal{P}_\zeta(k_*) \left(\frac{k}{k_*}\right)^{n_s - 1}$$

  where $n_s = 0.9603 \pm 0.0073$ *Planck 2013 Results. XXII*
  - Another observable: gravitational waves $r = \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} < 0.11$;

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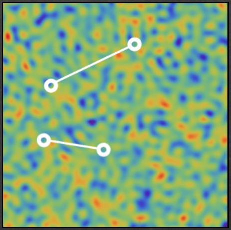

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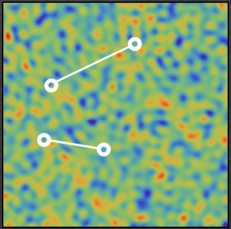

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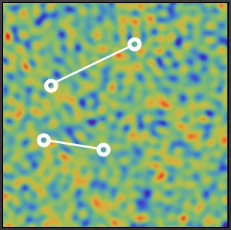

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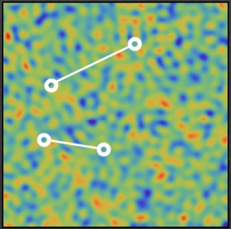

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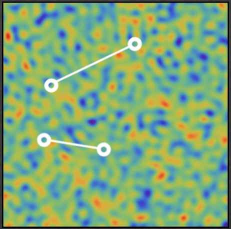
Simple Inflation Model

- $\mathcal{P}_\zeta$ and r gives information about the inflationary epoch



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$$V' \quad \Rightarrow \quad \epsilon \equiv \frac{m_{\rm Pl}^2}{2}\left(\frac{V'}{V}\right)^2$$

$$V'' \qquad \eta \equiv m_{\rm Pl}^2 \frac{V''}{V}$$

- For example
  - The energy scale of the inflation and the slope of the potential:

$$r = 16\epsilon \text{ and } \quad \mathcal{P}_\zeta(k_*) = \frac{1}{24\pi^2 m_{\rm Pl}^4}\frac{V}{\epsilon}$$

  - The curvature of the potential

$$\frac{\mathrm{d}\ln \mathcal{P}_\zeta}{\mathrm{d}\ln k} \equiv n_s - 1 = -6\epsilon + 2\eta$$

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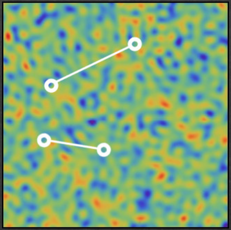

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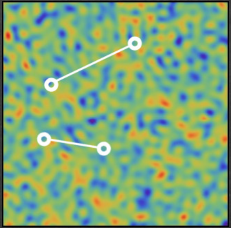

- Departures from the simplest scenarios $\Rightarrow$ more observables, e.g.:
  - features in the spectrum

$$\mathcal{P}_\zeta \neq \mathcal{P}_\zeta\left(k_*\right)\left(\frac{k}{k_*}\right)^{n_s-1}$$

  - non-Gaussianity;

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Reduced bispectrum

$$\frac{6}{5} f_{\rm NL} = \frac{B_\zeta\left(k, k', k''\right)}{P_\zeta\left(k\right) P_\zeta\left(k'\right) + 2\,\text{permutations}}$$

where e.g. $f_{\rm NL}^{\rm local} = 2.7 \pm 5.8$ *Planck 2013 Results. XXIV*

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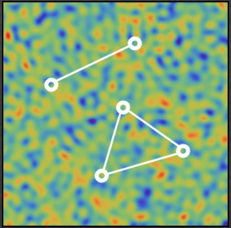

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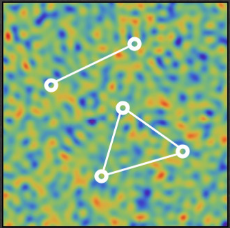

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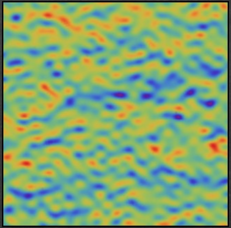

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Statistical Anisotropy

- In momentum space:
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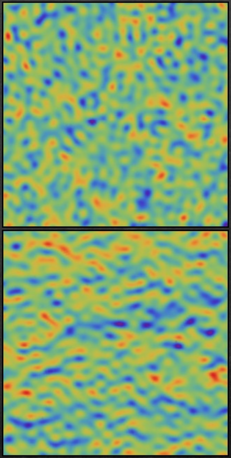

