Vacuum neutrino oscillations with relativistic wave packets in quantum field theory

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Based upon

- D. V. Naumov and V. A. Naumov, J. Phys. G **37** (2010) 105014
- V. A. Naumov and D. V. Naumov, Russian Phys. J. **53**, 6/1 (2010) 5
- D. V. Naumov, V. A. Naumov, [arXiv:1110.0989 [hep-ph]].

- Quantum-mechanical theory of neutrino oscillations is used to analyze the neutrino experimental data. However QM theory is contradictory and not complete. Used assumptions are questionable:
  - Why coherent state $|\nu_\alpha\rangle = \sum_i V_{\alpha i}|\nu_i\rangle$ can be produced in a reaction with ℓ_α while a state $|\ell_i\rangle = \sum_\alpha V^*_{\alpha i}|\alpha\rangle$ is likely to be incoherent?
  - Same $\boldsymbol{p}_\nu$ of all *massive* $|\nu_i\rangle$ is unphysical as it depends on reference frame.
  - Definite momentum $\boldsymbol{p}_\nu$ implies uncertain position of neutrino $\delta X_\nu \propto \infty$
  - Ultra-relativistic assumption is reference frame dependent and sometimes erroneously lead to factor two difference in oscillation phase
- The origin of problems of QM approach is due to ignoring of production and detection processes. Thus QM postulates the wave function of neutrino and this is the source of numerous paradoxes.

What can be found in the literature beyond naive QM approach?

- Neutrino is considered as wave packets either in QM or in QFT
  - This successfully resolves a number of problems of naive theory. But such an approach has its own problems:
  - ▷ the form of wave packet is a guess. It is hard to quantify the ”width” of neutrino wave packets
  - ▷ the wave packets used in the literature are essentially *non-relativistic* which is far from (almost) any experiment with neutrino.

- Creation and annihilation of particles is described in a consistent way in QFT
- Standard *S*-matrix theory of QFT works with states of definite momentum = states uniformly distributed over all space. This is a good approximation for microscopic scales. It is not valid for calculations of processes localized in space and time and separated by macroscopic intervals.
- To describe appropriate states one needs a theory of relativistic wave packets. We constructed such a theory
- Relativistic wave packets (RWP) — states described by mean momentum and coordinate and their corresponding «widths»
- The mean position of RWP follows *classical trajectory*
- A probability of collision of several RWP is given by

$$\int d^4x \prod_{\text{in,out}} |\text{wave function}_{\text{in}}(x - x_{\text{in}})|^2 |\text{wave function}_{\text{out}}(x - x_{\text{in}})|^2$$

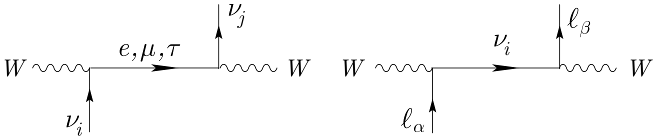
Our formalism generalizes for 4-D space the impact parameter which naturally describes suppression of collision probability with not overlaping RWPs

- QFT + localized in space-time relativistic wave packets in (*source and detector*)
- Macroscopic Feynman diagram (source and detector can be separated by thousands of km)
- Neutrino is a virtual particle (NO hypothesis about its 4-momentum, 4-coordinate, etc)
- Neutrino ”oscillations” are not mutual transformations $\nu_\alpha \leftrightarrow \nu_\beta$, but just a result of interference of diagrams with virtual massive neutrino ν_i as well as ”oscillations” of neutral kaons, D mesons and other similar known systems!

- Grimus, Stockinger 1996 Phys. Rev. D 54 3414 (arXiv:hep-ph/9603430)
- Cardall 2000 Phys. Rev. D 61 073006 (arXiv:hep-ph/9909332)
- Beuthe 2003 Phys. Rept. 375 105 (arXiv:hep-ph/0109119)
- D. V. Naumov and V. A. Naumov, J. Phys. G **37** (2010) 105014
- V. A. Naumov and D. V. Naumov, Russian Phys. J. **53**, 6/1 (2010) 5

Both diagrams are possible in the SM!

$$\mathcal{L} = -\sum_{i,\alpha} \frac{g}{\sqrt{2}} V_{\alpha i} \bar{l}_{\alpha L} \gamma_\mu \nu_{iL} W^\mu + h.c$$



An outlook on obtained results without equations

- Instead of postulating the neutrino wave function as done in QM approach, we *calculated* neutrino wave function — found to be (dispersing in space-time) RWP.
- Number of events of a macroscopic process is factorized into

$$\Phi \times \mathcal{P}_{\alpha\beta} \times \sigma$$

- Standard QM formula for ν-oscillations — is an approximation of a more general formula which depends
  - ▷ time intervals of neutrino source and detector (relevant for modern accelerator expeiments);
  - ▷ type of reaction of neutrino production and detection, its kinematics;
  - ▷ dimension of source and detector and distance between them.
- Our formula contains suppression of interference at distances larger than coherence length as well for incoherent superposition of states.

- Fock state:

$$|\boldsymbol{k}, s\rangle = \sqrt{2E_{\boldsymbol{k}}}\, a^{\dagger}_{\boldsymbol{k}s}|0\rangle,$$
$$E_{\boldsymbol{k}} = \sqrt{\boldsymbol{k}^2 + m^2}$$

- Singular normalization:

$$\langle \boldsymbol{q}, r|\boldsymbol{k}, s\rangle = (2\pi)^3 2E_{\boldsymbol{k}}\delta_{sr}\delta\left(\boldsymbol{k} - \boldsymbol{q}\right).$$

- Conventional commutation relations

$$\{a_{\boldsymbol{q}r}, a_{\boldsymbol{k}s}\} = \{a^{\dagger}_{\boldsymbol{q}r}, a^{\dagger}_{\boldsymbol{k}s}\} = 0,$$
$$\{a_{\boldsymbol{q}r}, a^{\dagger}_{\boldsymbol{k}s}\} = (2\pi)^3\delta_{sr}\delta\left(\boldsymbol{k} - \boldsymbol{q}\right).$$

- Wave packet

$$|\boldsymbol{p}, s, x\rangle = \int \frac{d\boldsymbol{k}\, \phi(\boldsymbol{k}, \boldsymbol{p}) e^{i(k-p)x}}{(2\pi)^3 2E_{\boldsymbol{k}}} |\boldsymbol{k}, s\rangle,$$

$\phi(\boldsymbol{k}, \boldsymbol{p})$ is Lorentz invariant, with a sharp peak at $\boldsymbol{k} = \boldsymbol{p}$ and a width σ

- Non singular normalization:

$$\langle \boldsymbol{q}, r, y|\boldsymbol{p}, s, x\rangle = \delta_{sr} e^{i(qy-px)} \mathcal{D}(\boldsymbol{p}, \boldsymbol{q}; x - y),$$

with relativistic-invariant function
$\mathcal{D}(\boldsymbol{p}, \boldsymbol{q}; x) = \int \frac{d\boldsymbol{k}}{(2\pi)^3 2E_{\boldsymbol{k}}} \phi(\boldsymbol{k}, \boldsymbol{p})\phi^*(\boldsymbol{k}, \boldsymbol{q}) e^{ikx}$

- $\phi(\boldsymbol{k}, \boldsymbol{p})$ can be thought as Fourie of the "source function" $j(x)$ in $(i\hat{\partial} - m)\psi(x) = j(x)$

Both Fock and Wave packet states transform in the same way under the Lorentz transformations.

- The normalization of the WP state is finite:

$$\langle \boldsymbol{p}, s, x | \boldsymbol{p}, s, x \rangle = \mathcal{D}(\boldsymbol{p}, \boldsymbol{p}; 0) = 2\overline{E}_{\boldsymbol{p}} \mathrm{V}(\boldsymbol{p}). \tag{1}$$

- The quantities $\overline{E}_{\boldsymbol{p}}$ and $\mathrm{V}(\boldsymbol{p})$ in (1) are, respectively, the mean energy and effective spatial volume of the packet, defined by

$$\overline{E}_{\boldsymbol{p}} = \frac{\int d\boldsymbol{x} \psi(\boldsymbol{p}, x) i\partial_0 \psi^*(\boldsymbol{p}, x)}{\int d\boldsymbol{x} |\psi(\boldsymbol{p}, x)|^2} = \frac{1}{\mathrm{V}(\boldsymbol{p})} \int \frac{d\boldsymbol{k} |\phi(\boldsymbol{k}, \boldsymbol{p})|^2}{4(2\pi)^3 E_{\boldsymbol{k}}}, \tag{2}$$

$$\mathrm{V}(\boldsymbol{p}) = \int d\boldsymbol{x} |\psi(\boldsymbol{p}, x)|^2 = \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{|\phi(\boldsymbol{k}, \boldsymbol{p})|^2}{(2E_{\boldsymbol{k}})^2} = \frac{\mathrm{V}(\boldsymbol{0})}{\Gamma_{\boldsymbol{p}}}, \tag{3}$$

where $\Gamma_{\boldsymbol{p}} = E_{\boldsymbol{p}}/m$. So, both $\overline{E}_{\boldsymbol{p}}$ and $\mathrm{V}(\boldsymbol{p})$, as well as the mean momentum $\overline{\boldsymbol{p}}$ defined by a relation similar to (2), are integrals of motion.

- The mean position of the packet follows the classical trajectory:

$$\overline{\boldsymbol{x}} = \frac{1}{\mathrm{V}(\boldsymbol{p})} \int d\boldsymbol{x} \psi^*(\boldsymbol{p}, x) \boldsymbol{x} \psi(\boldsymbol{p}, x) = \boldsymbol{v}_{\boldsymbol{p}} x_0. \tag{4}$$

Here $\boldsymbol{v}_{\boldsymbol{p}} = \overline{\boldsymbol{p}}/\overline{E}_{\boldsymbol{p}}$ is the mean group velocity of the packet, which coincides with the most probable velocity $\nabla_{\boldsymbol{p}} E_{\boldsymbol{p}} = \boldsymbol{p}/E_{\boldsymbol{p}}$.

- assume a simple form for:

$$\phi(\boldsymbol{k},\boldsymbol{p}) = \frac{2\pi^2}{\sigma^2 K_1(m^2/2\sigma^2)} \exp\left(-\frac{E_{\boldsymbol{k}}E_{\boldsymbol{p}} - \boldsymbol{k}\boldsymbol{p}}{2\sigma^2}\right) \stackrel{\text{def}}{=} \phi_G(\boldsymbol{k},\boldsymbol{p}), \quad (5)$$

where $K_1(t)$ is the modified Bessel function of the 3rd kind of order 1. ▸ More details

- then one can find:

$$\psi(\boldsymbol{p},x) = \frac{K_1(\zeta m^2/2\sigma^2)}{\zeta K_1(m^2/2\sigma^2)} \stackrel{\text{def}}{=} \psi_G(\boldsymbol{p},x), \text{ with } \zeta = \sqrt{1 - \frac{4\sigma^2}{m^2}\left[\sigma^2 x^2 + i(px)\right]}$$

which in the range $\sigma^2 (x_\star^0)^2 \ll m^2/\sigma^2, \sigma^2|\boldsymbol{x}_\star|^2 \ll m^2/\sigma^2$ reads:

$$\psi_G(\boldsymbol{p},x) = \exp\left(imx_\star^0 - \sigma^2 \boldsymbol{x}_\star^2\right) = \exp\left\{i(px) - \frac{\sigma^2}{m^2}\left[(px)^2 - m^2x^2\right]\right\}. \quad (6)$$

$|\psi_G(\boldsymbol{p},x)|$ is invariant under the transformations
$\{x_0 \longmapsto x_0 + \tau,\ \boldsymbol{x} \longmapsto \boldsymbol{x} + \boldsymbol{v}_{\boldsymbol{p}}\tau\}$.
▸ More details

S-matrix amplitude with wave-packets

- Initial and final states read

$$|i\rangle = |\mathbf{p}_1, x_1, s_1 \ldots \mathbf{p}_N, x_N, s_N\rangle, \quad |f\rangle = |\mathbf{k}_1, y_1, r_1 \ldots \mathbf{k}_n, y_n, r_n\rangle$$

- Dimensionless amplitude

$$\mathcal{A} = \frac{\langle f|S|i\rangle}{\sqrt{\langle i|i\rangle}\sqrt{\langle f|f\rangle}}$$

- *Microscopic* probability

$$P(p_i, x_i, s_i; k_f, y_f, r_f) \equiv |\mathcal{A}|^2$$

- *Macroscopically averaged probability* or number of events

$$dN = \sum_{s_i, r_f} \prod_i \frac{d\mathbf{p}_i d\mathbf{x}_i}{(2\pi)^3} \prod_f \frac{d\mathbf{p}_f d\mathbf{y}_f}{(2\pi)^3} P(p_i, x_i, s_i; k_f, y_f, r_f) \rho(p_i, x_i, s_i; k_f, y_f, r_f)$$

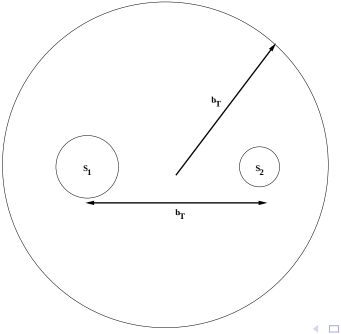
with ρ being the particles density distribution

An example. The cross-section

- Collision of two wavepackets with impact vector $\mathbf{b} = \mathbf{x}_2 - \mathbf{x}_1$ and $\mathbf{n} = \mathbf{v}_{12}/|\mathbf{v}_{12}|$
- Microscopic probability reads as ratio of two cross-sections:

$$P(\mathbf{b}, \mathbf{n}) = \frac{d\sigma}{S_{12}} e^{-\pi |\boldsymbol{b} \times \mathbf{n}|^2 / S_{12}},$$

with $S_{12} = S_1 + S_2$ and $S_i = \pi/2\sigma_i^2$ and σ is the usual cross-section with plane waves



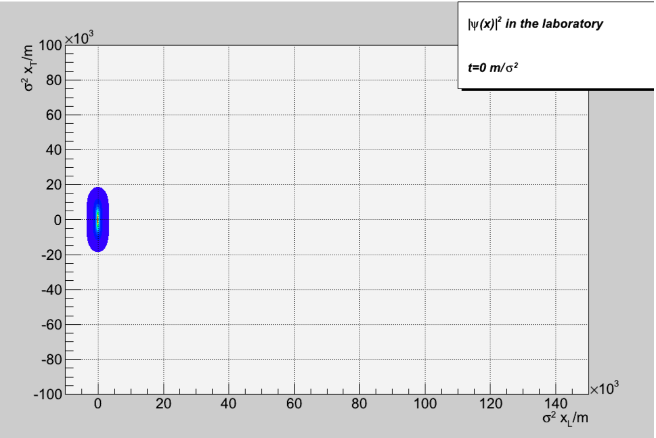
An example. The cross-section. Macroscopic averaging.

