Improving Naturalness in Warped Models

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Based on work with J. A. Cabrer and G. v. Gersdorff: arXiv:0907.5361; arXiv:1011.2205; arXiv:1103.1388; arXiv:1104.3149; arXiv:1104.5253

Outline

The outline of this talk is

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- Introduction
- Some general results
- RS model
- Our model
- Conclusion

Introduction

- The SM of EW interactions suffers from a naturalness problem as the Higgs mass is sensitive to UV physics scale Λ
- In particular the coupling of the top quark generates

Quadratic divergence $\Rightarrow$ sensitivity δ

$$\Delta m_H^2 \sim (3/4\pi^2)\Lambda^2 \Rightarrow \delta = 3\Lambda^2/(4\pi^2 m_H^2)$$

- Naturalness of the theory

$$\delta \sim 1 \Rightarrow \Lambda \lesssim 3.6\, m_H$$

$$M_H = [115, 600] \;\; GeV \Rightarrow \Lambda \lesssim [400, 2000] \;\; GeV$$

- In view of negative searches at LEP2, and the increasing bounds imposed by ongoing LHC searches it is thus interesting to consider possible solutions to the hierarchy problem able to accommodate a heavy Higgs

- Although the effective SM below a multi-TeV cutoff is more natural with a heavy Higgs, the present EWPT point towards a light Higgs
- In particular a χ^2 fit of all SM EWPO drives the 95% CL upper limit of the Higgs mass to $m_H \lesssim 150$ GeV
- It would then be necessary to introduce new physics to compensate for the contribution of a heavy Higgs

In fact...

we can consider the Higgs mass measurement at LHC as a good test for new physics: if LHC finds a heavy Higgs the SM would be excluded and new physics would be required motivating an upgrade of the LHC and/or the construction of other colliders

- We will consider models where EWPT are saved by the UV physics solving the hierarchy problem
- Since a heavy Higgs contributes negatively to the T parameter an obvious requirement is the presence of new states that violate custodial symmetry in such a way as to give a positive contribution to T
- Such states are naturally provided by the KK modes of the hypercharge gauge boson in 5D warped compactifications
- Warped models were originally proposed by Randall and Sundrum [1] for

AdS metric

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \quad A(y) = ky$$

and UV and IR boundaries located at $y = 0$ and $y = y_1$

[1] L. Randall and R. Sundrum, arXiv:hep-ph/9905221

General results

- We will then consider the SM propagating in a 5D space with an arbitrary metric $A(y)$
- 5D gauge fields $W^i_M(x,y)$ and $B_M(x,y)$ propagating in the bulk
- A stabilizing field $\phi(x,y)$ fixing the value of $A(y_1)$

A bulk SM Higgs

$$H(x,y) = \frac{1}{\sqrt{2}} e^{i\chi(x,y)} \begin{pmatrix} 0 \\ h(y) + \xi(x,y) \end{pmatrix}$$

- $\chi(x,y)$ contains the 4D Goldstone bosons
- $h(y)$ is the 5D Higgs background
- $\xi(x,y)$ describes the Higgs fluctuations

- We will consider for the moment an arbitrary background $h(y)$

- We will consider the 5D action

$$S_5 = \int d^5x\sqrt{-g}\left(-\frac{1}{4}\vec{W}_{MN}^2 - \frac{1}{4}B_{MN}^2 - |D_M H|^2 - \frac{1}{2}(D_M\phi)^2 - V(H,\phi)\right) - 2\sum_\alpha \int d^4x dy \sqrt{-g}\,(-1)^\alpha\,\lambda^\alpha(H,\phi)\delta(y-y_\alpha)$$

- With boundary potentials

$$\lambda^0(\phi_0,H) = M_0|H|^2\,, \quad -\lambda^1(\phi_1,H) = -M_1|H|^2 + \gamma|H|^4$$