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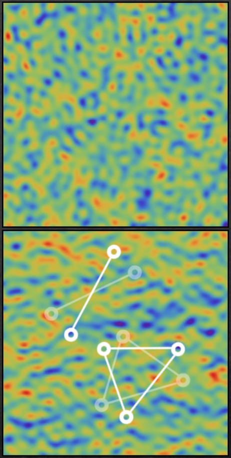

Observational constraints

- Statistical anisotropy:
  - WMAP9 data *Ramazanov & Rubtsov (1311.3272)* :

$$-0.046 < g_\zeta < 0.048 \quad \text{at} \quad 68\% \quad \text{CL}$$

  - Planck data *Kim & Komatsu (2013)* :

$$|g_\zeta| < 0.02 \quad \text{at} \quad 68\% \quad \text{CL}$$

- Two views:
  1. Still some space available (1st part of the talk);
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Sources of Statistical Anisotropy

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  - Remnant of initial stages of inflation; *Pereira et al. (2007)*
  - Three forms; *Urban (2013)*
  - (pseudo) Conformal Universe; *Libanov & Rubakov (2010)*
  - Vector fields; *Soda & Yokoyama (2008)*

Vector Fields

- Ubiquitous in particle physics models ⇒ relevant during inflation?
- EMT of (homogeneous) massless $U(1)$ vector field

$$T_\mu^\nu = \begin{pmatrix} \rho_W & & & 0 \\ & -p_W & & \\ & & -p_W & \\ 0 & & & +p_W \end{pmatrix}$$

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Conformal Invariance

- Massless $U(1)$ vector field $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ is invariant under

$$g_{\mu\nu}(x) \to \Omega(x)\, g_{\mu\nu}(x)$$

  - No classical perturbation;
  - $\rho_W \propto a^{-4}$;

Breaking Conformal Invariance

- Breaking conformal invariance: many examples in the literature on PMFs.

1. Modify potential term,
   1.1 $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{6}RA_\mu A^\mu \to$ ghost? *Himmetoglu et al. (2009)*, but *Karčiauskas & Lyth (2010)*
   1.2 $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2(t)\,A_\mu A^\mu$
2. Modify kinetic term, e.g.:

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Maxwell-type kinetic term: the only possibility without instabilities; *Carroll et al. (2009)*

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- EMT $T_\mu^\nu = \mathrm{diag}\left[\rho_W, -p_W, -p_W, +p_W\right] \Rightarrow$

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left(\mathrm{d}x^2 + \mathrm{d}y^2\right) - b^2(t)\,\mathrm{d}z^2$$

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- In this case $g_\zeta < 0$; *Gumrukcuoglu et al. (2010)*
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Direct Contribution to ζ

- $\delta W(\boldsymbol{x}) \Rightarrow \zeta_W(\boldsymbol{x})$;
- $\delta W(\boldsymbol{x})$ is statistically anisotropic

$$\langle \delta W_i(\boldsymbol{k})\, \delta W_j(\boldsymbol{k}') \rangle \propto T^{\mathrm{E}}_{ij}\left(\hat{\boldsymbol{k}}\right)\mathcal{P}_+(k) + iT^{\mathrm{O}}_{ij}\left(\hat{\boldsymbol{k}}\right)\mathcal{P}_-(k) + T^{\|}_{ij}\left(\hat{\boldsymbol{k}}\right)\mathcal{P}_{\|}(k)$$

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- The anisotropy in $\mathcal{P}_\zeta(\boldsymbol{k})$:

$$g_\zeta(k) = N_W^2 \frac{\mathcal{P}_{\|}(k) - \mathcal{P}_+(k)}{\mathcal{P}_\zeta^{\mathrm{iso}}(k)}$$

and preferred direction $\hat{\boldsymbol{n}} = \hat{\boldsymbol{W}}$.

