Random Walks in the Sky (remembering your steps!)

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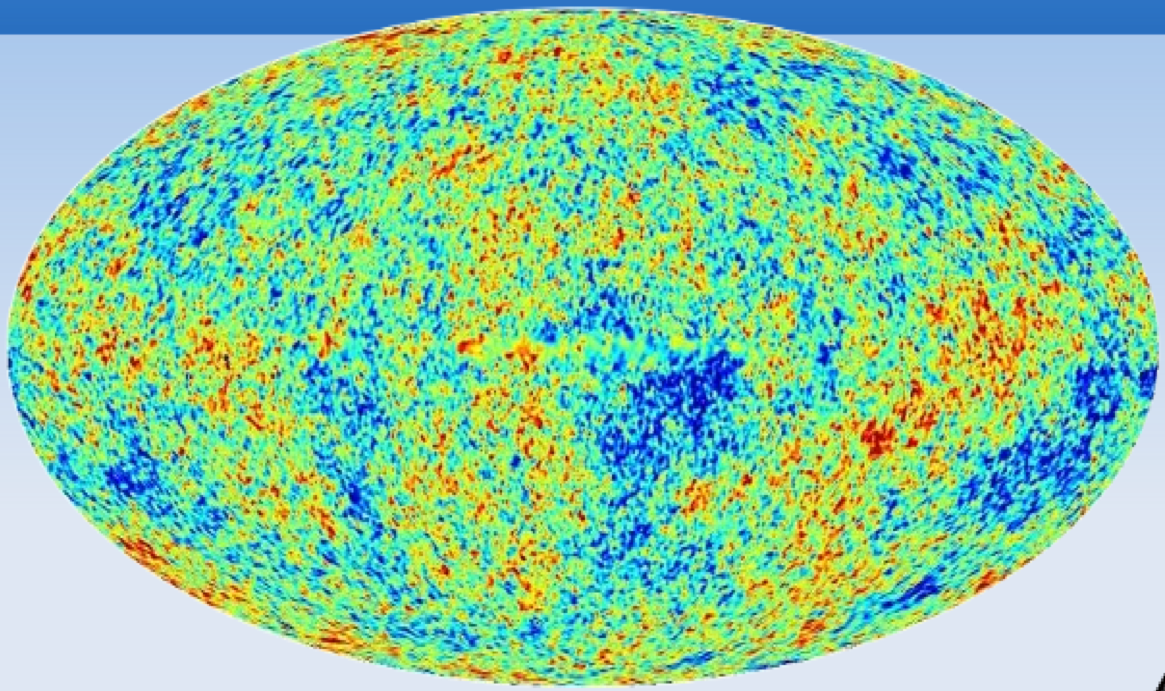
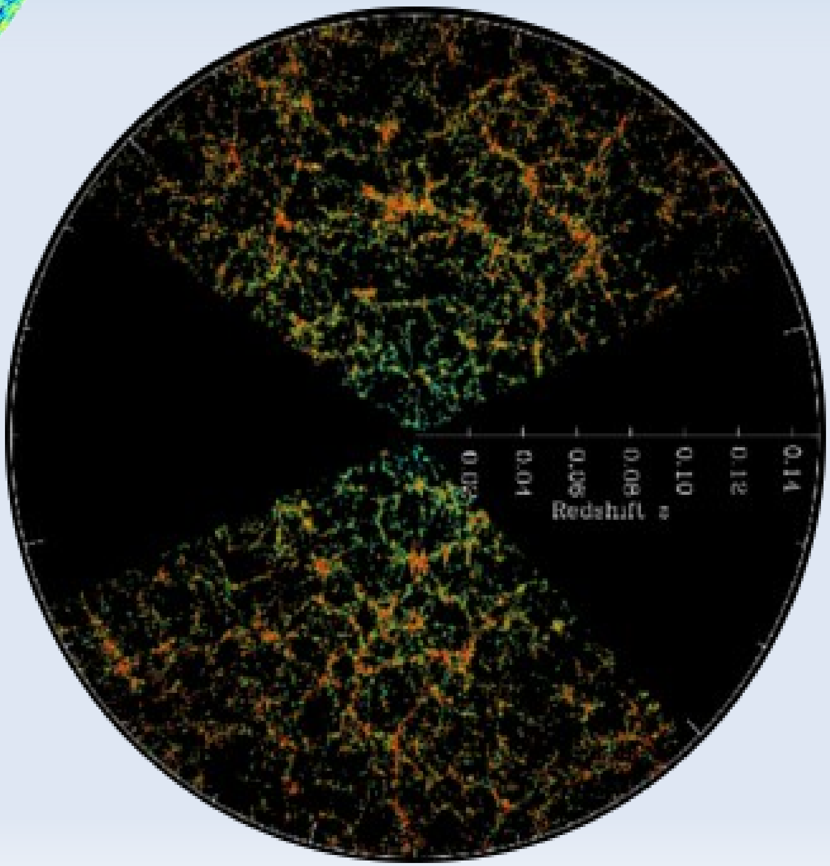
Formation of structures