- EWSB will be triggered by λ^1 on the IR boundary

- One can then construct the 4D effective theory by making the KK-mode expansion

$$A_\mu(x,y)=a_\mu(x)\cdot f_A(y)/\sqrt{y_1}$$

where $A=A^\gamma,Z,W^\pm$ and the dot product denotes an expansion in modes

- The functions f_A satisfy the EOM

$$m_{f_A}^2 f_A+(e^{-2A}f_A')'-M_A^2 f_A=0$$

$$M_W(y)=\frac{g_5}{2}h(y)e^{-A(y)}\ ,\qquad M_Z(y)=\frac{1}{c_W}M_W(y)$$

- and Neumann BC

$$f_A'\Big|_{y=0,y_1}=0.$$

- For the Higgs fluctuations one can write EOM and BC

$$\xi''(y) - 4A'\xi'(y) - \frac{\partial^2 V}{\partial h^2}\xi(y) + m_H^2 e^{2A}\xi(y) = 0$$

$$\frac{\xi'(y_\alpha)}{\xi(y_\alpha)} = \left.\frac{\partial^2 \lambda^\alpha(h)}{\partial h^2}\right|_{y=y_\alpha}$$

- The background $h(y)$ is determined from

$$h''(y) - 4A'h'(y) - \frac{\partial V}{\partial h} = 0\,, \quad h'(y_\alpha) = \left.\frac{\partial \lambda^\alpha}{\partial h}\right|_{y=y_\alpha}$$

- For a quadratic bulk Higgs potential the Higgs wave function $\xi(y)$ for $m_H = 0$ $(n = 0)$ is thus proportional to $h(y)$

The effective SM-like Lagrangian for the Higgs

$$\mathcal{L}_{\rm eff} = -|D_\mu \mathcal{H}|^2 + \mu^2|\mathcal{H}|^2 - \lambda|\mathcal{H}|^4$$

$$\mu^2 = (kZ)^{-1}\left(M_1 - \frac{h'(y_1)}{h(y_1)}\right)\rho^2, \quad \lambda = \frac{\gamma k^2}{Z^2}$$

- The IR scale ρ and dimensionless quantity Z are

Z is Higgs wave function renormalization

$$\rho = k e^{-A(y_1)}, \qquad Z = k\int_0^{y_1} dy\, \frac{h^2(y)}{h^2(y_1)} e^{-2A(y)+2A(y_1)}$$

- The physical Higgs mass is $m_H^2 = 2\mu^2 \sim 2Z^{-1}\rho^2$
- Radiative corrections in the effective theory below the scale $\Lambda \sim m_{KK}$ will tend to destabilize light Higgs masses: some degree of fine-tuning is needed to not spoil EWSB and to keep the Higgs light

- Electroweak precision measurements are commonly mapped to the set (T, S, W, Y) [2]
- They are defined as

Oblique observables

$$\alpha T = m_W^{-2}\left[c_W^2 \Pi_Z(0) - \Pi_W(0)\right]$$

$$\alpha S = 4 s_W^2 c_W^2 \left[\Pi'_Z(0) - \Pi'_\gamma(0)\right]$$

$$2 m_W^{-2} Y = s_W^2 \Pi''_Z(0) + c_W^2 \Pi''_\gamma(0)$$

$$2 m_W^{-2} W = c_W^2 \Pi''_Z(0) + s_W^2 \Pi''_\gamma(0)$$

- Associated with the coefficients of

Effective operators (d=6)

$$|H^\dagger D_\mu H|^2\,, \quad H^\dagger W_{\mu\nu} H B^{\mu\nu}\,, \quad (\partial_\rho B_{\mu\nu})^2\,, \quad (D_\rho W_{\mu\nu})^2$$