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$$\langle \delta W_i\left(\boldsymbol{k}\right) \delta W_j\left(\boldsymbol{k}'\right)\rangle \propto T^{\mathrm{E}}_{ij}\left(\hat{\boldsymbol{k}}\right)\mathcal{P}_+\left(k\right) + iT^{\mathrm{O}}_{ij}\left(\hat{\boldsymbol{k}}\right)\mathcal{P}_-\left(k\right) + T^{||}_{ij}\left(\hat{\boldsymbol{k}}\right)\mathcal{P}_{||}\left(k\right)$$

where $T^{\Lambda}_{ij}\left(\hat{\boldsymbol{k}}\right) = T^{\Lambda}_{ij}\left(\boldsymbol{e}^{\mathrm{R}}\left(\hat{\boldsymbol{k}}\right), \boldsymbol{e}^{\mathrm{L}}\left(\hat{\boldsymbol{k}}\right), \boldsymbol{e}^{||}\left(\hat{\boldsymbol{k}}\right)\right)$

- The anisotropy in $\mathcal{P}_\zeta\left(\boldsymbol{k}\right)$:

$$g_\zeta\left(k\right) = N_W^2 \frac{\mathcal{P}_{||}\left(k\right) - \mathcal{P}_+\left(k\right)}{\mathcal{P}_\zeta^{\mathrm{iso}}\left(k\right)}$$

and preferred direction $\hat{\boldsymbol{n}} = \hat{\boldsymbol{W}}$.

Avoiding Excessive Statistical Anisotropy

$$g_\zeta(k) = N_W^2 \frac{\mathcal{P}_{||}(k) - \mathcal{P}_+(k)}{\mathcal{P}_\zeta^{\rm iso}(k)}$$

1. Three orthogonal vector fields: $\sum_{i=3}\left(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{W}}_i\right)^2 = \text{const};$
2. Many randomly oriented vector fields: $\overline{\left(\hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{W}}_i\right)^2} \simeq \text{const};$
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- Non-Abelian vector fields *Karčiauskas (2012)*

$$\mathcal{L} = -\frac{1}{4} f^2(t) F^a_{\mu\nu} F_a^{\mu\nu},$$

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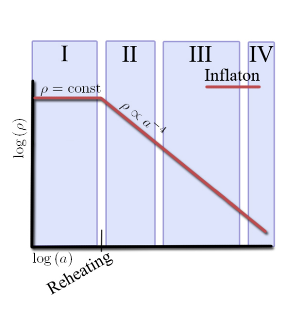

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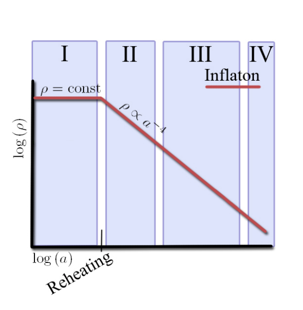

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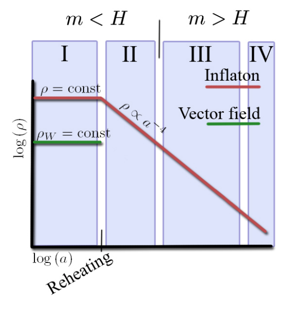

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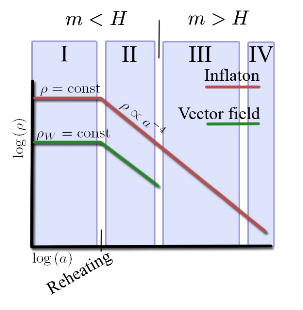

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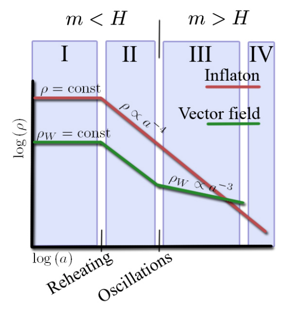

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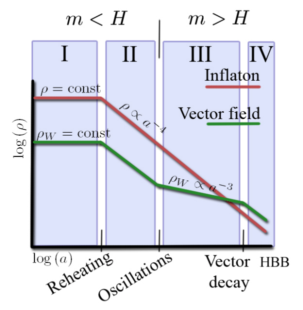

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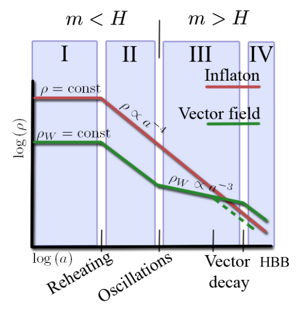