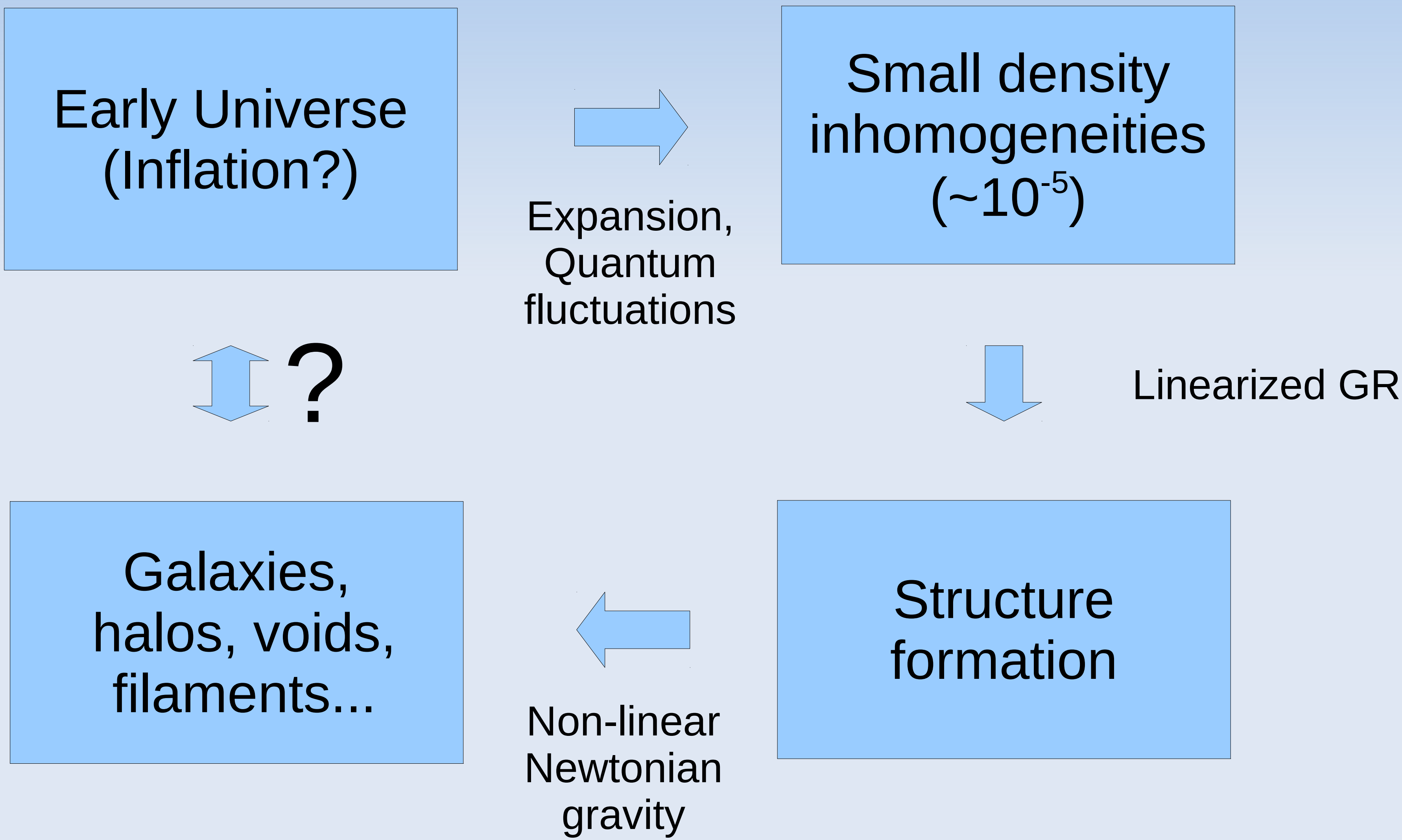


Halos evolve from critical overdensities



Can use halos to reconstruct the primordial distribution

Formation of structures



Halo Mass Function and Bias

- How many halos of given mass (mass function)
- Correlation of halo counts with underlying matter field (halo bias)

GOALS:

- Analytical predictions for halo statistics reproducing N-body sims
- Information on initial conditions and matter/energy balance from data:

Properties of Early Universe
Energy and matter content
Λ vs Quintessence vs ModGrav

Outline

Part ONE

- The excursion set approach
- Analytical progress for mass functions: the UPWARDS approximation

Part TWO

- Bias from excursion sets (a simple way!)