[2] R. Barbieri et al. hep-ph/0405040

- These observables can be computed as

Oblique observables

$$\alpha T = s_W^2 m_Z^2 \frac{I_2}{\rho^2} \frac{k y_1}{Z^2}$$

$$\alpha S = 8 s_W^2 c_W^2 m_Z^2 \frac{I_1}{\rho^2} \frac{1}{Z}$$

$$Y = W = c_W^2 m_Z^2 \frac{I_0}{\rho^2} \frac{1}{k y_1}$$

- where

$$I_n = k^3 \int_0^{y_1} (y_1 - y)^{2-n} u^n(y) e^{2A(y)-2A(y_1)}$$

$$u(y) = \int_y^{y_1} dy' \frac{h^2(y')}{h^2(y_1)} e^{-2A(y')+2A(y_1)}, \quad I_n/\rho^2 = \mathcal{O}(1/m_{KK}^2)$$

- T is volume enhanced and Z^2 suppressed

RS MODEL

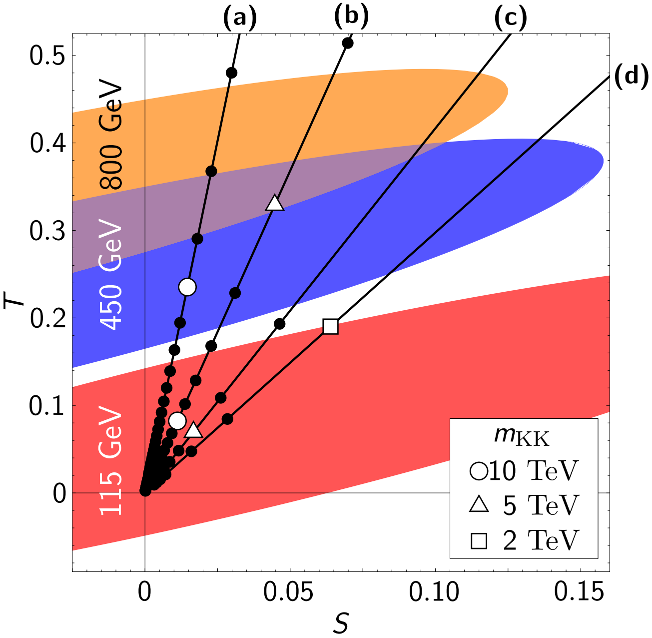
- In the RS model one introduces a 5D bulk Higgs mass: $M^2 = a(a-4)k^2$
- Holographic interpretation $a = \dim(\mathcal{O}_H)$
- Solution to the EOM: $h(y) \propto e^{aky}$
- No fine-tuning for $a > 2$: hierarchy solved with a composite Higgs
- $Z = \frac{1}{2(a-1)} < 1/2$ provides no suppression $\Rightarrow$ large m_{KK} & heavy Higgs & 5D Higgs as IR delocalized as possible
- For instance for $m_H = 450$ GeV the 95% CL window is