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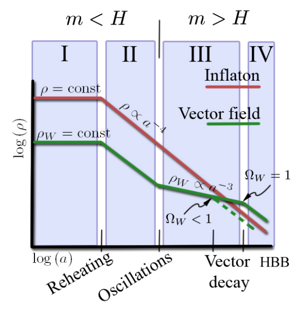

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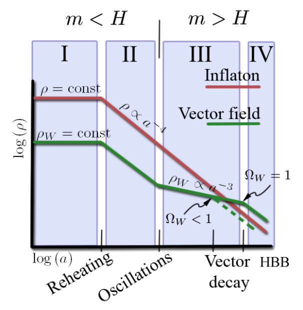

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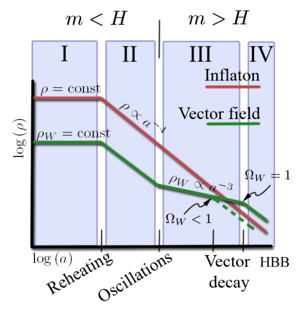


Subdominant Contribution: End-of-Inflation

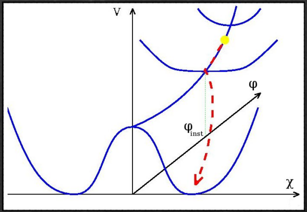
- End-of-Inflation Scenario *Yokoyama & Soda (2008)*

1. Hybrid inflation

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + V(\phi,\chi)$$

with

$$V(\phi,\chi) = \frac{1}{4}\lambda\left(\chi^2 - M^2\right)^2 + \frac{1}{2}\kappa^2\phi^2\chi^2 + V(\phi)$$



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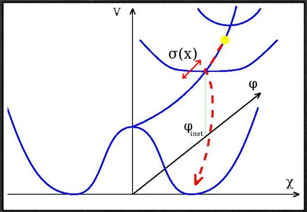

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$$g_\zeta \simeq \frac{\epsilon_*}{\epsilon_{\rm end}} \left(\frac{\kappa^2}{e^2} \frac{W}{\phi_{\rm end}} \right)^2$$

$$\frac{6}{5} f_{\rm NL}^{\rm local} \simeq \eta g_\zeta^2 \left[\left(C^2 - 1\right) \left(1 - W_\perp^2\right) - \frac{1}{4} \left(\sin\varphi\right)^2 W_\perp^4 \right]$$

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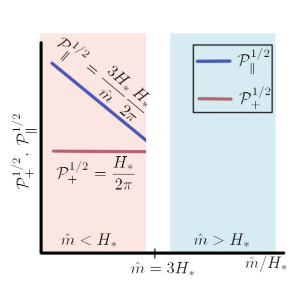

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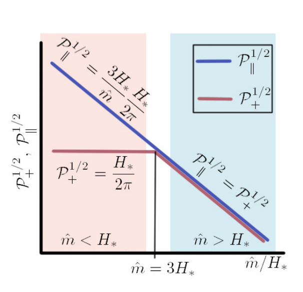

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Source Inflaton Perturbation

- If inflaton is coupled to the vector field

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} + V''(\phi)\,\delta\phi = -\left[V''(\phi, W)\,\delta\phi + \frac{\partial V'(\phi, W)}{\partial W_i}\delta W_i\right]$$

- Since δW_i statistically anisotropic $\Rightarrow$ $\delta\phi$ also statistically anisotropic;

Source Inflaton Perturbation: Example

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}$$

- Slowly rolling vector field

$$f(\phi) \propto a^{-2}$$

- Negligible backreaction

$$\frac{\rho_W}{\epsilon\rho} \ll 1$$

- We live in a typical location

$$W \gg H$$

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- Negligible backreaction

$$\frac{\rho_W}{\epsilon\rho} \ll 1$$

- We live in a typical location

$$W \gg H$$

- Inflaton perturbation

$$\delta\phi_{\boldsymbol{k}} = \delta\phi_k^0 + \frac{6N(k)}{\sqrt{2\epsilon}m_{\mathrm{Pl}}}\delta\left(W^2\right)_{\boldsymbol{k}}$$

Source Inflaton Perturbation: Example

- Anisotropy in the spectrum

$$g_\zeta\left(k_0\right) \simeq -1.3 \times 10^{-3}\left(\frac{N\left(k\right)}{60}\right)^2\left(\frac{\mathcal{P}_{\zeta_\phi}}{\mathcal{P}_\zeta\left(k_0\right)}\right)^2\left(\frac{W}{H}\right)^2,$$