Excursion set theory

- Halos from "dense enough" patches in the initial matter distribution δ_{in}
- Mean density δ_R ≡ [average of δ_{in} over volume R^3] ≥ threshold B

$$\delta_R(\mathbf{x}) \equiv \frac{1}{V_R}\int \mathrm{d}^3y\, W_R(\mathbf{y}-\mathbf{x})\delta(\mathbf{y}) \geq B$$

- Halo mass M proportional to the volume V ~ R^3 of the patch.

$$M = \bar{\rho}\, V_R \equiv \bar{\rho}\int \mathrm{d}^3y\, W_R(\mathbf{y})$$

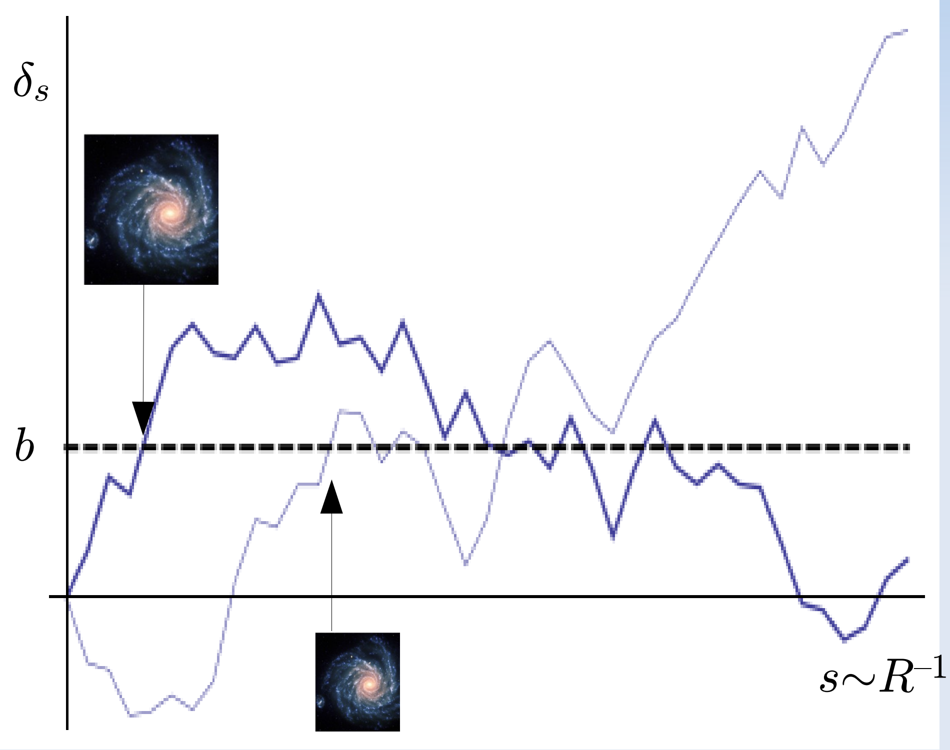
- If If B from spherical collapse in Einstein-DeSitter, then $B = \delta_c = 1.686$. But in general B is sensitive to Ω_m , Ω_Λ , w , baryons, neutrinos, modifications of gravity... It depends on scale and redshift
- Correlations depend on matter power spectrum $P(k)$ and choice of W

Excursion set theory

- Different locations realize different random walks: $s(M) \equiv \langle \delta_R^2(x) \rangle$

FIRST PASSAGE PROBLEM!

CORRELATED STEPS!
No known solution
NEED BETTER MATHS



- Abundance $n(M) \longleftrightarrow$ first crossing probability $f(s)$ at scale $s(M)$
- Oversimplified picture, working surprisingly well

First crossing distribution

- Probability of ANY crossing at s :

$$f(s) = \frac{\mathrm{d}}{\mathrm{d}s} \langle \vartheta(\delta - b(s)) \rangle$$

Press & Schechter (1974)

- Not any, but **FIRST** crossing (cloud-in-cloud problem); solution only for Gaussian uncorrelated steps

$$f(s) = \frac{\langle \vartheta(b_1 - \delta_1) \ldots \vartheta(b_{N-1} - \delta_{N-1}) \vartheta(\delta_N - b_N) \rangle}{\Delta s}$$

Bond et al. (1991)

First crossing distribution

- However: strongly correlated walks are less affected (less zig-zags)

Paranjape, Lam & Sheth (2011)