For localized 5D Higgs: $a \to \infty$

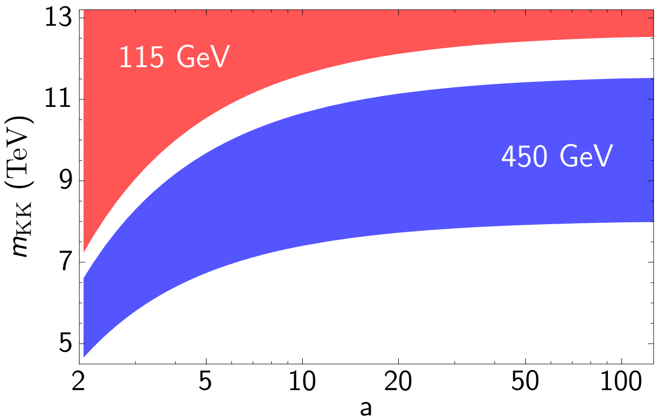
$$8 \lesssim m_{KK}/TeV \lesssim 11.5$$

For delocalized 5D Higgs: $a = 2.1$

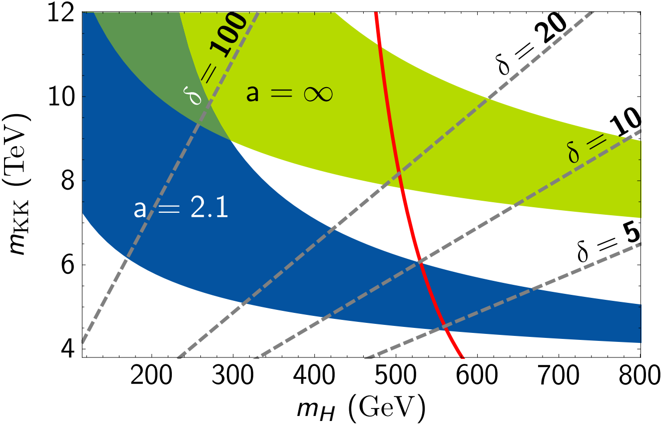
$$4.5 \lesssim m_{KK}/TeV \lesssim 6.5$$



95% CL regions. Ray (a) [(b)] is RS with a localized [bulk with a=2.1] Higgs



95% CL regions in the (a, m_{KK}) plane for RS and different values of the Higgs mass



95% CL regions in the (m_H, m_{KK}) plane for RS and the cases of a localized and a bulk Higgs with $a = 2.1$. Solid line is the perturbativity bound $\beta_\lambda^{(2)} = 0.5\ \beta_\lambda^{(1)}$

Our model

- We will consider a model with a conformal deformation in the IR
- It contains a stabilizing field ϕ which leads to

The metric

$$\phi(y) = -\frac{\sqrt{6}}{\nu}\log[\nu^2 bk(y_s - y)]$$

$$A(y) = ky - \frac{1}{\nu^2}\log\left(1 - y/(y_1 + \Delta)\right)\,, \quad \nu \in \mathbb{R}$$

- The metric has a spurious singularity located at $y_s = y_1 + \Delta$ outside the physical interval
- The dynamics of ϕ fixes y_1 $[A(y_1)]$ and Δ as in GW
- A 5D bulk Higgs mass: $M^2(\phi) = k^2[a(a-4) - be^{\nu\phi}]$ where a and b are arbitrary constants
- b can be absorbed by a shift of ϕ_0 at the UV: we fix it to $b = 1$

- The holographic interpretation of a is now a bit different from RS

In the IR

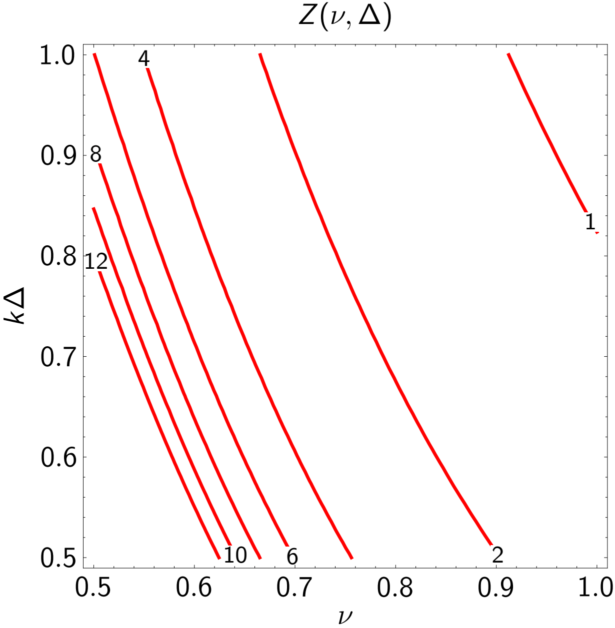
$$\dim(\mathcal{O}_H)^{IR} = \frac{\dim(\mathcal{O}_H)^{UV}}{1+\frac{1}{k\Delta\nu^2}}$$

- The solution to the EOM $h(y) = c_1 e^{aky} + c_2 \int^y e^{4A(y')-2aky'}$ imposes the constraint