- Introduce notation $\boldsymbol{b}_T = \boldsymbol{b} \times \mathbf{n}$
- Average over $\boldsymbol{b}_T$ assuming the flux does not depend on it:

$$dN = \langle P(\mathbf{b}, \mathbf{n}) \rangle = \int d\boldsymbol{b}_T \Phi \frac{d\sigma}{S_{12}} e^{-\pi \boldsymbol{b}_T^2 / S_{12}} = \Phi d\sigma.$$

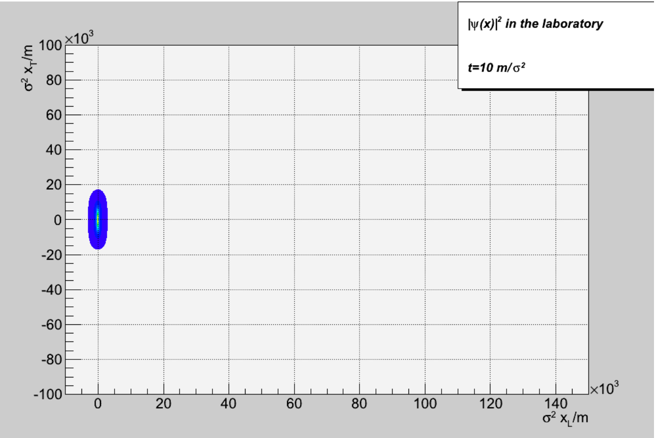
Evolution of wave packet

A numerical example: $\sigma^2/m^2 = 10^{-10}, \gamma = 10^5$



Evolution of wave packet

A numerical example: $\sigma^2/m^2 = 10^{-10}, \gamma = 10^5$



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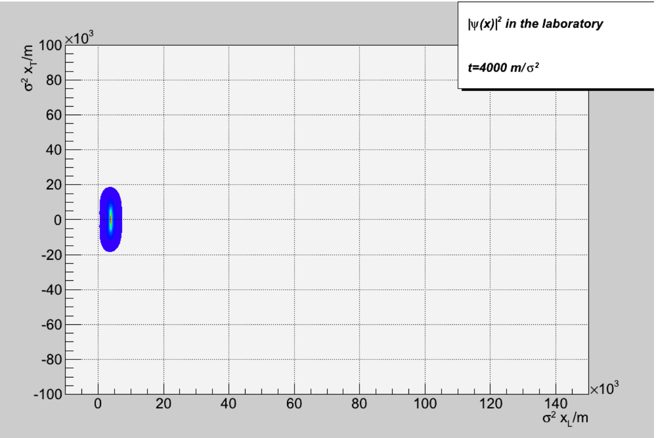

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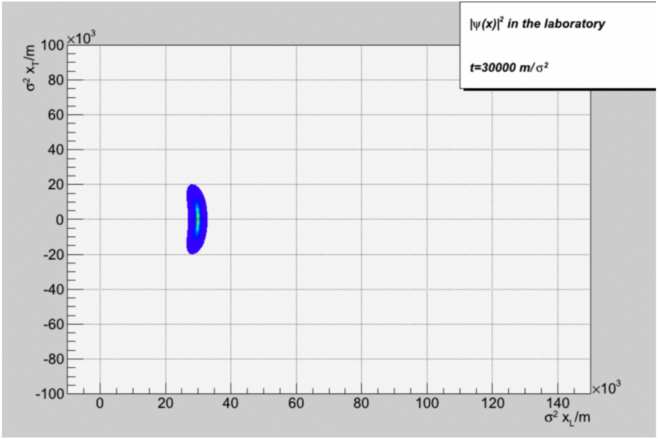

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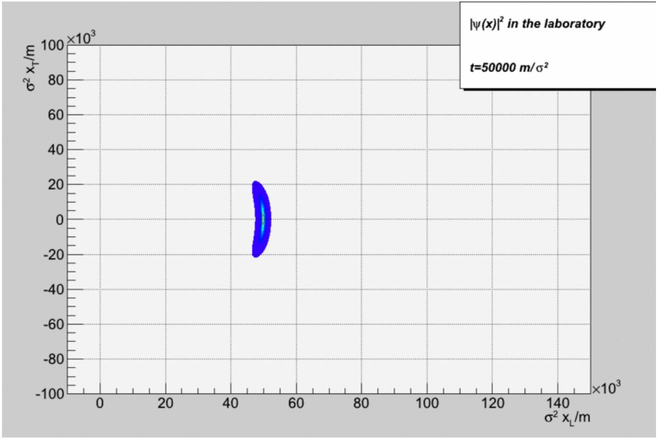

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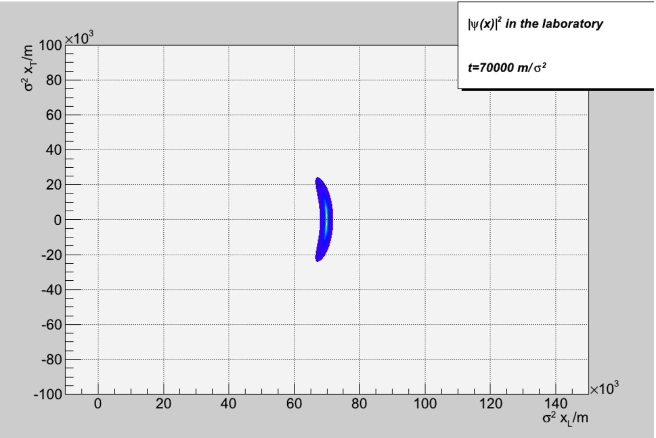
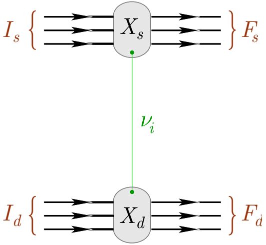


Dispersion

- Dispersion of wave-packet lead to a spherical like wave at large times relative to "dispersion time" m/σ^2
- The wave packet appears as a stable configuration over large times if it is ultra-relativistic

Particle	$\sigma_{\max}$, eV	$\Gamma/\sigma_{\max}$	$d_\star^{\min}$, cm
$\mu^\pm$	1.78×10^{-1}	1.68×10^{-9}	1.72×10^{-4}
$\tau^\pm$	2.01×10^{3}	1.13×10^{-6}	1.53×10^{-8}
$\pi^\pm$	1.88	1.35×10^{-8}	1.63×10^{-5}
π^0	3.25×10^{4}	2.41×10^{-4}	0.94×10^{-9}
$K^\pm$	5.12	1.04×10^{-8}	5.99×10^{-6}
K_S^0	6.05×10^{1}	1.22×10^{-7}	5.07×10^{-7}
K_L^0	2.53	5.08×10^{-9}	1.21×10^{-5}
$D^\pm$	1.09×10^{3}	5.82×10^{-7}	2.82×10^{-8}
D^0	1.73×10^{3}	9.28×10^{-7}	1.77×10^{-8}
$D_s^\pm$	1.61×10^{3}	8.18×10^{-7}	1.91×10^{-8}
$B^\pm$	1.46×10^{3}	2.76×10^{-7}	2.11×10^{-8}
B^0	1.51×10^{3}	2.86×10^{-7}	2.03×10^{-8}
B_s^0	1.55×10^{3}	2.89×10^{-7}	1.98×10^{-8}
n	2.64×10^{-5}	2.81×10^{-14}	1.16
Λ	5.28×10^{1}	4.74×10^{-7}	5.81×10^{-7}
$\Lambda_c^\pm$	2.74×10^{3}	1.87×10^{-6}	1.12×10^{-8}

- Calculation of macroscopic diagram with a block X_s of the "source" and a block X_d of the "detector". The blocks X_s and X_d are separated by macroscopically big space-time interval.
- The source emits neutrino with a time interval τ_s:
  - $\tau_s \to \infty$ corresponds to stationary source (Sun, reactor, atmosphere, ...)
  - $\tau_s \sim \mu s, ns$ — accelerator neutrino (T2K, Nova, OPERA, MINOS, K2K)
- The detector detects neutrino during a time interval τ_d:
  - $\tau_d \ll \tau_s$ corresponds to stationary source (Sun, reactor, atmosphere, ...)
  - $\tau_d \gtrsim \tau_s$ — accelerator neutrino (T2K, Nova, OPERA, MINOS, K2K)
- All external particles (incoming or outgoing) are relativistic wave packets
- Virtual neutrino is described by causial propagator



Feynman rules:

$$\begin{cases} e^{-ip_a(x_a-x)}\psi_a(\boldsymbol{p}_a, x_a - x) \\ e^{+ip_b(x_b-x)}\psi_b^*(\boldsymbol{p}_b, x_b - x), \end{cases}$$

where $\psi_\varkappa(\boldsymbol{p}_\varkappa, x)$ $(\varkappa = a, b)$ wave packets for every particle. Internal lines and loops remain unchanged.

Normalized amplitude is given by 4th order in perturbation theory in EW coupling constand g (strong and electromagnetic interactions are taken into account exactly — without perturbations!):

$$\begin{aligned}\mathcal{A}_{\beta\alpha} &= \langle\mathbf{out}|\mathbb{S}|\mathbf{in}\rangle\left(\langle\mathbf{in}|\mathbf{in}\rangle\langle\mathbf{out}|\mathbf{out}\rangle\right)^{-1/2} \\ &= \frac{1}{\mathcal{N}}\left(\frac{-ig}{2\sqrt{2}}\right)^4\langle F_s\oplus F_d|T\int dxdx'dydy'\ :j_\ell(x)W(x)::j_q(x')W^\dagger(x'): \\ &\quad\times :j_\ell^\dagger(y)W^\dagger(y)::j_q^\dagger(y')W(y'):\mathbb{S}_h|I_s\oplus I_d\rangle.\end{aligned}$$

- The hadronic parts of the amplitude can be factorized into a product of source and detector subblocks
- The $\int d\boldsymbol{q}$ in the neutrino propagator can be taking with help of Grimus-Stockinger (GS) theorem giving rise to $\propto \frac{e^{-iq_0 T + i\sqrt{q_0^2 - m_j^2} L}}{L}$ with $T = X_d^0 - X_s^0$, $\boldsymbol{L} = \boldsymbol{X_d} - \boldsymbol{X_s}$
- The remaining integration over $\int dq_0$ can be taken with help of the saddle-point approximation
- Assuming $m_j = 0$ in the matrix elements (not in the exponentials!) the total amplitude can be factorized into:

More details

$$\mathcal{A}_{\beta\alpha} = \frac{\mathfrak{D}|\mathbb{V}_s(p_\nu)\mathbb{V}_d(p_\nu)|M_s M_d^*}{i(2\pi)^{3/2}\mathcal{N}L} \sum_j V_{\alpha j}^* V_{\beta j}\, e^{-\Omega_j(T,L) - i\Theta_j}. \qquad (7)$$

where

- the matrix elements:

$$\begin{cases} M_s & \text{corresponds to reaction } I_s \to F_s' + \ell_\alpha^+ + \nu, \\ M_d & \text{corresponds to reaction } \nu + I_d \to F_d' + \ell_\beta^-. \end{cases}$$

- the phase

$$\Omega_j(T,L) = i(p_j X) + \left(2\widetilde{\mathfrak{D}}_j^2/E_\nu^2\right)\left[(p_j X)^2 - m_j^2 X^2\right], \qquad X = X_d - X_s.$$

- gauss dispersion of the neutrino energy $\widetilde{\mathfrak{D}}$ is a function of all σ

Given the factorized amplitude it turnes out that we *calculated* the amplitude of neutrino propagation (which coincides with its wave-function up to a factor $\sqrt{2E_\nu}$) of the outgoing neutrino $\psi_j^* = e^{-\Omega_j}$ with the explicitly invariant complex phase $\Omega_j(T, L)$:

$$\Omega_j(T, L) = i(p_j X) + \left(2\widetilde{\mathfrak{D}}_j^2 / E_\nu^2\right) \left[(p_j X)^2 - m_j^2 X^2\right], \qquad X = X_d - X_s. \tag{8}$$

- the ν WP is of CRGP form with the "width" $\Sigma_j = \sqrt{2}\widetilde{\mathfrak{D}}_j / \Gamma_j \qquad (\Gamma_j = E_\nu / m_j)$.
- Since Σ_j is a complex-valued function, the νWPs spreads with increase of $L = |\boldsymbol{X}|$. The spread effect can be important only at "cosmological" distances. Here we limit ourselves to the "terrestrial" conditions
- The relative energy-momentum uncertainty of the νWP is $\delta E_j / E_j \sim \delta P_j / P_j \sim \mathfrak{D} / E_\nu \lll 1$. The mean position of the νWPs evolves along the "classical trajectory" $\overline{\boldsymbol{L}} = \boldsymbol{v}_j T$

More details

Recall the dimensionless amplitude

$$\mathcal{A}_{\beta\alpha} = \frac{\mathfrak{D}|\mathbb{V}_s(p_\nu)\mathbb{V}_d(p_\nu)|M_s M_d^*}{i(2\pi)^{3/2}\mathcal{N}L} \sum_j V_{\alpha j}^* V_{\beta j}\, e^{-\Omega_j(T,L)-i\Theta}. \tag{9}$$

yielding the microscopic probability:

$$|\mathcal{A}_{\beta\alpha}|^2 = \frac{(2\pi)^4\delta_s(p_\nu - q_s)\mathrm{V}_s|M_s|^2}{\prod_{\varkappa\in S} 2E_\varkappa \mathrm{V}_\varkappa}\, \frac{(2\pi)^4\delta_d(p_\nu + q_d)\mathrm{V}_d|M_d|^2}{\prod_{\varkappa\in D} 2E_\varkappa \mathrm{V}_\varkappa} \times \frac{\mathfrak{D}^2}{(2\pi)^3 L^2}\left|\sum_j V_{\alpha j}^* V_{\beta j}\, e^{-\Omega_j(T,L)}\right|^2. \tag{10}$$

with

- $\delta_{s,d}$ - “smeared“ δ function
- $\mathrm{V}_{s,d}$ - 4-D overlap volume:

$$\mathrm{V}_{s,d} = \int dx \prod_{\varkappa\in S,D} |\psi_\varkappa(\boldsymbol{p}_\varkappa, x_\varkappa - x)|^2 = \frac{\pi^2}{4}|\mathfrak{R}_{s,d}|^{-1/2}\exp\left(-2\mathfrak{S}_{s,d}\right). \tag{11}$$

The microscopic probability depends on all individual mean coordinates of the wave packets, it should be averaged over ensembles in the source and detector to get the macroscopic probability or the number of events.