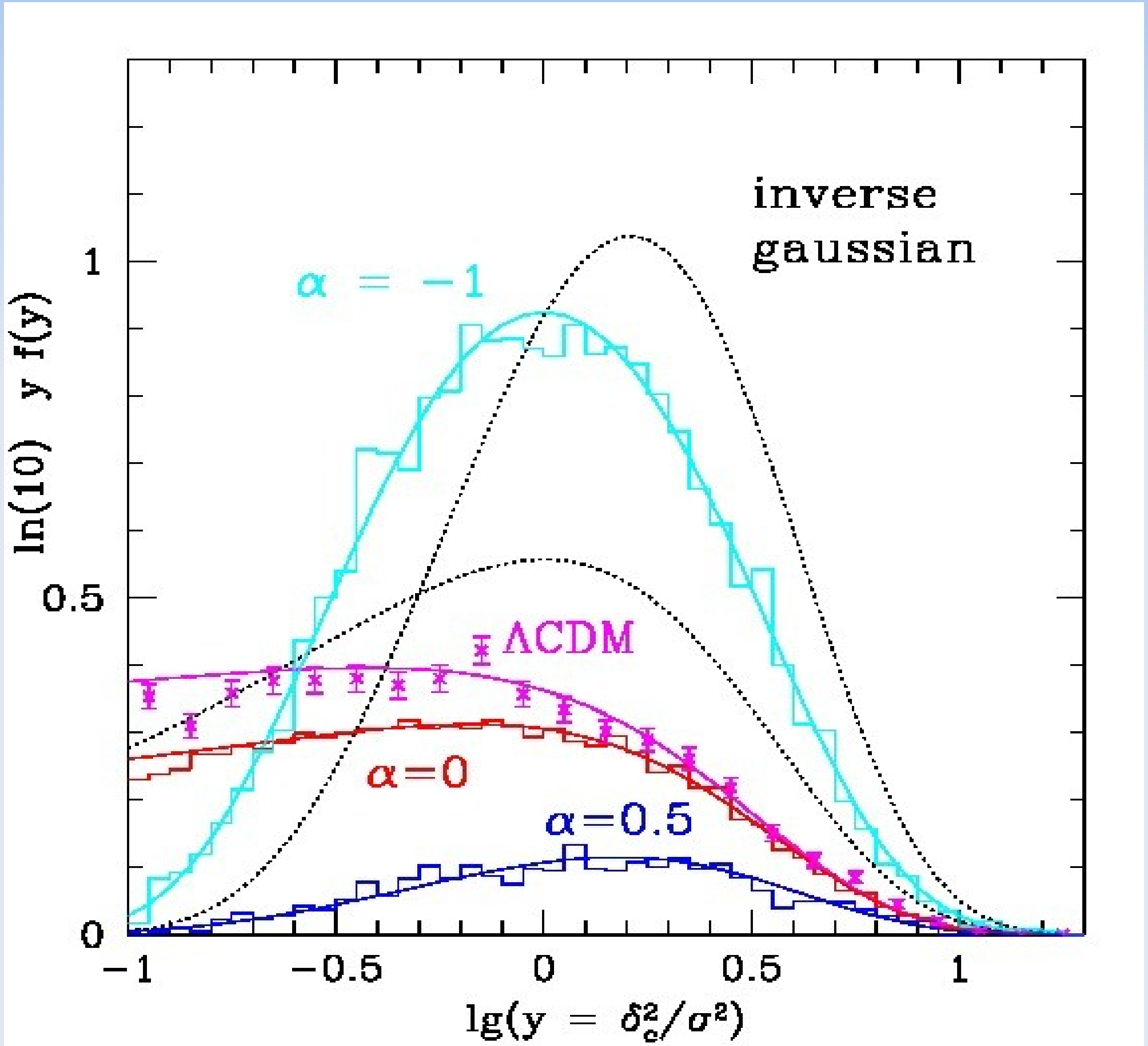
- Can relax FIRST into simply **UPWARDS**: $\delta = B$; $\delta' \geq B'$

$$f(s) = \left\langle \left[\frac{\mathrm{d}}{\mathrm{d}s} \vartheta(\delta_s - B) \right] \vartheta(\delta_s' - B') \right\rangle = \int_{B'}^{\infty} \mathrm{d}v \, (v - B') \, p(B, v; s)$$

MM & Sheth (2012)

$$f(s) = -\left(\frac{B}{\sqrt{s}} \right)' e^{-B^2/2s} \left[\frac{1 + \mathrm{erf}(X/\sqrt{2})}{2} + \frac{e^{-X^2/2}}{2X\sqrt{2\pi}} \right]$$

First crossing distribution



MM & Sheth (2012)

Upward mobility, back-substitution

- To reach any $\delta > b$ at s, must cross at some $S < s$:

$$p(\delta \geq b, s) = \int_0^s \mathrm{d}S\, f(S) p(\delta \geq b, s | \mathrm{first}, S)$$