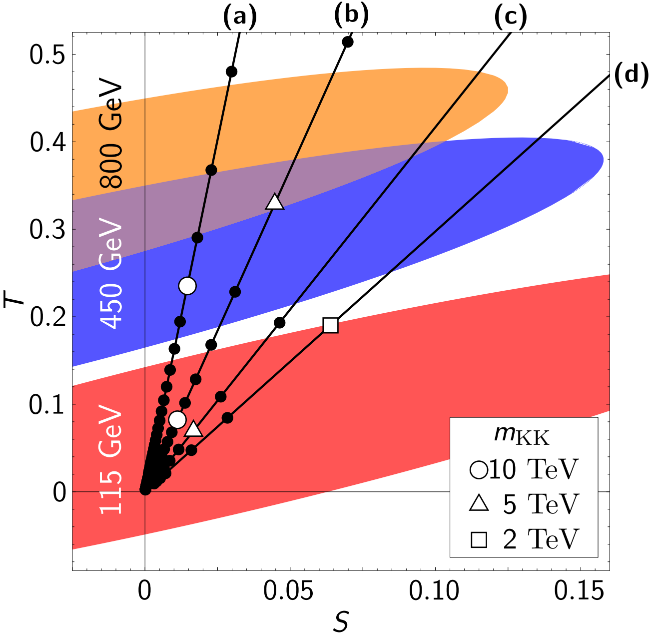
Hierarchy condition ($a > 2$ for RS)

$$a \gtrsim a_0 = 2A_1/ky_1$$

- In many cases $Z \gg 1$ which softens the bounds on EWPO



Contour lines of fixed $Z(\nu,\Delta)$ *where* $a = a_0(\nu,\Delta)$ *and* $A(y_1) = 35$



95% CL regions. Ray (c) [(d)] is our model with $k\Delta = 1$ and $\nu = 0.7$ [$\nu = 0.6$]

- For instance for $m_H = 450$ GeV and $a = a_0$ the 95% CL window is

For $k\Delta = 1$, $\nu = 0.7$

$$2.1 \lesssim m_{KK}/TeV \lesssim 2.9$$

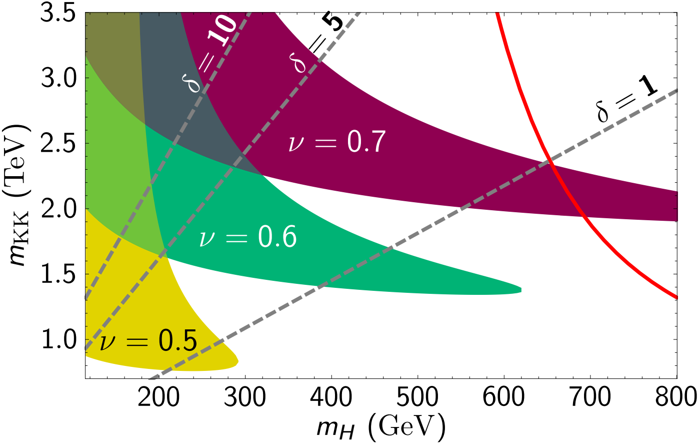
For $k\Delta = 1$, $\nu = 0.6$

$$1.4 \lesssim m_{KK}/TeV \lesssim 1.7$$

This shows that :

  - A heavy Higgs can be consistent with KK-modes accessible at LHC energies
  - The measurement of the Higgs mass at LHC should constrain the model parameters

- These two features are exhibited in the next plot



95% CL regions in the (m_H, m_{KK}) plane for our model with $k\Delta = 1$ and $\nu = 0.7, 0.6, 0.5$. Solid line corresponds to the perturbativity bound $\beta_\lambda^{(2)} = 0.5\ \beta_\lambda^{(1)}$

- The impact of the KK modes on the EWPO depends crucially on how strongly the former couple to the Higgs currents
- Writing the coupling as

$$\mathcal{L}=\sum_n \alpha_n (g\, W_\mu^n j_\mu^L + g'\, B_\mu^n j_\mu^Y)$$

- We can express these couplings in terms of the Z factors as

$$\alpha_n=\frac{k}{Z}\int_0^{y_1} dy\,\frac{h^2(y)}{h^2(y_1)}e^{-2A(y)+2A(y_1)}f_n(y)$$