We assume

- particles distribution functions independent on time within time intervals $\tau_s = x_2^0 - x_1^0$ and $\tau_d = y_2^0 - y_1^0$ (could be astronomically large):

$$f_a(\mathbf{p}_a, s_a; x) = \theta\left(x^0 - x_1^0\right)\theta\left(x_2^0 - x^0\right)\bar{f}_a(\mathbf{p}_a, s_a; \mathbf{x}) \text{ for } a \in I_s,$$
$$f_a(\mathbf{p}_a, s_a; y) = \theta\left(y^0 - y_1^0\right)\theta\left(y_2^0 - y^0\right)\bar{f}_a(\mathbf{p}_a, s_a; \mathbf{y}) \text{ for } a \in I_d.$$

- source and detectors have finite 3D volumes $V_{\mathcal{S}}, V_{\mathcal{D}}$

Then:

$$\langle\langle|\mathcal{A}_{\beta\alpha}|^2\rangle\rangle \equiv dN_{\alpha\beta} = \int \prod_{\varkappa\in I_s, I_d, F_s, F_d} \frac{d\mathbf{x}_\varkappa d\mathbf{p}_\varkappa f_\varkappa(\mathbf{p}_\varkappa, s_\varkappa, \mathbf{x}_\varkappa)}{(2\pi)^3 2E_\varkappa \mathrm{V}_\varkappa} \langle|\mathcal{A}_{\beta\alpha}|^2\rangle$$
$$= \frac{\tau_d}{V_{\mathcal{D}} V_{\mathcal{S}}} \int d\mathbf{x} \int d\mathbf{y} \int d\Phi_\nu \int d\sigma_{\nu\mathcal{D}} \mathcal{P}_{\alpha\beta}(E_\nu, |\mathbf{y} - \mathbf{x}|).$$

$$\langle\langle|\mathcal{A}_{\beta\alpha}|^2\rangle\rangle \equiv dN_{\alpha\beta} = \int \prod_{\varkappa\in I_s, I_d, F_s, F_d} \frac{d\boldsymbol{x}_\varkappa d\boldsymbol{p}_\varkappa f_\varkappa(\boldsymbol{p}_\varkappa, s_\varkappa, \boldsymbol{x}_\varkappa)}{(2\pi)^3 2E_\varkappa \mathrm{V}_\varkappa} \langle|\mathcal{A}_{\beta\alpha}|^2\rangle$$

$$= \frac{\tau_d}{V_{\mathcal{D}} V_{\mathcal{S}}} \int d\boldsymbol{x} \int d\boldsymbol{y} \int d\Phi_\nu \int d\sigma_{\nu\mathcal{D}} \mathcal{P}_{\alpha\beta}(E_\nu, |\boldsymbol{y} - \boldsymbol{x}|).$$

where

- flux density of neutrinos in $\mathcal{D}$, produced through the processes $I_s \to F_s' \ell_\alpha^+ \nu$ in $\mathcal{S}$ reads:

$$\frac{d\boldsymbol{x}}{V_{\mathcal{S}}} \int \frac{d\Phi_\nu}{dE_\nu}$$

- the differential cross section of the neutrino scattering off the detector as a whole.

$$\frac{1}{V_{\mathcal{D}}} \int d\boldsymbol{y} d\sigma_{\nu\mathcal{D}}$$

▸ More details

▷ Main result of our work is the most general formula for "oscillation probability":

$$\mathcal{P}_{\alpha\beta}(E_\nu, L) = \sum_{ij} V^*_{\alpha i} V_{\alpha j} V_{\beta i} V^*_{\beta j} S_{ij} \exp\left(i\frac{2\pi L}{L_{ij}} - \mathscr{A}^2_{ij} \right). \tag{12}$$

• This probability coincides with QM formula if $S_{ij} = 1$ and $\mathscr{A}_{ij} = 0$. Violation of these conditions limits region of applicability of standard formula. One needs to satisfy two conflicting requirements on neutrino energy dispersion $\delta E_\nu = \mathfrak{D}$

- coherence condition at production (could be violated):

$$\delta E_\nu \gg \frac{\pi \mathfrak{n}}{2L_{ij}}$$

- destruction of coherence (will be violated at large distances):

$$\delta E_\nu \ll E_\nu \frac{L_{ij}}{2\pi L}$$

The S_{ij} is a quite complicated object:

$$S_{ij} = \frac{\exp\left(-\mathscr{B}_{ij}^2\right)}{4\tau_d\mathfrak{D}} \sum_{l,l'=1}^{2} (-1)^{l+l'+1} \mathrm{Ierf}\left[2\mathfrak{D}\left(x_l^0 - y_{l'}^0 + \frac{L}{v_{ij}}\right) - i\mathscr{B}_{ij}\right], \tag{13}$$

$$\mathscr{A}_{ij} = (v_j - v_i)\mathfrak{D}L = \frac{2\pi\mathfrak{D}L}{E_\nu L_{ij}}, \qquad \mathscr{B}_{ij} = \frac{\Delta E_{ji}}{4\mathfrak{D}} = \frac{\pi\mathfrak{n}}{2\mathfrak{D}L_{ij}}, \tag{14}$$

$$\varphi_{ij} = \frac{2\pi L}{L_{ij}}, \qquad L_{ij} = \frac{4\pi E_\nu}{\Delta m_{ij}^2}, \qquad \frac{1}{v_{ij}} = \frac{1}{2}\left(\frac{1}{v_i} + \frac{1}{v_j}\right),$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2, \qquad \Delta E_{ij} = E_i - E_j,$$

$$\mathrm{Ierf}(z) = \int_0^z dz' \mathrm{erf}(z') + \frac{1}{\sqrt{\pi}} = z\,\mathrm{erf}(z) + \frac{1}{\sqrt{\pi}} e^{-z^2},$$

which depends on:

- ν energy-momentum uncertainty $\mathfrak{D}$, its mean energy E_ν, masses m_i, m_j, correction to energy and momentum $\mathfrak{n}$
- time intervals of the source τ_s and the detector τ_d
- length between the source and the detector L and time difference between two time windows in the source and the detector T. There is no reason *apriori* that $L \approx T$, these are two *independent* parameters

In the asymptotic regime $t = \tau_s \mathcal{D} \to \infty$ (valid for atmosperic, solar, reactor ν):

$$S(t, t', b) \sim \exp(-b^2)$$

the “probability“ (12) takes on the form already known from the literature,[1]

$$\begin{aligned}
\mathcal{P}_{\alpha\beta}(E_\nu, L) &= \sum_{ij} V^*_{\alpha i} V_{\alpha j} V_{\beta i} V^*_{\beta j} \exp\left(i\frac{2\pi L}{L_{ij}} - \mathscr{A}^2_{ij} - \mathscr{B}^2_{ij} \right), \\
&= \sum_{ij} V^*_{\alpha i} V_{\alpha j} V_{\beta i} V^*_{\beta j} \exp\left(i\frac{2\pi L}{L_{ij}} - \left(\frac{2\pi \mathfrak{D} L}{E_\nu L_{ij}}\right)^2 - \left(\frac{\pi \mathfrak{n}}{2\mathfrak{D} L_{ij}}\right)^2 \right) \\
&= \sum_{ij} V^*_{\alpha i} V_{\alpha j} V_{\beta i} V^*_{\beta j} \exp\left(i\frac{2\pi L}{L_{ij}} - \left(\frac{L}{L^{coh}_{ij}}\right)^2 - \left(\frac{1}{\mathfrak{D} L^{int}_{ij}}\right)^2 \right)
\end{aligned}$$

with

- “coherence length“ $L^{\rm coh}_{ij} = \frac{1}{\Delta v_{ij} \mathfrak{D}}$ $\quad (\Delta v_{ij} = |v_j - v_i|)$,
- “interference length“ $L^{\rm int}_{ij} = \frac{1}{4\Delta E_{ij}} = \frac{2L_{ij}}{\pi \mathfrak{n}}$.

[1] See. e.g., C. Giunti C and C. W. Kim, *Fundamentals of Neutrino Physics and Astrophysics* (Oxford University Press Inc., New York, 2007); M. Beuthe, *Oscillations of neutrinos and mesons in quantum field theory*, Phys. Rept. **375** (2003) 105 [arXiv:hep-ph/0109119]; M. Beuthe, *Towards a unique formula for neutrino oscillations in vacuum*, Phys. Rev. D **66** (2002) 013003 [arXiv:hep-ph/0202068].

- The factors $\exp\left(-\mathscr{A}_{ij}^2\right)$ (with $i{\neq}j$) suppress the interference terms at the distances exceeding the "coherence length"

$$L_{ij}^{\rm coh} = \frac{1}{\Delta v_{ij}\mathfrak{D}} \gg |L_{ij}| \qquad (\Delta v_{ij} = |v_j - v_i|),$$

when the νWPs ψ_i^* and ψ_j^* are strongly separated in space and do not interfere anymore. Clearly $L_{ij}^{\rm coh} \to \infty$ in the plane-wave limit.

- The suppression factors $\exp\left(-\mathscr{B}_{ij}^2\right)$ ($i{\neq}j$) work in the opposite situation, when the external packets in $\mathcal{S}$ or $\mathcal{D}$ (or in both $\mathcal{S}$ and $\mathcal{D}$) are strongly delocalized
The gross dimension of the the neutrino production and absorption regions in $\mathcal{S}$ and $\mathcal{D}$ is of the order of $1/\mathfrak{D}$. The interference terms vanish if this scale is large compared to the "interference length"

$$L_{ij}^{\rm int} = \frac{1}{4\Delta E_{ij}} = \frac{2L_{ij}}{\pi\mathfrak{n}}.$$

In other words, the QFT approach predicts vanishing of neutrino oscillations in the plane-wave limit. In this limit, the flavor transition probability does not depend on L, E_ν, and neutrino masses m_i and becomes

$$\mathcal{P}_{\alpha\beta}^{\rm PWL} = \sum_i |V_{\alpha i}|^2 |V_{\beta i}|^2 \leq 1.$$

Thereby, a nontrivial interference of the diagrams with the intermediate neutrinos of different masses is only possible if $\mathfrak{D} \neq 0$.

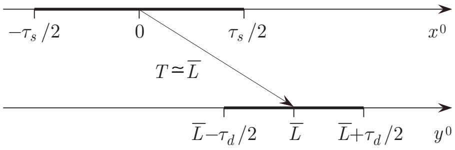
Current accelerator experiments detect neutrinos within tiny time windows:

	T2K	OPERA	MINOS	MiniBooNe
			Nova	SciBooNe
τ_s	$50ns$	$10.5\mu s$	$10\mu s$	$4ns$
τ_d	$500ns$	$20\mu s$	$100\mu s$	$\#ns$

Our formula predicts *vanishing* of the number of detected events if the time windows are not synchronized.

Let us consider further the case of "synchronized" measurements, in which

$$x^0_{1,2} = \mp\frac{\tau_s}{2}, \quad y^0_{1,2} = \overline{L} \mp \frac{\tau_d}{2}.$$

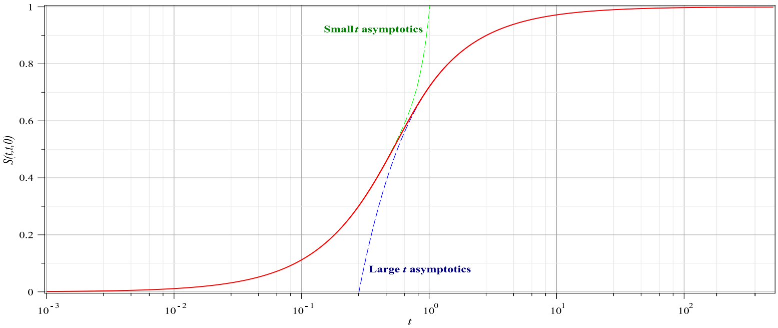


Thus the factor S_{ij} can be expressed through a real-valued function $S(t, t', b)$ of three dimensionless variables:

$$S_{ij} = S\left(\mathfrak{D}\tau_s, \mathfrak{D}\tau_d, \mathscr{B}_{ij}\right),$$
$$2t' S(t, t', b) = \exp\left(-b^2\right) \operatorname{Re}\left[\operatorname{Ierf}\left(t + t' + ib\right) - \operatorname{Ierf}\left(t - t' + ib\right)\right].$$

- The diagonal decoherence function $0 \le S_{ii} = S\left(\mathfrak{D}\tau_s, \mathfrak{D}\tau_d, 0\right) \le 1$ suppresses the total event rate
- The non-diagonal decoherence functions $0 \le S_{ij} = S\left(\mathfrak{D}\tau_s, \mathfrak{D}\tau_d, \mathscr{B}_{ij}\right) \le 1$ $[i \ne j]$ suppress the interference ("oscillations")

The diagonal decoherence function

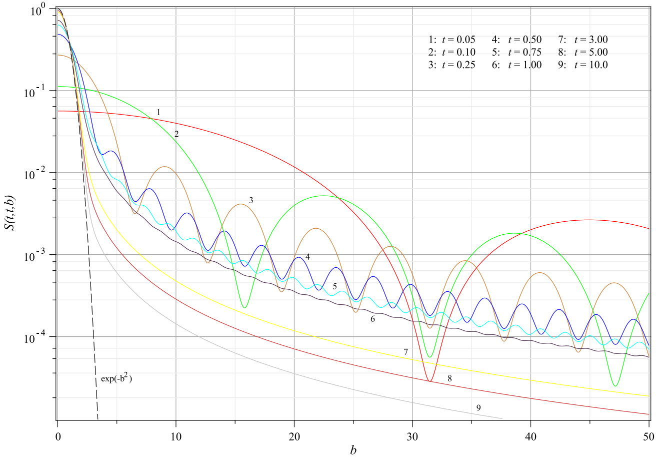


- $S(t,t,0) \to 0$ for $\tau_s \ll \tau_\nu \sim 1/\mathfrak{D}$. This is natural to obtain zero events measuring much shorter times than the neutrino wave packet time width
- $S(t,t,0) \to 1$ for $\tau_s \gg \tau_\nu \sim 1/\mathfrak{D}$. No suppression is exected for measurement time intervals much longer than the neutrino wave packet time width
  - Note however that the effect of finite the neutrino wave packet time width τ_ν persists up to $\tau_s/\tau_\nu \sim 10-100$. This provides a hint to *measure neutrino wave packet time width!*
  - Another way to measure τ_ν is measure the event rate as a function of τ_d at fixed τ_s

The details

Nondiagonal decoherence function

The decoherence function $S(t, t', b)$ at $b \neq 0$ is much more involved.