- Do the **UPWARDS** approximation $\delta'(S) \geq b'(S)$ and solve for $f(S)$

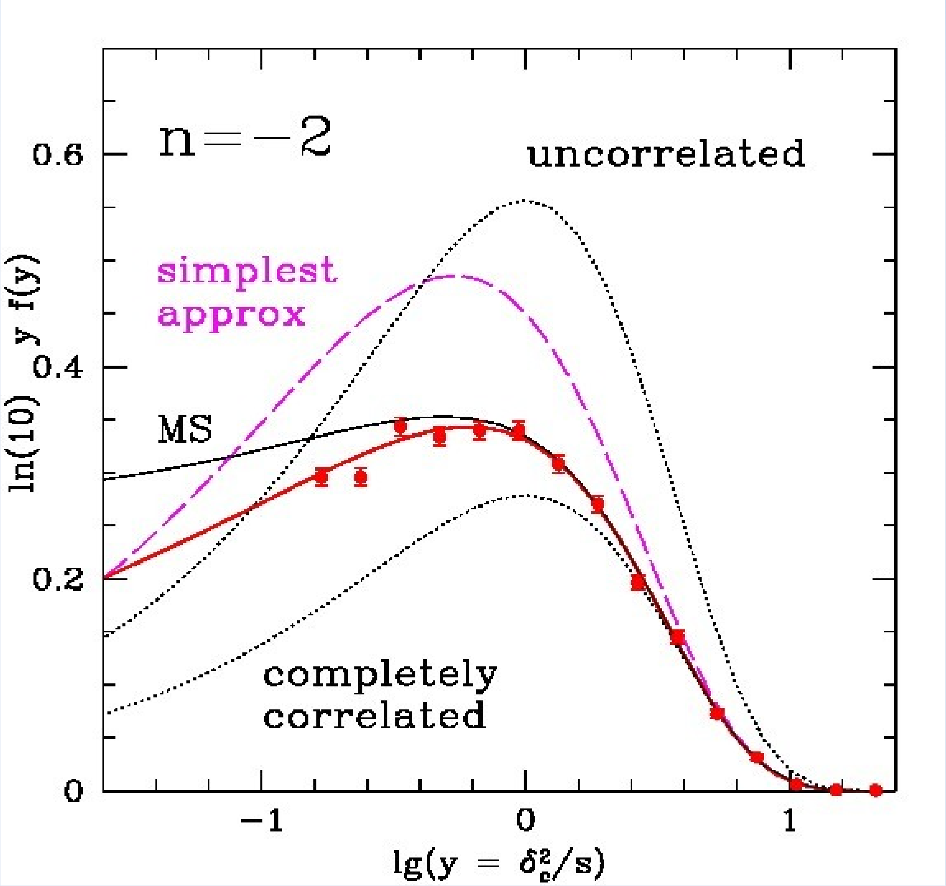
$$p(\delta \geq b, s | \mathrm{first}, S) \qquad \Rightarrow \qquad p(\delta \geq b, s | \mathrm{up}, S)$$

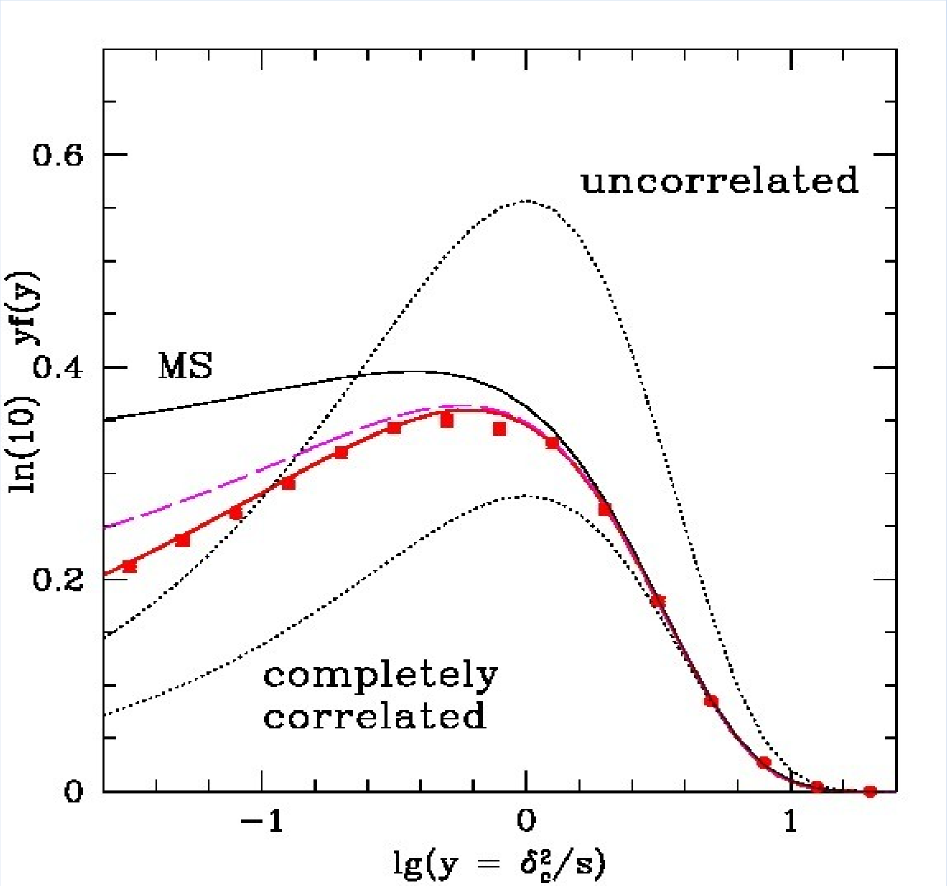
- Now solve for $f(S)$

MM & Sheth (2013)

Upward mobility, back-substitution

$$p(\delta \geq b, s) = \int_0^s \mathrm{d}S f(S) p(\delta \geq b, s | \mathrm{up}, S)$$





MM & Sheth (2013)

Upward mobility, back-substitution

$$p(\delta \geq b, s) = \int_0^s \mathrm{d}S f(S) p(\delta \geq b, s | \mathrm{up}, S)$$

- Full understanding of the correlated random walk problem, with any power spectrum and barrier!