- Enhanced Z factors will reduce these couplings, provided the integrals stay approximately constant

- A useful approximation for the KK wave function is given in terms of Bessel functions

$$f_n(y) = z\left[Y_0(m_n z_0)J_1(m_n z) - J_0(m_n z_0)Y_1(m_n z)\right]$$

- where

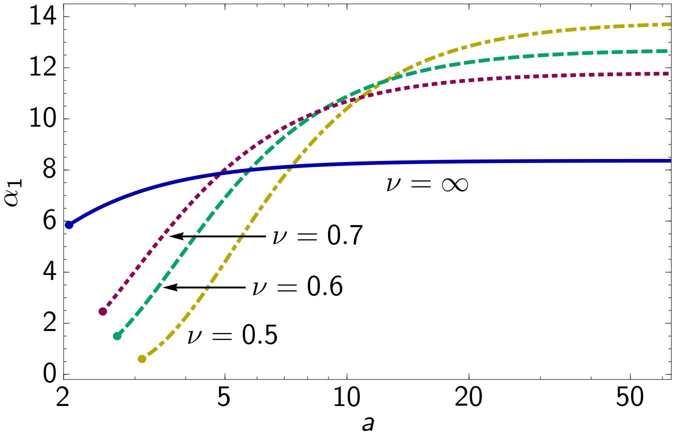
$$z(y) \equiv \int^y e^A \approx e^A/A'$$

are the conformal coordinates

- Note that A' can become large and hence e^A can be very different from z
- The mass eigenvalues are approximated by

$$m_n \approx j_{0,n} A'(y_1)\rho/k$$

where $j_{0,n}$ is n^{th} zero of the Bessel function $J_0(x)$



Plot of the coupling α_1 as a function of a

Some benchmark points

ν	$k\Delta$	a	Z	ky_1	$\frac{m^{(KK)}_{\rm gauge}}{\rm TeV}$	$\frac{m^{(KK)}_{\rm Higgs}}{\rm TeV}$	$\frac{m_{\rm rad}}{\rm TeV}$
0.48	1.0	3.2	6.6	22	0.82	0.92	0.28
0.55	1.3	2.8	2.1	25	2.4	2.9	0.84
0.64	1.6	2.5	1.2	28	4.0	4.9	1.3
0.73	1.7	2.4	0.86	30	5.2	6.6	1.6
∞	∞	2.1	0.47	35	7.5	12	0

Table: *Values of different relevant quantities at some of the points where $\delta = 0.1$ and $A(y_1) = 35$. The stabilization mechanism in our model dissapears when we take the RS limit, leading to vanishing radion mass as expected.*

Conclusions

- We have considered models where the 5D SM gauge and Higgs bosons propagate in the bulk
- In the RS model a heavy Higgs is more natural than a light one and EWPT are satisfied for lighter values of the KK mass

RS window for $m_H = 450$ GeV

$$4.5 \text{ TeV} \lesssim m_{KK} \lesssim 6.5 \text{ TeV}$$

- In the model with an IR deformation of the conformal symmetry the KK spectrum can be accessible to LHC both for light and heavy Higgs and satisfying the naturalness condition
- In most of the parameter space the fine-tuning is better than 10%
- The Higgs discovery at LHC will just constrain the model parameters

- Fermion localized in the bulk with 5D masses can be localized towards the UV/IR brane in a way similar to RS models
- A careful consideration of quarks in the bulk should enable us to describe non-oblique observables and fit e.g. [3]

Third generation quark observables

$$R_b,\ A^b_{FB}\ (LEP),\ A^t_{FB}\ (CDF), \ldots$$

[3] J.A. Cabrer, G. Gersdorff, M.Q., Work in preparation