At very large t, the function $S(t, t, b)$ becomes nearly independent on t, slowly approaching the asymptotic behavior $S(t, t, b) \sim \exp(-b^2)$ $(t, t' \to \infty)$.

Nondiagonal decoherence function

- At finite τ_s, τ_d the suppression is smaller than the asymptotic limit $\lim_{t\to\infty} S(t,t,b) \sim \exp(-b^2)$. Why?
- The reason is that *a measurement at finite time intervals introduces an additional uncertainty into energy-momentum thus making the coherence of states more probable*

▸ The details

Where one can observe a difference between QM and QFT approaches?

- At accelerators one can expect:
  - *decrease* in number of events in near detector due to rather small time intervals τ_s, τ_d
  - *modification* of ”oscillation” signal in far detector
  - However, taking into account of found corrections might get $\Delta m^2, \sin^2 2\theta$ right
- Intense (Mega-Curie) sources of (anti)neutrino with narrow energy lines $\delta E_\nu \ll E_\nu$ mostl likely will be *incoherent* according to QFT and *coherent* in QM theory.
- Cosmological neutrinos are *incoherent* in QFT based approach
- Light and Heavy (but still ultra-relativistic) neutrinos will not show an ”oscillotary” behaviour as they produced *inchorently* in QFT while they will ”oscillate” in QM theory
- Our approach gives a completely different picture about sterile neutrinos compared to QM

Sterile neutrino — is a popular explanation of experimental ”anomalies” (new degree of freedom — an additional Δm^2)

- LSND anomaly
- MiniBooNE anomaly
- reactor anomaly

- ▷ Basic question: What it is «sterile neutrino»?
- ▷ Traditional answer: A state of neutrino which *does not interact* but a state into which *active* = *interacting* neutrinos could ”oscillate”
- ▷ However the effect of ”oscillations” is NOT due to mutual transformation of neutrinos into each other, rather it is a result of interference of diagrams with interacting neutrinos.

Could sterile neutrinos exist in QFT?

In SM there are "sterile" states — right neutrinos ν_R. However they are sterile until neutrino is considered massless. Non zero neutrino mass means that ν_L, ν_R interact with Higgs field. There are two scenarios:

1. Number of fields ν_R equals three: diagonalization $\lambda_{\ell\ell'}\bar{\nu}_{\ell L}\phi\nu_{\ell' R} + \text{h.c.}$ gives three massive fields of neutrinos. No sterile neutrinos.
2. Number of fields ν_R is larger than three. Then one has to diagonalize the most general mass lagrangian:

$$\mathcal{L}_m = -\frac{1}{2}\left(\overline{\nu_L}, \overline{(\nu_R)^c}\right)\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}\begin{pmatrix} (\nu_L)^c \\ \nu_R \end{pmatrix} + h.c.$$

where $\nu_L^f = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L})^T$, $(\nu_R)^c = ((\nu_{eR})^c, (\nu_{\mu R})^c, (\nu_{\tau R})^c, \ldots)^T$.

$\mathcal{L}_m$ is diagonal in terms of new fields ν_L^m, N_L:

$$\begin{pmatrix} \nu_L^f \\ (\nu_R)^c \end{pmatrix} = \begin{pmatrix} V & M \\ K & U \end{pmatrix}\begin{pmatrix} \nu_L^m \\ N_L \end{pmatrix}$$

The fields ν_L^m, N_L interact with W, Z — not really sterile!

The unitarity of mixing matrix gives:

$$VV^\dagger + MM^\dagger = 1, \quad KK^\dagger + UU^\dagger = 1, \quad VK^\dagger + MU^\dagger = 0,$$

however $VV^\dagger \neq 1, UU^\dagger \neq 1$!.

Existence of additional neutrino fields lead to a statement that PMNS mixing matrix could not non unitary!

- Interaction with W:

$$\mathcal{L}_{cc}^{SM/EW} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \ell_L^\alpha \gamma_\mu \nu_L^\alpha W^\mu + h.c. \to$$

$$-\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{3} V_{\alpha i} \ell_L^\alpha \gamma_\mu \nu_{iL}^m W^\mu - \frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{i=1}^{3} M_{\alpha i} \ell_L^\alpha \gamma_\mu N_{iL} W^\mu + h.c.$$

- Interaction with Z non diagonal:

$$\mathcal{L}_{nc}^{SM/EW} = -\frac{g}{\cos\theta_W} \sum_{\ell=e,\mu,\tau} \bar{\nu}_{\ell L} \gamma_\mu \nu_L Z^\mu \to$$

$$-\frac{g}{\cos\theta_W} \left(V^\dagger V \bar{\nu}_L^m \gamma_\mu \nu_L^m + M^\dagger M \bar{N}_L \gamma_\mu N_L + V^\dagger M \bar{\nu}_L^m \gamma_\mu N_L + M^\dagger V \bar{N}_L \gamma_\mu \nu_L^m \right) Z^\mu$$

There are at least two data sets which puts limits:

- LEP from decays $Z \to \text{invisible}$ limits $M^\dagger M \ll 1$.
- Tritium decays also limits $m_e = \sum_{i=1}^{3} |V_{ei}|^2 m_i + \sum_k |M_{ek}|^2 m_k$

One can think about possible scenaries:

- If $m_N \gg m_\nu$, then no oscillations with Δm^2_{ster} could be expected because of coherence suppression. The effect can be seen in mixing matrix non-unitarity and as a result fewer number of events.
- If $m_N \simeq m_\nu$, the interference is possible while the contribution from Δm^2_{ster} will be suppressed due to smallness of matrix M. It is hard to the interference (possible only at comparable neutrino masses) + small contributon to Z boson width + small contribution to mass m_e + small neutrino mass (cosmology) simultaneosly. This makes our approach very different from QM (thus our theory is falsifiable)

In a consistent QFT based theory of neutrino oscillations the expected effect of ”sterile” neutrino is significantly different from QM approach.

Within QFT and relativistic wave functions of external states we considered a macroscopic process with lepton number violation. As a result one can find

- calculated neutrino wave function - which turned out to be (in general) a spreading with time-distance wave packet.
- the number of events of macroscopic process is factorized into a product $\Phi\times\ \mathcal{P}_{\alpha\beta}\times\sigma$
- The standard QM ν-oscillation formula is an approximation of the more general formula which depends essentialy on
  - ▷ the source “machine ” and detector exposure time;
  - ▷ reaction types in the neutrino production and absorption regions and phase-space domains of these reactions;
  - ▷ dimensions of the source and detector and distance between them.
- The QFT formula as a by product explains a lost of coherence for charged leptons
- We argue that the neutrino wave function can be measured in experiments with short beam pulses (T2K, OPERA, MINOS, MiniBooNe, SciBooNe, Nova, ...)

Appendices. Additional information

We consider a simple model of the state – relativistic Gaussian packet (RGP) with:

$$\phi(\boldsymbol{k},\boldsymbol{p}) = \frac{2\pi^2}{\sigma^2 K_1(m^2/2\sigma^2)} \exp\left(-\frac{E_{\boldsymbol{k}}E_{\boldsymbol{p}} - \boldsymbol{kp}}{2\sigma^2}\right) \stackrel{\text{def}}{=} \phi_G(\boldsymbol{k},\boldsymbol{p}), \tag{15}$$

where $K_1(t)$ is the modified Bessel function of the 3rd kind of order 1. In what follows we assume

$$\sigma^2 \ll m^2 \tag{16}$$

Then the function (15) can be rewritten as an asymptotic expansion:

$$\phi_G(\boldsymbol{k},\boldsymbol{p}) = \frac{2\pi^{3/2}}{\sigma^2}\frac{m}{\sigma} \exp\left[\frac{(k-p)^2}{4\sigma^2}\right]\left[1 + \frac{3\sigma^2}{4m^2} + \mathcal{O}\left(\frac{\sigma^4}{m^4}\right)\right]. \tag{17}$$

The nonrelativistic limit of the function (17) coincides, up to a normalization factor, with the usual (noncovariant) Gaussian distribution:

$$\varphi_G(\boldsymbol{k}-\boldsymbol{p}) \propto \exp\left[-\frac{(\boldsymbol{k}-\boldsymbol{p})^2}{4\sigma^2}\right].$$

But it is not the case at relativistic and especially ultrarelativistic momenta.

Return

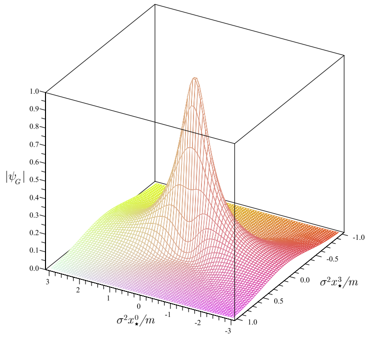
Example: in the vicinity of the maximum $\boldsymbol{k} = \boldsymbol{p}$

$$\phi_G(\boldsymbol{k}, \boldsymbol{p}) \approx \frac{2\pi^{3/2}}{\sigma^2} \frac{m}{\sigma} \exp\left[-\frac{(\boldsymbol{k} - \boldsymbol{p})^2}{4\sigma^2 \Gamma_{\boldsymbol{p}}^2} \right] \qquad (\boldsymbol{k} \sim \boldsymbol{p}).$$

We see that in this case the relativistic effect consists in a “renormalization” of the WP width ($\sigma \to \sigma\Gamma_{\boldsymbol{p}}$). This renormalization is very essential for the neutrino production and detection processes involving relativistic particles.

Return

Function $\psi_G(\boldsymbol{p},x)$.



A 3D plot of $|\psi_G(\mathbf{0},x_\star)|$ as a function of $\sigma^2 x^0_\star/m$ and $\sigma^2 x^3_\star/m$ (assuming that $\boldsymbol{x}_\star = (0,0,x^3_\star)$). The calculations are done for $\sigma/m = 0.1$.

$$\psi(\boldsymbol{p},x) = \frac{K_1(\zeta m^2/2\sigma^2)}{\zeta K_1(m^2/2\sigma^2)} \stackrel{\text{def}}{=} \psi_G(\boldsymbol{p},x),$$

where we have defined the dimensionless Lorentz-invariant complex variable

$$\zeta = \sqrt{1 - \frac{4\sigma^2}{m^2}\left[\sigma^2 x^2 + i(px)\right]};$$

$$|\zeta|^4 = \left[1 - \frac{4\sigma^4 x^2}{m^2}\right]^2 + \frac{16\sigma^4 (px)^2}{m^4},$$

$$\varphi = -\frac{1}{2}\arcsin\left[\frac{4\sigma^2(px)}{m^2|\zeta|^2}\right].$$

It can be proved that for any $\boldsymbol{p}$ and x

$$|\zeta| \geq 1 \quad \text{and} \quad |\varphi| < \pi/2.$$

Return

An analysis of the asymptotic expansion of $\ln\left[\psi_G(\mathbf{0}, x_\star)\right]$ in powers of $\sigma^2/(m^2\zeta)$ provides the following (necessary and sufficient) conditions of the nondiffluent behavior:

$$\sigma^2 (x_\star^0)^2 \ll m^2/\sigma^2, \qquad \sigma^2 |\mathbf{x}_\star|^2 \ll m^2/\sigma^2. \tag{18a}$$

They can be rewritten in the equivalent but explicitly Lorentz-invariant form:

$$(px)^2 \ll m^4/\sigma^4, \qquad (px)^2 - m^2x^2 \ll m^4/\sigma^4. \tag{18b}$$

Under these conditions $\psi_G(\mathbf{p}, x)$ reduces to the simple form:

$$\psi_G(\mathbf{p}, x) = \exp\left(imx_\star^0 - \sigma^2 \mathbf{x}_\star^2\right) = \exp\left\{i(px) - \frac{\sigma^2}{m^2}\left[(px)^2 - m^2x^2\right]\right\}. \tag{19}$$

Some properties of CRGP:

1. The mean coordinate of the packet follows the classical trajectory (CT) $\mathbf{x} = \mathbf{v_p} x_0$.
2. $|\psi_G(\mathbf{p}, x)| = 1$ along the CT and $|\psi_G(\mathbf{p}, x)| < 1$ with any deviation from it.
3. $|\psi_G(\mathbf{p}, x)|$ is invariant under the transformations $\{x_0 \longmapsto x_0 + \tau, \ \mathbf{x} \longmapsto \mathbf{x} + \mathbf{v_p}\tau\}$.
4. In the nonrelativistic limit, the wave function (19) takes the form:

$$\psi_G(\mathbf{p}, x) \approx \exp\left[im\left(x_0 - \mathbf{v_p}\mathbf{x}\right) - \sigma^2\left|\mathbf{x} - \mathbf{v_p}x_0\right|^2\right].$$

◂ Return

The additional factors (1) provide the following two common multipliers in the integrand of the scattering amplitude [we will call these the overlap integrals]:

$$\mathbb{V}_s(q) = \int dx e^{+iqx} \left[\prod_{a \in I_s} e^{-ip_a x_a} \psi_a \left(\boldsymbol{p}_a, x_a - x \right) \right] \left[\prod_{b \in F_s} e^{ip_b x_b} \psi_b^* \left(\boldsymbol{p}_b, x_b - x \right) \right],$$
$$\mathbb{V}_d(q) = \int dx e^{-iqx} \left[\prod_{a \in I_d} e^{-ip_a x_a} \psi_a \left(\boldsymbol{p}_a, x_a - x \right) \right] \left[\prod_{b \in F_d} e^{ip_b x_b} \psi_b^* \left(\boldsymbol{p}_b, x_b - x \right) \right]. \quad (20)$$

The function $\mathbb{V}_s$ ($\mathbb{V}_d$) characterizes the 4D overlap of the “in” and “out” wave-packet states in the source (detector) vertex Return

In the plane-wave limit ($\sigma_\varkappa \to 0, \forall \varkappa$)

$$\mathbb{V}_s(q) \to (2\pi)^4 \delta\left(q - q_s\right) \quad \text{and} \quad \mathbb{V}_d(q) \to (2\pi)^4 \delta\left(q + q_d\right),$$

where q_s and q_d are the 4-momentum transfers defined by

$$q_s = \sum_{a \in I_s} p_a - \sum_{b \in F_s} p_b \quad \text{and} \quad q_d = \sum_{a \in I_d} p_a - \sum_{b \in F_d} p_b.$$

The δ functions provide the energy-momentum conservation in the vertices s and d (that is in the “subprocesses” $I_s \to F_s + \nu_i^*$ and $\nu_i^* + I_d \to F_d$) and, as a result, – in the whole process:

$$I_s \oplus I_d \to F_s \oplus F_d : \qquad \sum_{a \in I_s \oplus I_d} p_a = \sum_{b \in F_s \oplus F_d} p_b.$$

Information about the space-time coordinates of the interacting packets is completely lost.

In the general case of *nonzero* spreads $\sigma_\varkappa$, we may expect no more than an approximate conservation of energy and momentum and lack of any singularities. To quantify this expectation, we apply the CRGP model.

$$T_\varkappa^{\mu\nu} = \sigma_\varkappa^2 \left(u_\varkappa^\mu u_\varkappa^\nu - g^{\mu\nu}\right), \qquad [u_\varkappa = p_\varkappa / m_\varkappa = \Gamma_\varkappa(1, \boldsymbol{v}_\varkappa)]$$

$$\Downarrow$$

$$\mathbb{V}_{s,d}(q) = \int dx \exp\left[i\left(\pm qx - q_{s,d}x\right) - \sum_{\varkappa \in S,D} T_\varkappa^{\mu\nu} \left(x_\varkappa - x\right)_\mu \left(x_\varkappa - x\right)_\nu\right],$$

where $S = I_s \oplus F_s$ and $D = I_d \oplus F_d$. It is useful to define also the tensors

$$\mathfrak{R}_s^{\mu\nu} = \sum_{\varkappa \in S} T_\varkappa^{\mu\nu} \quad \text{and} \quad \mathfrak{R}_d^{\mu\nu} = \sum_{\varkappa \in D} T_\varkappa^{\mu\nu}.$$

A crucial property:

$$T_\varkappa^{\mu\nu} x_\mu x_\nu = \sigma_\varkappa^2 \left[(u_\varkappa x)^2 - x^2\right] = \sigma_\varkappa^2 \boldsymbol{x}_\star^2 \geq 0 \quad \Longrightarrow \quad \mathfrak{R}_{s,d}^{\mu\nu} x_\mu x_\nu > 0.$$

Consequently, there exist the positive-definite tensors $\widetilde{\mathfrak{R}}_s^{\mu\nu}$ and $\widetilde{\mathfrak{R}}_d^{\mu\nu}$ such that

$$\widetilde{\mathfrak{R}}_{s,d} = ||\widetilde{\mathfrak{R}}_{s,d}^{\mu\nu}|| = g\, \mathfrak{R}_{s,d}^{-1}\, g, \quad \mathfrak{R}_{s,d} = ||\mathfrak{R}_{s,d}^{\mu\nu}||.$$

The explicit form and properties of these tensors and relevant convolutions are established (studied in detail for the most important reactions $1 \to 2,\ 1 \to 3,\ 2 \to 2$). Return

The overlap integrals In CRGP are the 4D Gaussian integrals in Minkowski space.

$$\mathbb{V}_{s,d}(q) = (2\pi)^4 \widetilde{\delta}_{s,d}\left(q \mp q_{s,d}\right) \exp\left[-\mathfrak{S}_{s,d} \pm i\left(q \mp q_{s,d}\right) \cdot X_{s,d}\right],$$

$$\widetilde{\delta}_{s,d}(K) = (4\pi)^{-2} |\Re_{s,d}|^{-1/2} \exp\left(-\frac{1}{4}\widetilde{\Re}^{\mu\nu}_{s,d} K_\mu K_\nu\right),$$

$$\mathfrak{S}_{s,d} = \sum_{\varkappa,\varkappa'}\left(\delta_{\varkappa\varkappa'} T^{\mu\nu}_{\varkappa} - T^{\mu}_{\varkappa\mu'} \widetilde{\Re}^{\mu'\nu'}_{s,d} T^{\nu}_{\varkappa'\nu'}\right) x_{\varkappa\mu} x_{\varkappa'\nu},$$

$$X^{\mu}_{s,d} = \widetilde{\Re}^{\mu\nu}_{s,d} \sum_{\varkappa} T^{\lambda}_{\varkappa\nu} x_{\varkappa\lambda}.$$

Physical meaning of $\widetilde{\delta}_{s,d}$, $\mathfrak{S}_{s,d}$, and $X_{s,d}$.

- From the integral representation $\widetilde{\delta}_{s,d}(K) = \int \frac{dx}{(2\pi)^4} \exp\left(-\Re^{\mu\nu}_{s,d} x_\mu x_\nu + iKx\right)$ it follows that $\widetilde{\delta}_{s,d}(K) \to \delta(K)$ in the plane-wave limit $[\sigma_\varkappa \to 0,\ \forall\varkappa \Rightarrow \widetilde{\Re}^{\mu\nu}_{s,d} \to 0]$.
  $\Longrightarrow$ just the factors $\widetilde{\delta}_s\left(q - q_s\right)$ and $\widetilde{\delta}_d\left(q + q_d\right)$ are responsible for the approximate energy–momentum conservation (with the accuracy governed by the momentum spreads of the interacting packets) in the neutrino production and detection points.

Return

- The functions $\exp(-\mathfrak{S}_s)$ and $\exp(-\mathfrak{S}_d)$ are the geometric suppression factors conditioned by a partial overlap of the in and out WPs in the space-time regions of their interaction in the source and detector.

This can be seen after converting $\mathfrak{S}_{s,d}$ to the form[2]

$$\mathfrak{S}_{s,d} = \sum_{\varkappa} T_{\varkappa}^{\mu\nu} \left(x_{\varkappa} - X_{s,d}\right)_{\mu} \left(x_{\varkappa} - X_{s,d}\right)_{\nu} \tag{21}$$

and taking into account that both $\mathfrak{S}_{s,d}$ and $X_{s,d}$ are invariants under the group of uniform rectilinear motions (here, $\tau_{\varkappa}$ are arbitrary real time parameters)

$$\left\{x_{\varkappa}^0 \longmapsto \tilde{x}_{\varkappa}^0 = x_{\varkappa}^0 + \tau_{\varkappa},\ \boldsymbol{x}_{\varkappa} \longmapsto \tilde{\boldsymbol{x}}_{\varkappa} = \boldsymbol{x}_{\varkappa} + \boldsymbol{v}_{\varkappa}\tau_{\varkappa}\right\}$$

Due to this symmetry, (21) can be rewritten as

$$\mathfrak{S}_{s,d} = \sum_{\varkappa} \sigma_{\varkappa}^2 \left[\left(\Gamma_{\varkappa}^2 - 1\right)\left(b_{\varkappa}^0\right)^2 + \boldsymbol{b}_{\varkappa}^2\right] = \sum_{\varkappa} \sigma_{\varkappa}^2 |\boldsymbol{b}_{\varkappa}^{\star}|^2,$$

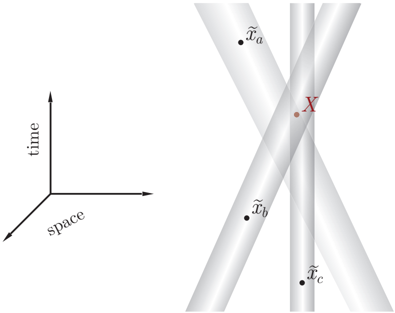
$$\begin{aligned} b_{\varkappa}^0 &= \left(x_{\varkappa}^0 - X_{s,d}^0\right) - |\boldsymbol{v}_{\varkappa}|^{-1}\boldsymbol{n}_{\varkappa}\cdot\left(\boldsymbol{x}_{\varkappa} - \boldsymbol{X}_{s,d}\right), \\ \boldsymbol{b}_{\varkappa} &= \left(\boldsymbol{x}_{\varkappa} - \boldsymbol{X}_{s,d}\right) - \left[\boldsymbol{n}_{\varkappa}\cdot\left(\boldsymbol{x}_{\varkappa} - \boldsymbol{X}_{s,d}\right)\right]\boldsymbol{n}_{\varkappa}, \end{aligned} \qquad \left[\boldsymbol{n}_{\varkappa} = \begin{cases} \boldsymbol{v}_{\varkappa}/|\boldsymbol{v}_{\varkappa}|, & \text{for } \boldsymbol{v}_{\varkappa} \neq 0, \\ 0, & \text{for } \boldsymbol{v}_{\varkappa} = 0. \end{cases}\right]$$

The 4-vector $b_{\varkappa} = (b_{\varkappa}^0, \boldsymbol{b}_{\varkappa})$ is a relativistic analog of the usual impact parameter, so it is natural to call it the impact vector. [Note that $|\boldsymbol{b}_{\varkappa}| = \left|\boldsymbol{n}_{\varkappa} \times \left(\boldsymbol{x}_{\varkappa} - \boldsymbol{X}_{s,d}\right)\right|$ for $\boldsymbol{v}_{\varkappa} \neq 0$.]

The 4-vectors $X_{s,d}$ can be called, accordingly, the impact points. ◂ Return

[2] In this derivation we have used the translation invariance of the functions $\mathfrak{S}_{s,d}$.

The suppression of the overlap integral caused by the factors $\exp(-\mathfrak{S}_s)$ and $\exp(-\mathfrak{S}_d)$ can be small only if all the world tubes in the source/detector inter-cross each other.



Artistic view of the "classical world tubes" of interacting wave packets. The tubes reproduce the space-time cylindrical volumes swept by the classically moving spheroids which represent the wave-packet patterns. The impact point X is defined by the velocities $\boldsymbol{v}_\varkappa$ of the packets and the space-time points $\tilde{x}_\varkappa = \tilde{x}_\varkappa(\tau_\varkappa)$ arbitrarily chosen on the axes of the tubes. The axes do not necessarily cross the point X.

The interacting packets behave, bluntly speaking, like colliding interpenetrative cloudlets. Return.

1. Quark-lepton blocks. We use the Standard Model (SM) phenomenologically extended by inclusion of a neutrino mass term. The quark-lepton blocks are described by the Lagrangian

$$\mathcal{L}_W(x) = -\frac{g}{2\sqrt{2}}\left[j_\ell(x)W(x) + j_q(x)W(x) + \text{H.c.}\right],$$

where g is the $SU(2)$ coupling constant, j_ℓ and j_q are the weak charged currents:

$$j_\ell^\mu(x) = \sum_{\alpha i} V_{\alpha i}^* \overline{\nu}_i(x) O^\mu \ell_\alpha(x), \qquad j_q^\mu(x) = \sum_{qq'} V_{qq'}'^{*} \overline{q}(x) O^\mu q'(x), \qquad [O^\mu = \gamma^\mu(1-\gamma_5)].$$

Here $V_{\alpha i}$ ($\alpha = e, \mu, \tau$; $i = 1, 2, 3$) and $V'_{qq'}$ ($q = u, c, t$; $q' = d, s, b$) are the elements of the neutrino and quark mixing matrices ($\boldsymbol{V}$ and $\boldsymbol{V}'$, respectively).
The normalized amplitude is given by the 4th order of the perturbation theory in g:

$$\begin{aligned}\mathcal{A}_{\beta\alpha} &= \langle\mathbf{out}|\mathbb{S}|\mathbf{in}\rangle\left(\langle\mathbf{in}|\mathbf{in}\rangle\langle\mathbf{out}|\mathbf{out}\rangle\right)^{-1/2}\\ &= \frac{1}{\mathcal{N}}\left(\frac{-ig}{2\sqrt{2}}\right)^4 \langle F_s \oplus F_d|T\int dxdx'dydy' \,:j_\ell(x)W(x)::j_q(x')W^\dagger(x'):\\ &\quad \times :j_\ell^\dagger(y)W^\dagger(y)::j_q^\dagger(y')W(y'):\,\mathbb{S}_h|I_s\oplus I_d\rangle. \end{aligned} \tag{22}$$

The normalization factor $\mathcal{N}$ in the CRGP approximation is given by

$$\mathcal{N}^2 = \langle\mathbf{in}|\mathbf{in}\rangle\langle\mathbf{out}|\mathbf{out}\rangle = \prod_{\varkappa \in I_s \oplus I_d \oplus F_s \oplus F_d} 2E_\varkappa \mathrm{V}_\varkappa(\boldsymbol{p}_\varkappa).$$

2. Hadronic blocks. The strong and (possibly) electromagnetic interactions responsible for nonperturbative processes of fragmentation and hadronization are described by the hadronic (QCD) interaction Lagrangian $\mathcal{L}_{\text{h}}(x)$ and the corresponding part of the full S-matrix is

$$\mathbb{S}_h = \exp\left[i \int dz \mathcal{L}_{\text{h}}(z)\right].$$

The following factorization theorem can be proved

$$\begin{aligned}
\langle F'_s \oplus F'_d | T \left[: j_q^{\mu}(x) : \ \mathbb{S}_h : j_q^{\dagger\nu}(y) : \right] | I_s \oplus I_d \rangle &= \mathcal{J}_s^{\mu}(p_S) \mathcal{J}_d^{\nu\dagger}(p_D) \\
\times \left[\prod_{a \in I_s} e^{-ip_a x_a} \psi_a(\boldsymbol{p}_a, x_a - x) \right] &\left[\prod_{b \in F'_s} e^{ip_b x_b} \psi_b^*(\boldsymbol{p}_b, x_b - x) \right] \\
\times \left[\prod_{a \in I_d} e^{-ip_a x_a} \psi_a(\boldsymbol{p}_a, x_a - y) \right] &\left[\prod_{b \in F'_d} e^{ip_b x_b} \psi_b^*(\boldsymbol{p}_b, x_b - y) \right].
\end{aligned}$$

Here $\mathcal{J}_s(p_S)$ and $\mathcal{J}_d(p_D)$ are the c-number hadronic currents in which the strong interactions are taken into account nonperturbatively, and p_S and p_D denote the sets of the momentum and spin variables of the hadronic states.