- Observations $|g_\zeta| < 0.02$: if $\mathcal{P}_{\zeta_\phi} = \mathcal{P}_\zeta$ the condition $W \gg H$ is only marginally allowed.
- non-Gaussianity

$$f_{\rm NL}^{\rm iso} = -10N\left(k_0\right)g_\zeta\left(k_0\right)\frac{\mathcal{P}_{\zeta_\phi}}{\mathcal{P}_\zeta}.$$

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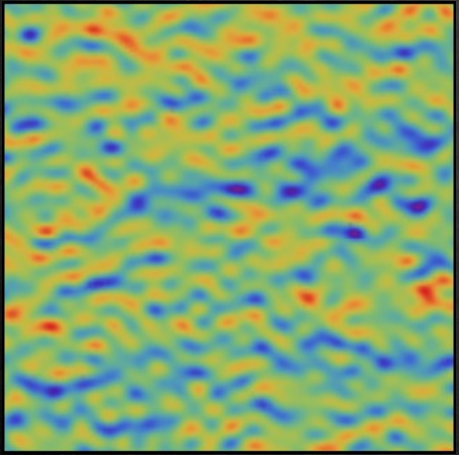
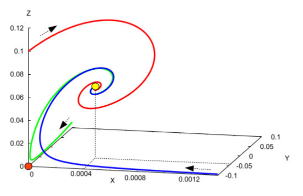
Vector fields for statistical anisotropy



Vector fields for slowly rolling inflaton



Equations of Motion I

- The work of *Wagstaff, J. M. and Dimopoulos, K. (2011)*
- The action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}m_{\mathrm{Pl}}^2R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V\left(\phi\right) - \frac{1}{4}f^2\left(\phi\right)F_{\mu\nu}F^{\mu\nu}\right],$$

where

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2\left(t\right)\left[\mathrm{e}^{2\sigma(t)}\left(\mathrm{d}x^2 + \mathrm{d}y^2\right) + \mathrm{e}^{-4\sigma(t)}\mathrm{d}z^2\right]$$

Equations of Motion I

- This gives

$$
\begin{aligned}
H^2\left(1-\Sigma^2\right) &= \frac{\frac{1}{2}\dot{\phi}^2+V+\rho_{\mathrm{kin}}}{3m_{\mathrm{Pl}}^2}, \\
\dot{H}+3H^2 &= \frac{3V+\rho_{\mathrm{kin}}}{3m_{\mathrm{Pl}}^2}, \\
H\dot{\Sigma}+\Sigma\dot{H}+3H^2\Sigma &= \frac{2\rho_{\mathrm{kin}}}{3m_{\mathrm{Pl}}^2}, \\
\ddot{\phi}+3H\dot{\phi}+V'-\rho_{\mathrm{kin}}' &= 0, \\
\ddot{A}_i+H\left(1+4\Sigma+2\frac{\dot{f}}{Hf}\right)\dot{A}_i &= 0,
\end{aligned}
$$

where

$$
H \equiv \frac{\dot{a}}{a}, \quad \Sigma \equiv \frac{\dot{\sigma}}{H}, \quad \rho_{\mathrm{kin}} \equiv \frac{1}{2}\left(f\frac{\dot{A}_i}{a}\right)^2
$$

Equations of Motion II

- It is useful to introduce

$$x \equiv \frac{\dot{\phi}}{\sqrt{6}m_{\rm Pl}H}, \quad y \equiv \frac{\sqrt{V}}{\sqrt{3}m_{\rm Pl}H}, \quad z \equiv \frac{\sqrt{\rho_{\rm kin}}}{\sqrt{3}m_{\rm Pl}H}$$