MM & Sheth (2013)

- Does this $f(s)$ reproduce N-body mass function? Not really...

- Not surprising, space correlations are neglected. HOWEVER...

Upward mobility, back-substitution

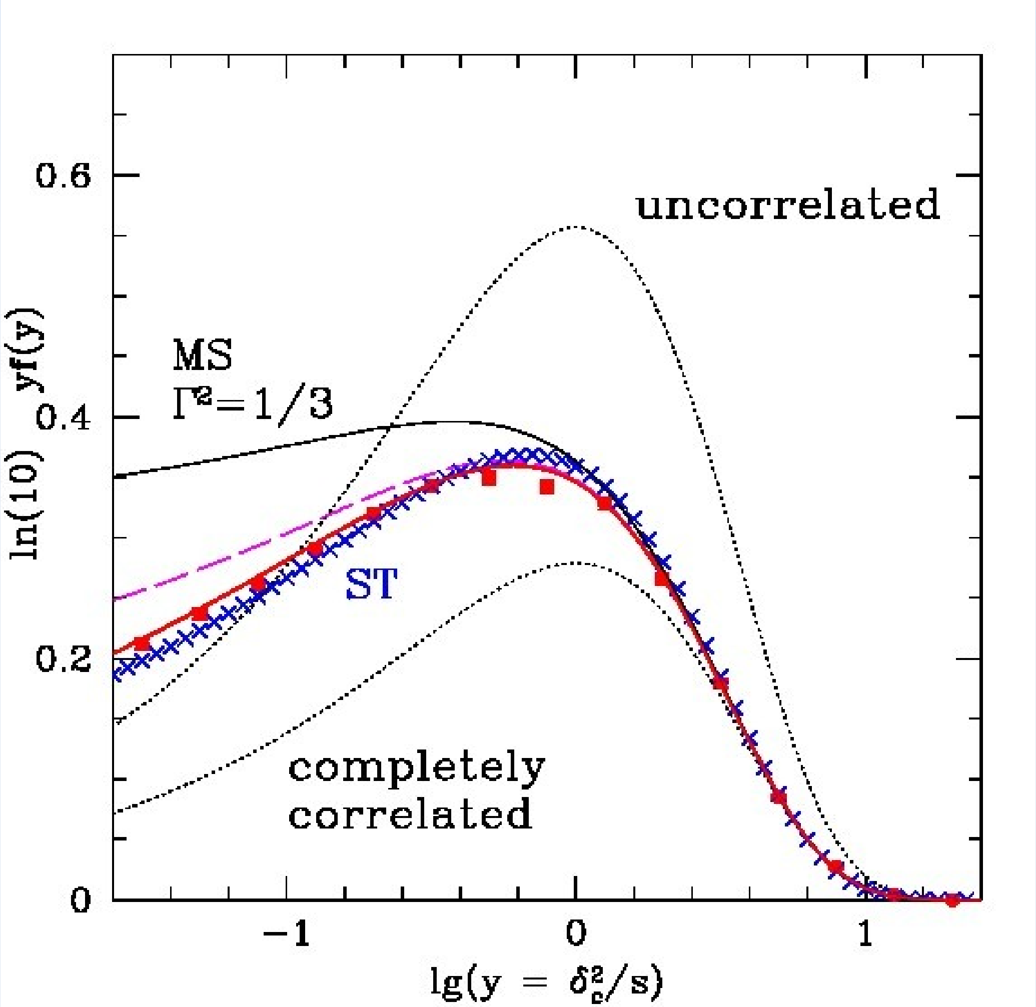
$$p(\delta \geq b, s) = \int_0^s \mathrm{d}S f(S) p(\delta \geq b, s|\mathrm{up}, S)$$

HOWEVER:

- Simple model with Markovian velocities (not heights!)
- Rescaled constant barrier:

$$\delta_c \to \sqrt{0.7}\delta_c$$

IT WORKS!!



MM & Sheth (2013)

Adding non-Gaussianity: MF

Same formalism for non-Gaussian initial conditions:

$$f(s) = \left[\frac{\mathrm{d}}{\mathrm{d}s}\int_{b(s)}^{\infty}\mathrm{d}\delta\, p(\delta;s)\right]\frac{1+\mathrm{erf}(X/\sqrt{2})}{2} - B'p(B;s)\left[\frac{e^{-X^2/2}}{2X\sqrt{2\pi}} + \dots\right]$$

old PS result

full NG pdf

Gaussian factors

small!