The proof is based on the assumed narrowness of the WPs in the momentum space, macroscopic remoteness of the interaction regions in the source and detector vertices, and the consideration of translation invariance.

The explicit form of the hadronic currents $\mathcal{J}_s$ and $\mathcal{J}_d$ is not needed for our purposes.

Now, by applying Wick's theorem, factorization theorem, and the known properties of the leptonic WPs, the amplitude (22) can be rewritten in the following way:

$$\mathcal{A}_{\beta\alpha} = \frac{g^4}{64\mathcal{N}} \sum_j V_{\beta j} \mathcal{J}_d^{\nu\dagger} \overline{u}(\boldsymbol{p}_\beta) O_{\nu'} \mathbb{G}_{\nu\mu}^{j\nu'\mu'}(\{\boldsymbol{p}_\varkappa, x_\varkappa\}) O_{\mu'} v(\boldsymbol{p}_\alpha) \mathcal{J}_s^{\mu} V_{\alpha j}^*, \qquad (23)$$

$$\mathbb{G}_{\nu\mu}^{j\nu'\mu'}(\{\boldsymbol{p}_\varkappa, x_\varkappa\}) = \int \frac{dq}{(2\pi)^4} \mathbb{V}_d(q) \Delta_\nu^{\nu'}(q - p_\beta) \Delta^j(q) \Delta_\mu^{\mu'}(q + p_\alpha) \mathbb{V}_s(q). \qquad (24)$$

Here $\mathbb{V}_s(q)$ and $\mathbb{V}_d(q)$ are the overlap integrals; Δ^j and Δ_μ^ν are the propagators of, respectively, the massive neutrino ν_j and W boson.[3]

$$\Delta^j(q) = i\left(\hat{q} - m_j + i0\right)^{-1}.$$

3. Large-distance asymptotics. At large values of $|\boldsymbol{X}_s - \boldsymbol{X}_d|$, the integral (24) can be evaluated by means of the Grimus-Stockinger (GS) theorem.[4]

> Let $F(\boldsymbol{q})$ be a thrice continuously differentiable function such that F itself and its 1st and 2nd derivatives decrease not slowly than $|\boldsymbol{q}|^{-2}$ as $|\boldsymbol{q}| \to \infty$. Then in the asymptotic limit of $L = |\boldsymbol{L}| \to \infty$,
>
> $$\int \frac{d\boldsymbol{q}}{(2\pi)^3} \frac{F(\boldsymbol{q}) e^{i\boldsymbol{q}\boldsymbol{L}}}{s - \boldsymbol{q}^2 + i0} \sim \begin{cases} -\frac{1}{4\pi L} F\left(\sqrt{s}\,\boldsymbol{L}/L\right) \exp\left(i\sqrt{s}L\right) + \mathcal{O}\left(L^{-3/2}\right) & \text{at } s > 0, \\ \mathcal{O}\left(L^{-2}\right) & \text{at } s < 0. \end{cases}$$

The integrand in (24) satisfies the formulated requirements.

Return

[3]The explicit form of $\Delta_{\mu\nu}$ is not used below. So $\Delta_{\mu\nu}$ can be thought of as *renormalized* propagator.

[4]W. Grimus and P. Stockinger, *Real oscillations of virtual neutrinos*, Phys. Rev. D **54** (1996) 3414 [arXiv:hep-ph/9603430].

4. Integration in q_0. The integral over q_0, which remains after applying the GS theorem can be evaluated by the regular saddle-point method.
In the ultrarelativistic approximation ($q_s^0 \approx -q_d^0 \gg m_j, j = 1, 2, 3$) the stationary saddle point $q_0 = E_j$ can be found as a series in powers of the small parameter $r_j = m_j^2/(2E_\nu^2)$. We obtain

$$q_0 \equiv E_j = E_\nu \left[1 - \mathfrak{n} r_j - \mathfrak{m} r_j^2 + \mathcal{O}\left(r_j^3\right)\right],$$

$$|\boldsymbol{q}_j|_{q_0=E_j} \equiv P_j = E_\nu \left[1 - (\mathfrak{n}+1) r_j - \left(\mathfrak{n} + \mathfrak{m} + \frac{1}{2}\right) r_j^2 + \mathcal{O}\left(r_j^3\right)\right],$$

$$\frac{P_i}{E_j} \equiv v_j = 1 - r_j - \left(2\mathfrak{n} + \frac{1}{2}\right) r_j^2 + \mathcal{O}\left(r_j^3\right), \quad E_j^2 - P_j^2 = m_j^2;$$

$$\mathfrak{n} = \frac{\boldsymbol{Y}\boldsymbol{l}}{Yl}, \quad \mathfrak{m} = \mathfrak{n}\left(\frac{3}{2} + 2\mathfrak{n}\right) + \frac{1}{R} \sum_{n=1,2,3} \left(\widetilde{\Re}_s^{0n} + \widetilde{\Re}_d^{0n}\right) l_n,$$

$$Y^\mu = \widetilde{\Re}_s^{\mu\nu} q_{s\nu} - \widetilde{\Re}_d^{\mu\nu} q_{d\nu}, \quad R = \left(\widetilde{\Re}_s^{\mu\nu} + \widetilde{\Re}_d^{\mu\nu}\right) l_\mu l_\nu,$$

$$E_\nu = \frac{Yl}{R}, \quad l = (1, \boldsymbol{l}), \quad \boldsymbol{l} = \frac{\boldsymbol{L}}{L}, \quad \boldsymbol{L} = \boldsymbol{X}_d - \boldsymbol{X}_s.$$

Return

The quantities E_j, $\boldsymbol{P}_j = P_j \boldsymbol{l}$ and $\boldsymbol{v}_j = v_j \boldsymbol{l}$ can naturally be treated as, respectively, the effective energy, momentum and velocity of the virtual neutrino ν_j.

The ultrarelativistic approximation is, of course, reference-frame dependent. That is why the obtained result is not explicitly Lorentz-invariant.

In the limit of $m_j = 0$ and assuming the exact energy-momentum conservation,

$$E_j = P_j = E_\nu = q_s^0 = -q_d^0.$$

But, in the general case, the effective neutrino 4-momentum $p_j = (E_j, \boldsymbol{p}_j)$ is determined by the mean momenta and momentum spreads of the external WPs involved in the process.

> Below, we'll limit ourselves to the 1st order of the expansion in r_j. However, the next-order corrections are needed to define properly the range of applicability of the obtained result.

Finally, by introducing the notation

$$\Omega_j(T,L) = i\left(E_jT - P_jL\right) + 2\left(\widetilde{\mathfrak{D}}_j/P_j\right)^2\left(P_jT - E_jL\right)^2,$$

$$\Theta = X_s q_s + X_d q_d, \quad L = |\boldsymbol{X}_d - \boldsymbol{X}_s|, \quad T = X_d^0 - X_s^0,$$

$$\widetilde{\mathfrak{D}}_j = \mathfrak{D}_j\left(1 + \frac{8ir_jE_\nu^2\mathfrak{D}_j^2L}{P_j^3}\right)^{-1/2}, \quad \mathfrak{D}_j = \frac{1+\mathfrak{n}r_j}{\sqrt{2R}},$$

Return

we arrive at the saddle-point estimate of the function (24):

$$\mathbb{G}_{\nu\mu}^{j\nu'\mu'} = \Delta_\nu^{\nu'}(p_j - p_\beta)(\hat{p}_j + m_j)\Delta_\mu^{\mu'}(p_j + p_\alpha)|\mathbb{V}_d(p_j)\mathbb{V}_s(p_j)|\,\frac{\widetilde{\mathfrak{D}}_j e^{-\Omega_j(T,L)-i\Theta}}{i(2\pi)^{3/2}L}. \qquad (25)$$

This formula can be (and must be) somewhat simplified by putting $r_j = 0$ everywhere wherever it is not a factor multiplying L or T (whose values can be arbitrary large).

Then the 4-vector p_j is replaced by the light-cone 4-vector

Excursus: Neutrino wave packet (νWP).

The complex phase $\Omega_j(T,L)$ can be written in an explicitly invariant form:

$$\Omega_j(T,L) = i(p_jX) + \left(2\widetilde{\mathfrak{D}}_j^2/E_\nu^2\right)\left[(p_jX)^2 - m_j^2X^2\right], \qquad X = X_d - X_s. \tag{26}$$

Comparing the factor $\psi_j^* = e^{-\Omega_j}$ in (25) with the generic CRGP wave function (19), we conclude that ψ_j^* can be treated as the (outgoing) νWPs in which the role of the parameter σ is played by the function

$$\Sigma_j = \sqrt{2}\widetilde{\mathfrak{D}}_j/\Gamma_j, \qquad (\Gamma_j = E_\nu/m_j).$$

Since Σ_j is a complex-valued function, the νWPs spreads with increase of $L = |\boldsymbol{X}|$. The spread effect can be important only at "cosmological" distances. Here we limit ourselves to the "terrestrial" conditions, for which it is *a fortiori* possible to judge that

$$E_jL \ll \left(\Gamma_jE_j/2\mathfrak{D}_j\right)^2.$$

$$\Downarrow$$

$$\widetilde{\mathfrak{D}}_j \simeq \mathfrak{D}_j \simeq 1/\sqrt{2R} \equiv \mathfrak{D} \quad \text{and} \quad \Sigma_j \simeq \sqrt{2}\mathfrak{D}/\Gamma_j = 1/(\Gamma_j\sqrt{R}).$$

The relative energy-momentum uncertainty of the νWP is $\delta E_j/E_j \sim \delta P_j/P_j \sim \mathfrak{D}/E_\nu \lll 1$. Of course, the mean position of the νWPs evolves along the "classical trajectory" $\overline{\boldsymbol{L}} = \boldsymbol{v}_jT$, the quantum deviations from which, $\delta\boldsymbol{L}$, are suppressed by the factor

$$\exp\left\{-2\mathfrak{D}^2\left[\delta\boldsymbol{L}^2/\Gamma_j^2 + (\boldsymbol{l}\delta\boldsymbol{L})^2\right]\right\}.$$

5. Source-detector factorization. Now, by applying the identity

$$P_-\hat{p}_\nu P_+ = P_- u_-(\boldsymbol{p}_\nu)\overline{u}_-(\boldsymbol{p}_\nu)P_+,$$

where $u_-(\boldsymbol{p}_\nu)$ is the Dirac bispinor for the left-handed massless neutrino and $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$, we define the matrix elements

$$M_s = (g^2/8)\,\overline{u}_-(\boldsymbol{p}_\nu)\mathcal{J}_s^\mu \Delta_\mu^{\mu'}(p_\nu + p_\alpha)O_{\mu'}u(\boldsymbol{p}_\alpha),$$
$$M_d^* = (g^2/8)\,\overline{v}(\boldsymbol{p}_\beta)O_{\mu'}\Delta_\mu^{\mu'}(p_\nu - p_\beta)\mathcal{J}_d^{\mu\dagger}u_-(\boldsymbol{p}_\nu),$$

which describe the reactions with production and absorption of a real massless neutrino ν:

$$\begin{cases} M_s \text{ corresponds to reaction } I_s \to F_s' + \ell_\alpha^+ + \nu, \\ M_d \text{ corresponds to reaction } \nu + I_d \to F_d' + \ell_\beta^-. \end{cases}$$

The final expression for the amplitude (23) is

$$\mathcal{A}_{\beta\alpha} = \frac{\mathfrak{D}|\mathbb{V}_s(p_\nu)\mathbb{V}_d(p_\nu)|M_s M_d^*}{i(2\pi)^{3/2}\mathcal{N}L}\sum_j V_{\alpha j}^* V_{\beta j}\, e^{-\Omega_j(T,L)-i\Theta}. \tag{27}$$

• The form of eq. (27) suggests that it is common for essentially any class of macrodiagrams, with exchange of virtual neutrinos between the source and detector, until we do not specify the explicit form of the matrix elements M_s and M_d.

• To obtain similar answer for the macrodiagrams with an exchange of virtual antineutrinos, one has to replace (besides the matrix elements) $\boldsymbol{V} \longmapsto \boldsymbol{V}^\dagger$.