- Then Equations of motion become

$$\begin{aligned}
1 &= x^2 + y^2 + z^2 + \Sigma^2 \\
\frac{\mathrm{d}\ln H}{\mathrm{d}N} &= 3\left(x^2 + \Sigma^2\right) + 2z^2 \\
\Sigma\frac{\mathrm{d}\ln \Sigma}{\mathrm{d}N} &= \Sigma\left[3\left(1 - x^2 - \Sigma^2\right) - 2z^2\right] - 2z^2 \\
\frac{\mathrm{d}x}{\mathrm{d}N} &= x\left[3\left(1 - x^2 - \Sigma^2\right) - 2z^2\right] + \left(1 - x^2 - z^2 - \Sigma^2\right)\lambda - z^2\Gamma \\
\frac{\mathrm{d}z}{\mathrm{d}N} &= z\left[3\left(1 - x^2 - \Sigma^2\right) - 2z^2\right] + \left(4\Sigma + x\Gamma - 1\right)z
\end{aligned}$$

where

$$\lambda \equiv \sqrt{\frac{3}{2}}m_{\rm Pl}\frac{V'}{V} = \sqrt{3\epsilon} \quad \Gamma \equiv \sqrt{6}m_{\rm Pl}\frac{f'}{f}$$

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$$\begin{aligned}
1 &= x_c^2 + y_c^2 + z_c^2 + \Sigma_c^2 \\
\epsilon_H &= 3\left(x_c^2 + \Sigma_c^2\right) + 2 z_c^2 \\
0 &= \Sigma_c \left[3\left(1 - x_c^2 - \Sigma_c^2\right) - 2 z_c^2\right] - 2 z_c^2 \\
0 &= x_c \left[3\left(1 - x_c^2 - \Sigma_c^2\right) - 2 z_c^2\right] + \left(1 - x_c^2 - z_c^2 - \Sigma_c^2\right) \lambda - z_c^2 \Gamma \\
0 &= z_c \left[3\left(1 - x_c^2 - \Sigma_c^2\right) - 2 z_c^2\right] + \left(4\Sigma_c + x_c \Gamma - 1\right) z_c
\end{aligned}$$

- Take $\lambda \simeq \text{constant}$, $\Gamma \simeq \text{constant}$;

Fixed Points

1. Standard slow-roll solution

$$x_c = -\frac{\lambda}{3}, \quad y_c = \sqrt{1 - \frac{\lambda^2}{9}}, \quad z_c = 0, \quad \Sigma_c = 0$$

   stable fixed point for $\lambda < 3$ and $\lambda\left(\lambda + \Gamma\right) < 6$

2. Anisotropic kination solution

$$x_c = x, \quad y_c = 0, \quad z_c = 0, \quad \Sigma_c = \sqrt{1 - x^2}$$

3. Vector scaling solution

$$x_c \simeq -\frac{2}{\Gamma}, \quad y_c \simeq 1, \quad z_c \simeq \frac{\sqrt{\lambda\Gamma - 6}}{\Gamma}, \quad \Sigma_c \simeq \frac{2}{3}\frac{\lambda\Gamma - 6}{\Gamma^2}$$

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Scale Invariant Spectrum

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- $x_c \simeq -\frac{2}{\Gamma} \Rightarrow \frac{\dot{f}}{f} = -2H \Rightarrow f \propto a^{-2}$;
  - vector field is slowly-rolling;
  - scale invariant perturbation spectrum:

$$\mathcal{P}_+ \simeq \left(\frac{H}{2\pi}\right)^2$$

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- *Demozzi, Mukhanov & Rubinstein (2009)* the f^2F^2 model suffers from the backreaction problem;
- The background anisotropy

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The η problem

- A generic SUGRA inflation model gives $|\eta| \gtrsim 1$

$$V(\phi) = V(0)\left[1 + \frac{|\phi|^2}{m_{\mathrm{Pl}}^2} + \frac{1}{2}\frac{|\phi|^4}{m_{\mathrm{Pl}}^4} + \dots\right]$$

This gives $V''(\phi) \simeq V''(0) + 3H^2$

1. Tuned duration of inflation;
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Solution of the η problem

- Remember

$$\ddot{\phi} + 3H\dot{\phi} + V' - \rho'_{\rm kin} = 0$$

- In the attractor

$$V'_{\rm eff} \equiv V' - \rho'_{\rm kin} \simeq \frac{6}{\lambda\Gamma} V'$$

- Even if V' is large ϕ rolls slowly!
- A concrete example in *Dimopoulos, Lazarides & Wagstaff (2012)* for SUGRA hybrid inflation.

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Conclusions

- Vector fields can play a role in inflationary dynamics;
- If so, statistical anisotropy is likely;
- Can help solving the η problem;