- Non-perturbative in NG parameters: $p(\delta;s)$ is the exact pdf!

$$p(B;s) = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{B^2}{2} + \mu\frac{B^3}{3!} + \dots\right] \qquad \left[\mu = \frac{\langle\delta^3\rangle}{s^{3/2}} \sim f_{\mathrm{NL}}\right]$$

- Residual NG corrections are small: OK as perturbations

MM & Sheth (2013)

Light vs Mass: Halo Bias

- Relation between halo abundance δ_h and underlying DM density
- Usually, computed from $p(\delta_h;s|\delta_0;s_0)$. Is there an easier way?
- Yes, there is. Expanding halo correlation functions in terms of DM correlation functions

Light vs Mass: Halo Bias

- Take the most generic dependence on the matter field:

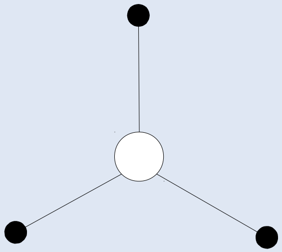
$$\delta_h(\mathbf{x}) = \sum_{k=0}^{\infty} \frac{1}{k!} \int \mathrm{d}^3 y_1 \ldots \mathrm{d}^3 y_k \, b_k(\mathbf{x}-\mathbf{y}_1, \ldots, \mathbf{x}-\mathbf{y}_k) \delta(\mathbf{y}_1) \ldots \delta(\mathbf{y}_k)$$

e.g. Matsubara (2011)

- Compute connected correlation function of δ_h and δ :

$$\langle \delta_h(\mathbf{x}) \delta(\mathbf{z}_1) \ldots \delta(\mathbf{z}_n) \rangle_c = \int \mathrm{d}^3\mathbf{x}_1 \ldots \mathrm{d}^3\mathbf{x}_n \underbrace{\left\langle \frac{\delta^n \delta_h(\mathbf{x})}{\delta\delta(\mathbf{x}_1) \ldots \delta\delta(\mathbf{x}_n)} \right\rangle}_{\text{Bias functions}} \prod_{j=1}^{n} \langle \delta(\mathbf{x}_j) \delta(\mathbf{z}_j) \rangle$$

- δ_h acts as an effective vertex for δ :



Halo Bias from Excursion Sets

- Need a prediction for δ_h. Can get it from excursion sets:

$$1+\delta_h(m) = \frac{\vartheta(B_1-\delta_1)\dots\vartheta(B_{N-1}-\delta_{N-1})\vartheta(\delta_N-B_N)}{\langle\vartheta(B_1-\delta_1)\dots\vartheta(B_{N-1}-\delta_{N-1})\vartheta(\delta_N-B_N)\rangle}$$

$$b_n(\mathbf{x}-\mathbf{y}_1,\dots,\mathbf{x}-\mathbf{y}_n) = \sum_{i_1,\dots,i_n}^{N}\left\langle\frac{\partial^n\delta_h(m)}{\partial\delta_{i_1}\cdots\partial\delta_{i_n}}\right\rangle W_{i_1}(\mathbf{x}-\mathbf{y}_1)\dots W_{i_n}(\mathbf{x}-\mathbf{y}_n)$$

$$\left\langle\frac{\partial^n\delta_h}{\partial\delta_{i_1}\cdots\partial\delta_{i_n}}\right\rangle = \frac{(-1)^n}{f(s)}\frac{\partial^n f(s)}{\partial B_{i_1}\cdots\partial B_{i_n}}$$

- Still a bit complicated...

$$\langle\delta_h\delta_0\rangle = \sum_{i=1}^{N}\left\langle\frac{\partial\delta_h}{\partial\delta_i}\right\rangle\langle\delta_i\delta_0\rangle \qquad \langle\delta_h\delta_0^2\rangle = \sum_{i,j=1}^{N}\left\langle\frac{\partial^2\delta_h}{\partial\delta_i\partial\delta_j}\right\rangle\langle\delta_i\delta_0\rangle\langle\delta_j\delta_0\rangle$$

Halo Bias from Excursion Sets

- Use the UPWARDS approximation. Only two variables!

$$1+\delta_h = \frac{1}{f(s)}\left[\frac{\mathrm{d}}{\mathrm{d}s}\vartheta(\delta_s - B)\right]\vartheta(\delta_s' - B')$$

- The real space bias functions become easy:

$$b_1(\mathbf{x}-\mathbf{y}) = -\frac{1}{f(s)}\left[W_R(\mathbf{x}-\mathbf{y})\frac{\partial}{\partial B} + W_R'(\mathbf{x}-\mathbf{y})\frac{\partial}{\partial B'}\right]f(s)$$

$$b_n(\mathbf{x}-\mathbf{y}_1,\ldots,\mathbf{x}-\mathbf{y}_n) = \frac{(-1)^n}{f(s)}\prod_{i=1}^{n}\left[W_R(\mathbf{x}-\mathbf{y}_i)\frac{\partial}{\partial B} + W_R'(\mathbf{x}-\mathbf{y}_i)\frac{\partial}{\partial B'}\right]f(s)$$

MM, Paranjape & Sheth (to appear)

Halo Bias from Excursion Sets

- What should one measure?

$$\langle \delta_h \delta_0 \rangle = b_{10}^{(f)} \langle \delta \delta_0 \rangle + b_{11}^{(f)} \langle \delta' \delta_0 \rangle$$

$$\left\langle \delta_h \delta_0^2 \right\rangle_c = b_{20}^{(f)} \langle \delta \delta_0 \rangle^2 + 2 b_{21}^{(f)} \langle \delta \delta_0 \rangle \langle \delta' \delta_0 \rangle + b_{22}^{(f)} \langle \delta' \delta_0 \rangle^2$$