Return

It can be shown that

$$|\mathbb{V}_{s,d}(p_\nu)|^2 = (2\pi)^4 \delta_{s,d}(p_\nu \mp q_{s,d}) \mathrm{V}_{s,d}, \tag{28}$$

where $\delta_{s,d}$ are the "smeared" δ functions (analogous to the functions $\widetilde{\delta}_{s,d}$) and $\mathrm{V}_{s,d}$ are the effective 4D overlap volumes of the external packets in the source and detector;

$$\delta_{s,d}(K) = (2\pi)^{-2} |\mathfrak{R}_{s,d}|^{-1/2} \exp\left(-\frac{1}{2} \widetilde{\mathfrak{R}}^{\mu\nu}_{s,d} K_\mu K_\nu \right), \tag{29}$$

$$\mathrm{V}_{s,d} = \int dx \prod_{\varkappa \in S,D} |\psi_\varkappa(\boldsymbol{p}_\varkappa, x_\varkappa - x)|^2 = \frac{\pi^2}{4} |\mathfrak{R}_{s,d}|^{-1/2} \exp\left(-2\mathfrak{S}_{s,d}\right). \tag{30}$$

$$\Downarrow$$

$$|\mathcal{A}_{\beta\alpha}|^2 = \frac{(2\pi)^4 \delta_s(p_\nu - q_s) \mathrm{V}_s |M_s|^2}{\prod_{\varkappa \in S} 2E_\varkappa \mathrm{V}_\varkappa} \frac{(2\pi)^4 \delta_d(p_\nu + q_d) \mathrm{V}_d |M_d|^2}{\prod_{\varkappa \in D} 2E_\varkappa \mathrm{V}_\varkappa} \times \frac{\mathfrak{D}^2}{(2\pi)^3 L^2} \left| \sum_j V^*_{\alpha j} V_{\beta j} e^{-\Omega_j(T,L)} \right|^2. \tag{31}$$

This is the microscopic probability dependent on the mean momenta $\boldsymbol{p}_\varkappa$, initial coordinates $x_\varkappa$, masses $m_\varkappa$, and parameters $\sigma_\varkappa$ of all external wave packets participated in the reaction.

By using the explicit form of the functions $\delta_{s,d}$ and $\mathfrak{D}$, one can prove the following approximate relation:

$$2\sqrt{\pi}\mathfrak{D}\delta_s\left(p_\nu - q_s\right)\delta_d\left(p_\nu + q_d\right)F(p_\nu) = \int dE'_\nu \delta_s\left(p'_\nu - q_s\right)\delta_d\left(p'_\nu + q_d\right)F(p'_\nu), \tag{32}$$

where $F(p_\nu)$ is an arbitrary slowly varying function and $p'_\nu = (E'_\nu, \mathbf{p}'_\nu) = E'_\nu l$. The relation is valid with the same accuracy with which the amplitude (27) itself has been deduced that is, with the accuracy of the saddle-point method. With help of (32) the squared amplitude (31) transforms to

$$|\mathcal{A}_{\beta\alpha}|^2 = \int dE_\nu \frac{(2\pi)^4\delta_s(p_\nu - q_s)\mathrm{V}_s|M_s|^2}{\prod_{\varkappa\in S} 2E_\varkappa \mathrm{V}_\varkappa} \frac{(2\pi)^4\delta_d(p_\nu + q_d)\mathrm{V}_d|M_d|^2}{\prod_{\varkappa\in D} 2E_\varkappa \mathrm{V}_\varkappa} \times \frac{\mathfrak{D}}{2\sqrt{\pi}(2\pi)^3L^2}\left|\sum_j V^*_{\alpha j}V_{\beta j}\, e^{-\Omega_j(T,L)}\right|^2. \tag{33}$$

The probability (33) is the most general result of this work. However, it is too general to be directly applied to the contemporary neutrino oscillation experiments.

To obtain the observable quantities, the probability must be averaged/integrated over all the unmeasurable or unused variables of incoming/outgoing WP states.

A thought experiment:

Assume that the statistical distributions of the incoming WPs $a \in I_{s,d}$ over the mean momenta, spin projections, and space-time coordinates in the source and detector "devices" can be described by the one-particle distribution functions $f_a(\boldsymbol{p}_a, s_a, x_a)$. It is convenient to normalize each function f_a to the total number, $N_a(x_a^0)$, of the packets a at a time x_a^0:

$$\sum_{s_a} \int \frac{d\boldsymbol{x}_a d\boldsymbol{p}_a}{(2\pi)^3} f_a(\boldsymbol{p}_a, s_a, x_a) = N_a(x_a^0) \qquad (a \in I_{s,d}).$$

For clarity purposes, we (re)define the terms "source" and "detector":

$$\mathcal{S} = \operatorname{supp}_{\{x_a;\ a \in I_s\}} \prod_a f_a(\boldsymbol{p}_a, s_a, x_a), \quad \mathcal{D} = \operatorname{supp}_{\{x_a;\ a \in I_d\}} \prod_a f_a(\boldsymbol{p}_a, s_a, x_a).$$

We'll use the same terms and notation $\mathcal{S}$ and $\mathcal{D}$ also for the corresponding devices.

Return

Innocent assumptions:

[1] $\mathcal{S}$ and $\mathcal{D}$ are finite and mutually disjoint within the space domain.

[2] Effective spatial dimensions of $\mathcal{S}$ and $\mathcal{D}$ are small compared to the mean distance between them but very large compared to the effective dimensions ($\sim \sigma_{\varkappa}^{-1}$) of all WPs in $\mathcal{S}$ and $\mathcal{D}$.

[3] The experiment measures only the momenta of the secondaries in $\mathcal{D}$ and (due to [2]) the background events caused by the secondaries falling into $\mathcal{D}$ from $\mathcal{S}$ can be neglected.

[4] The detection efficiency in $\mathcal{D}$ is 100%.

With all these assumptions, the macroscopically averaged probability reads

$$\begin{aligned}\langle\!\langle|\mathcal{A}_{\beta\alpha}|^2\rangle\!\rangle \equiv dN_{\alpha\beta} = &\sum_{\text{spins}}\int\prod_{a\in I_s}\frac{d\mathbf{x}_a d\mathbf{p}_a f_a(\mathbf{p}_a, s_a, \mathbf{x}_a)}{(2\pi)^3 2E_a \mathrm{V}_a}\int\prod_{b\in F_s}\frac{d\mathbf{x}_b d\mathbf{p}_b}{(2\pi)^3 2E_b \mathrm{V}_b}\mathrm{V}_s \\ &\times\int\prod_{a\in I_d}\frac{d\mathbf{x}_a d\mathbf{p}_a f_a(\mathbf{p}_a, s_a, \mathbf{x}_a)}{(2\pi)^3 2E_a \mathrm{V}_a}\int\prod_{b\in F_d}\frac{d\mathbf{x}_b [d\mathbf{p}_b]}{(2\pi)^3 2E_b \mathrm{V}_b}\mathrm{V}_d \\ &\times\int dE_\nu (2\pi)^4\delta_s(p_\nu - q_s)|M_s|^2(2\pi)^4\delta_d(p_\nu + q_d)|M_d|^2 \\ &\times\frac{\mathfrak{D}}{2\sqrt{\pi}(2\pi)^3 L^2}\left|\sum_j V_{\alpha j}^* V_{\beta j}\, e^{-\Omega_j(T,L)}\right|^2. \end{aligned} \tag{34}$$

- $\sum_{\text{spins}}$ denotes the averaging/summation over the spin projections of the in/out states.
- Symbol $[d\mathbf{p}_b]$ indicates that integration in variable $\mathbf{p}_b$ is not performed, i.e., $\int [d\mathbf{p}_b] = d\mathbf{p}_b$.

Clearly, $\langle\!\langle|\mathcal{A}_{\beta\alpha}|^2\rangle\!\rangle$ represents the total number, $dN_{\alpha\beta}$, of the events recorded in $\mathcal{D}$ and consisted of the secondaries $b\in F_d$ having the mean momenta between $\mathbf{p}_b$ and $\mathbf{p}_b + d\mathbf{p}_b$.

Under additional assumptions, the unwieldy expression (34) can be simplified in a few steps.

Step 1: Multidimensional integration in WP positions.

Supposition 5: The distribution functions $f_a(\boldsymbol{p}_a, s_a, x_a)$, as well as the factors $e^{-\Omega_j-\Omega_i^*}/L^2$ vary at large (macroscopic) scales.

The integrand $\prod_\varkappa |\psi_\varkappa(\boldsymbol{p}_\varkappa, x_\varkappa - x)|^2$ in the integral representation of the overlap volumes (30) is essentially different from zero only if the classical word lines of all packets $\varkappa$ pass through a small (though not necessarily microscopic) vicinity of the integration variable.

Supposition 6: The edge effects can be neglected (a harmless extension of supposition [2]).

As a result, expression (34) is reduced to the following:

$$dN_{\alpha\beta} = \sum_{\text{spins}} \int dx \int dy \int d\mathfrak{P}_s \int d\mathfrak{P}_d \int dE_\nu \frac{\mathfrak{D}\left|\sum_j V_{\alpha j}^* V_{\beta j} e^{-\Omega_j(T,L)}\right|^2}{16\pi^{7/2}|\boldsymbol{y}-\boldsymbol{x}|^2}, \tag{35}$$

where $T = y_0 - x_0$, $L = |\boldsymbol{y} - \boldsymbol{x}|$ and we have defined the differential forms

$$d\mathfrak{P}_s = \prod_{a\in I_s} \frac{d\boldsymbol{p}_a f_a(\boldsymbol{p}_a, s_a, x)}{(2\pi)^3 2E_a} \prod_{b\in F_s} \frac{d\boldsymbol{p}_b}{(2\pi)^3 2E_b} (2\pi)^4 \delta_s(p_\nu - q_s)|M_s|^2, \tag{36a}$$

$$d\mathfrak{P}_d = \prod_{a\in I_d} \frac{d\boldsymbol{p}_a f_a(\boldsymbol{p}_a, s_a, y)}{(2\pi)^3 2E_a} \prod_{b\in F_d} \frac{[d\boldsymbol{p}_b]}{(2\pi)^3 2E_b} (2\pi)^4 \delta_d(p_\nu + q_d)|M_d|^2. \tag{36b}$$

Step 2: Integration in time variables.

Supposition 7: During the experiment, the distribution functions f_a in $\mathcal{S}$ and $\mathcal{D}$ vary slowly enough with time so that they can be modelled by the "rectangular ledges"

$$\begin{aligned} f_a(\boldsymbol{p}_a, s_a; x) &= \theta\left(x^0 - x_1^0\right) \theta\left(x_2^0 - x^0\right) \bar{f}_a(\boldsymbol{p}_a, s_a; \boldsymbol{x}) \text{ for } a \in I_s, \\ f_a(\boldsymbol{p}_a, s_a; y) &= \theta\left(y^0 - y_1^0\right) \theta\left(y_2^0 - y^0\right) \bar{f}_a(\boldsymbol{p}_a, s_a; \boldsymbol{y}) \text{ for } a \in I_d. \end{aligned} \tag{37}$$

Supposition 8: The time intervals needed to switch on and switch off the source and detector are negligibly small in comparison with periods of stationarity $\tau_s = x_2^0 - x_1^0$ and $\tau_d = y_2^0 - y_1^0$.

In case of detector, the step functions in (37) can be thought as the "hardware" or "software" trigger conditions. The periods of stationarity τ_s and τ_d can be astronomically long, as it is for the solar and atmospheric neutrino experiments ($\tau_s \ggg \tau_d$ in these cases), or very short, like in the experiments with short-pulsed accelerator beams (when usually $\tau_s \lesssim \tau_d$).

Within the model (37), the only time-dependent factor in the integrand of (35) is $e^{-\Omega_j - \Omega_i^*}$. So the problem is reduced to the (comparatively) simple integral

$$\int_{y_1^0}^{y_2^0} dy^0 \int_{x_1^0}^{x_2^0} dx^0\, e^{-\Omega_j(y^0 - x^0, L) - \Omega_i^*(y^0 - x^0, L)} = \frac{\sqrt{\pi}}{2\mathfrak{D}} \tau_d \exp\left(i\varphi_{ij} - \mathscr{A}_{ij}^2\right) S_{ij}. \tag{38}$$

In this relation, we have adopted the following notation:

$$S_{ij} = \frac{\exp\left(-\mathscr{B}_{ij}^2\right)}{4\tau_d\mathfrak{D}} \sum_{l,l'=1}^{2} (-1)^{l+l'+1} \mathrm{Ierf}\left[2\mathfrak{D}\left(x_l^0 - y_{l'}^0 + \frac{L}{v_{ij}}\right) - i\mathscr{B}_{ij}\right], \tag{39}$$

$$\mathscr{A}_{ij} = (v_j - v_i)\mathfrak{D}L = \frac{2\pi\mathfrak{D}L}{E_\nu L_{ij}}, \qquad \mathscr{B}_{ij} = \frac{\Delta E_{ji}}{4\mathfrak{D}} = \frac{\pi\mathfrak{n}}{2\mathfrak{D}L_{ij}}, \tag{40}$$

$$\varphi_{ij} = \frac{2\pi L}{L_{ij}}, \qquad L_{ij} = \frac{4\pi E_\nu}{\Delta m_{ij}^2}, \qquad \frac{1}{v_{ij}} = \frac{1}{2}\left(\frac{1}{v_i} + \frac{1}{v_j}\right),$$

$$\Delta m_{ij}^2 = m_i^2 - m_j^2, \qquad \Delta E_{ij} = E_i - E_j,$$

$$\mathrm{Ierf}(z) = \int_0^z dz'\,\mathrm{erf}(z') + \frac{1}{\sqrt{\pi}} = z\,\mathrm{erf}(z) + \frac{1}{\sqrt{\pi}} e^{-z^2},$$

For a more realistic description of the beam pulse experiments, the model (37) could be readily extended by inclusion of a series of rectangular ledges followed by pauses during which $f_a = 0$.