MM, Paranjape & Sheth (2012)

- The coefficients are straightforward:

$$b_{nk}^{(f)} = \frac{(-1)^n}{f(s)} \frac{\partial^{n-k}}{\partial B^{n-k}} \frac{\partial^k}{\partial B'^k} f(s)$$

with $f(s) = \int_{B'}^{\infty} \mathrm{d}v (v - B') p(B, v)$

MM, Paranjape & Sheth (to appear)

Halo Bias from Excursion Sets

- Linear bias in Fourier space:

$$b_1(k) = b_{10}^{(f)} \underbrace{W(kR)}_{\sim 1} + b_{11}^{(f)} \underbrace{2sW'(kR)}_{\sim k^2 R^2}$$

- Quadratic bias:

$$b_2(k_1, k_2) \simeq b_{20}^{(f)} + b_{21}^{(f)}(k_1^2 + k_2^2)R^2 + b_{22}^{(f)} k_1^2 \, k_2^2 R^4$$

- Numerical predictions for the coefficients $b_{nj}(m)$ from $f(s)$; b_{n0} is the same as in peak-background split

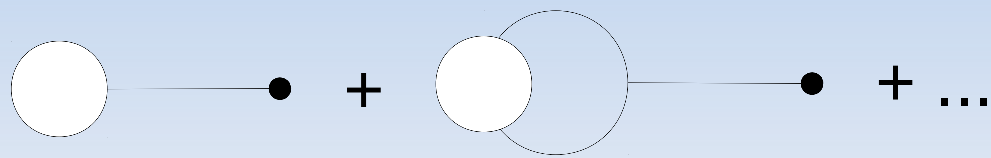
- k-dependence!

MM, Paranjape & Sheth (2012)

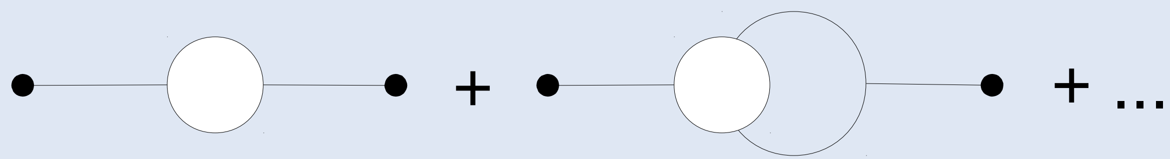
Adding non-Gaussianity: Bias

- Expand halo-matter n-point functions in matter polyspectra

$$\langle \delta_h(\mathbf{x}) \delta(\mathbf{z}_1) \rangle_c =$$



$$\langle \delta_h(\mathbf{x}) \delta(\mathbf{z}_1) \delta(\mathbf{z}_2) \rangle_c =$$



- Generic excursion set bias for non-Gaussian walks:

$$\langle \delta_h \delta_0 \rangle = \sum_{i=1}^{N} \left\langle \frac{\partial \delta_h}{\partial \delta_i} \right\rangle \langle \delta_i \delta_0 \rangle + \frac{1}{2} \sum_{i,j=1}^{N} \left\langle \frac{\partial^2 \delta_h}{\partial \delta_i \partial \delta_j} \right\rangle \langle \delta_i \delta_j \delta_0 \rangle + \ldots$$

Adding non-Gaussianity: Bias

- With the two-step approximation:

$$\langle\delta_h\delta_0\rangle \simeq b_{10}^{(f)}\langle\delta\delta_0\rangle + b_{11}^{(f)}\langle\delta'\delta_0\rangle + \frac{1}{2}\left[b_{20}^{(f)}\left\langle\delta^2\delta_0\right\rangle + 2\,b_{21}^{(f)}\langle\delta'\delta\delta_0\rangle + b_{22}^{(f)}\left\langle\delta'^2\delta_0\right\rangle\right] + \dots$$

- Same definition of b_{nj} as before, but now with non-Gaussian $f(s)$

$$\Rightarrow \quad \Delta b_1(k) = \frac{2f_{\rm NL}^{\rm local}}{k^2T(k)}\left[s\,b_{20}^{(f)} + b_{21}^{(f)} + \left\langle(\delta')^2\right\rangle b_{22}^{(f)} + \mathcal{O}(k^2)\right]$$

- All coefficients matter at small k. Possibly, also some effects at $k \sim R$ (where the leading term starts decaying). Equilateral NG?

MM, Paranjape & Sheth (to appear)

Conclusions

- Accurate solution of first passage of correlated random walks
- Full understanding of the excursion set approach to structure formation
- Simple rescaling of the spherical collapse barrier reproduces correctly the Gaussian mass function
- Straightforward non-perturbative inclusion of NG (can do Eulerian field!)
- Self-consistent predictions of bias functions and coefficients, new strategies to measure them in simulations
- To do: check against N-body simulations, generalization to excursion set theory of peaks
- Interesting possibilities (?) for primordial non-Gaussianity

Thanks!!