Then substituting (38) into (35) we obtain:

$$dN_{\alpha\beta} = \tau_d \sum_{\text{spins}} \int d\mathbf{x} \int d\mathbf{y} \int d\mathfrak{P}_s \int d\mathfrak{P}_d \int dE_\nu \frac{\mathcal{P}_{\alpha\beta}(E_\nu, |\mathbf{y}-\mathbf{x}|)}{4(2\pi)^3|\mathbf{y}-\mathbf{x}|^2}, \tag{41a}$$

$$\equiv \frac{\tau_d}{V_\mathcal{D} V_\mathcal{S}} \int d\mathbf{x} \int d\mathbf{y} \int d\Phi_\nu \int d\sigma_{\nu\mathcal{D}} \mathcal{P}_{\alpha\beta}(E_\nu, |\mathbf{y}-\mathbf{x}|). \tag{41b}$$

Explanation of the factors in eq. (41b).

▷ $V_{\mathcal{S}}$ and $V_{\mathcal{D}}$ are the spatial volumes of the source and detector, respectively.

▷ The differential form $d\Phi_\nu$ is defined in such a way that the integral

$$\frac{d\mathbf{x}}{V_{\mathcal{S}}}\int\frac{d\Phi_\nu}{dE_\nu} = d\mathbf{x}\sum_{\text{spins}\in S}\int\frac{d\mathfrak{P}_s E_\nu}{2(2\pi)^3|\mathbf{y}-\mathbf{x}|^2} \tag{42}$$

is the flux density of neutrinos in $\mathcal{D}$, produced through the processes $I_s \to F_s' \ell_\alpha^+ \nu$ in $\mathcal{S}$.

More precisely, it is the number of neutrinos appearing per unit time and unit neutrino energy in an elementary volume $d\mathbf{x}$ around the point $\mathbf{x}\in\mathcal{S}$, travelling within the solid angle $d\Omega_\nu$ about the flow direction $\mathbf{l} = (\mathbf{y}-\mathbf{x})/|\mathbf{y}-\mathbf{x}|$ and crossing a unit area, placed around the point $\mathbf{y}\in\mathcal{D}$ and normal to $\mathbf{l}$.

Return

- The differential form $d\sigma_{\nu\mathcal{D}}$ is defined in such a way that

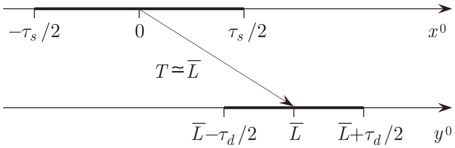
$$\frac{1}{V_{\mathcal{D}}}\int d\mathbf{y}d\sigma_{\nu\mathcal{D}} = \sum_{\text{spins}\in D}\int \frac{d\mathbf{y}d\mathfrak{P}_d}{2E_\nu} \tag{43}$$

represents the differential cross section of the neutrino scattering off the detector as a whole.

> In the particular (and the most basically important) case of neutrino scattering in the reaction $\nu a \to F_d'\ell_\beta^-$, provided that the momentum distribution of the target scatterers a is sufficiently narrow, the differential form $d\sigma_{\nu\mathcal{D}}$ becomes exactly the elementary differential cross section of this reaction multiplied by the total number of the particles a in $\mathcal{D}$.

Return

Let us now return to the decoherence factor, limiting ourselves to a consideration of “synchronized” measurements, in which



$$x^0_{1,2} = \mp \frac{\tau_s}{2}, \quad y^0_{1,2} = \overline{L} \mp \frac{\tau_d}{2}.$$

With certain technical simplifications, the factor (39) can be expressed through a real-valued function $S(t, t', b)$ of three dimensionless variables, namely:

$$S_{ij} = S\left(\mathfrak{D}\tau_s, \mathfrak{D}\tau_d, \mathscr{B}_{ij}\right),$$

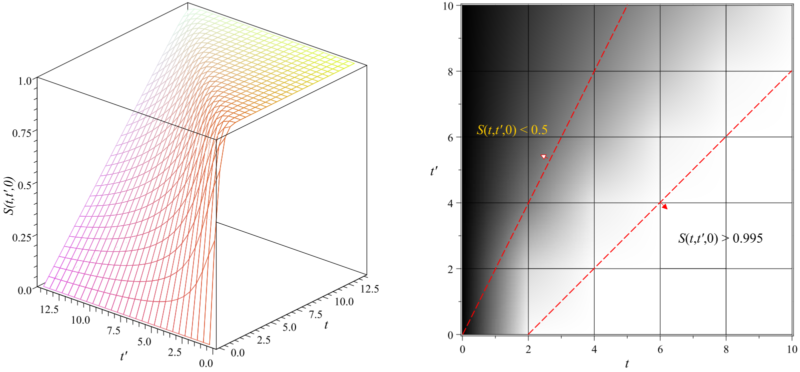
$$2t'S(t, t', b) = \exp\left(-b^2\right) \operatorname{Re}\left[\operatorname{Ierf}\left(t + t' + ib\right) - \operatorname{Ierf}\left(t - t' + ib\right)\right].$$

Diagonal decoherence function

$$S(t, t', 0) = \frac{1}{2t'}\left[\operatorname{Ierf}\left(t + t'\right) - \operatorname{Ierf}\left(t - t'\right)\right] \equiv S_0(t, t'), \tag{44}$$

This function corresponds to the noninterference (neutrino mass independent) decoherence factors S_{ii}. The following inequalities can be proved:

$$0 < S_0(t, t') < 1, \quad S_0(t, t') < t/t' \text{ for } t' \geq t, \quad S_0(t + \delta t, t) > \operatorname{erf}(\delta t) \text{ for } \delta t > 0.$$

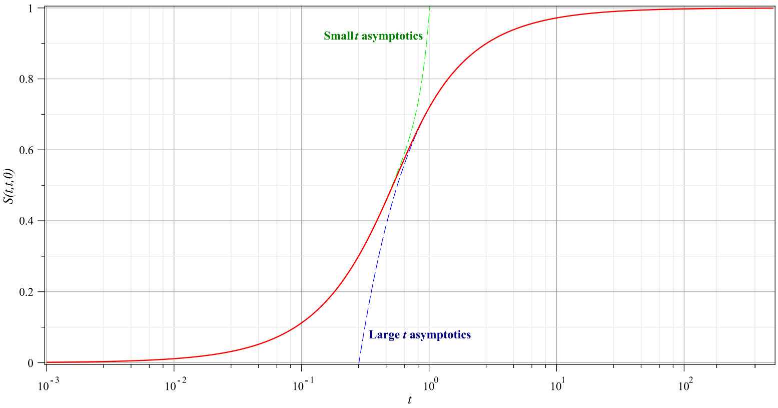


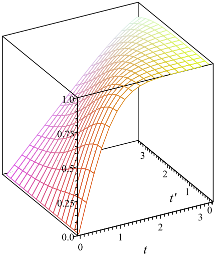
The strong dependence of the common suppression factor $S_0(t,t')$ on its arguments at $t \lesssim t'$ provides a potential possibility of an experimental estimation of the function $\mathfrak{D}$ (or, rather, of its mean values within the phase spaces), based on the measuring the count rate $dR_{\alpha\beta} = dN_{\alpha\beta}/\tau_d$ as a function of τ_d and τ_s (at fixed $\overline{L}$) and comparing the data with the results of Monte-Carlo simulations.

The optimal strategy of such an experiment should be a subject of a dedicated analysis.

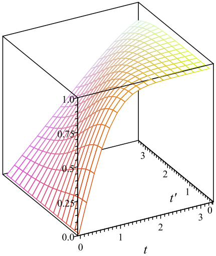
For the important special case, $t' = t$ (representative, in particular, for the experiments with accelerator neutrino beams), we find

$$S_0(t,t) = \mathrm{erf}(2t) - \frac{1-e^{-4t^2}}{2\sqrt{\pi}t} \approx \begin{cases} \frac{2t}{\sqrt{\pi}}\left(1 - \frac{2t^2}{3} + \frac{8t^4}{15}\right) & \text{for } t \ll 1, \\ 1 - \frac{1}{2\sqrt{\pi}t} & \text{for } t \gg 1. \end{cases} \tag{45}$$

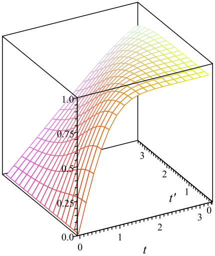




$S(t, t', 0.1)$.

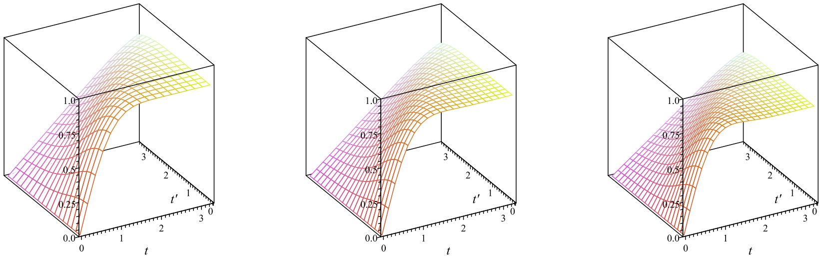


$S(t, t', 0.2)$.



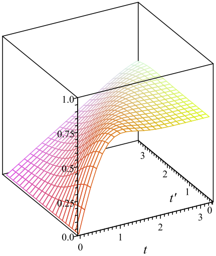
$S(t, t', 0.3)$.

Return

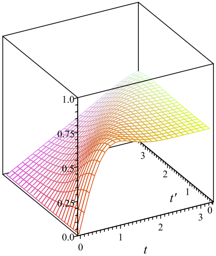


$S(t, t', 0.4)$. $S(t, t', 0.5)$. $S(t, t', 0.6)$.

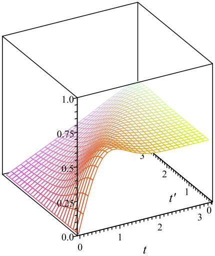
Return



$S(t, t', 0.7)$.



$S(t, t', 0.8)$.



$S(t, t', 0.9)$.

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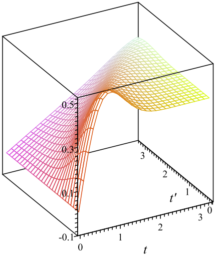
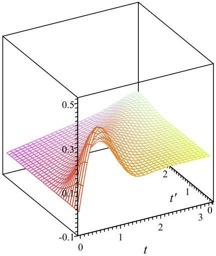
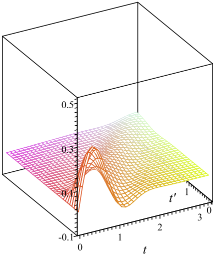


$S(t, t', 1.0)$.

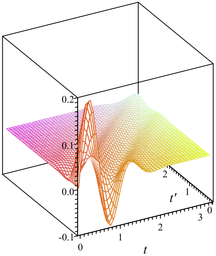


$S(t, t', 1.5)$.

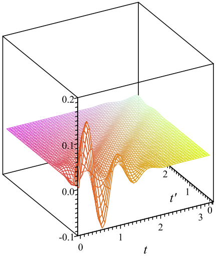


$S(t, t', 2.0)$.

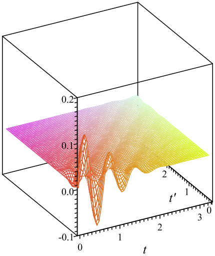
Return



$S(t, t', 3.0)$.

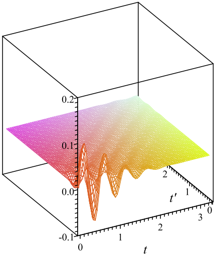


$S(t, t', 4.0)$.

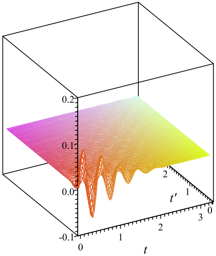


$S(t, t', 5.0)$.

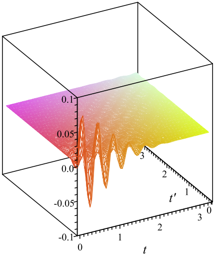
Return



$S(t, t', 6.0)$.



$S(t, t', 7.0)$.



$S(t, t', 8.0)$.

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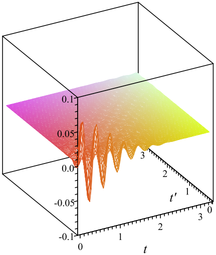
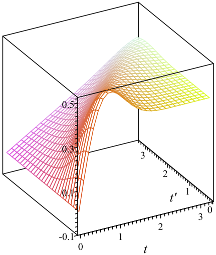
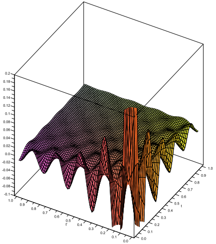


$S(t, t', 9.0)$.



$S(t, t', 10.0)$.



$S(t, t', 15.0)/S_0(t, t')$.

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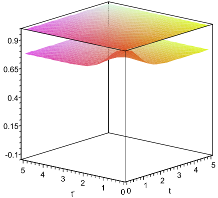
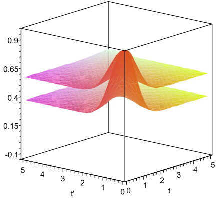
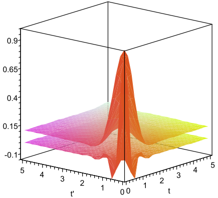


$S(t,t',0.10)/S_0(t,t')$,
$S(t,t',0.50)/S_0(t,t')$.



$S(t,t',0.75)/S_0(t,t')$,
$S(t,t',1.00)/S_0(t,t')$.



$S(t,t',1.50)/S_0(t,t')$,
$S(t,t',4.00)/S_0(t,t')$.

Return

Summary of OPERA results

- Advance arrival of ν_μ from CERN to LNGS

$$\delta t = 57.8 \pm 7.8(\text{stat.})^{+8.3}_{-5.9}(\text{sys.})$$

- this corresponds to

$$(v - c)/c = (2.37 \pm 0.32(\text{stat.})^{+0.34}_{-0.24}(\text{sys.})) \times 10^{-5}$$

- These results were reproduced by a test performed with a beam with a short-bunch time-structure allowing measuring the neutrino time of flight at the single interaction level.

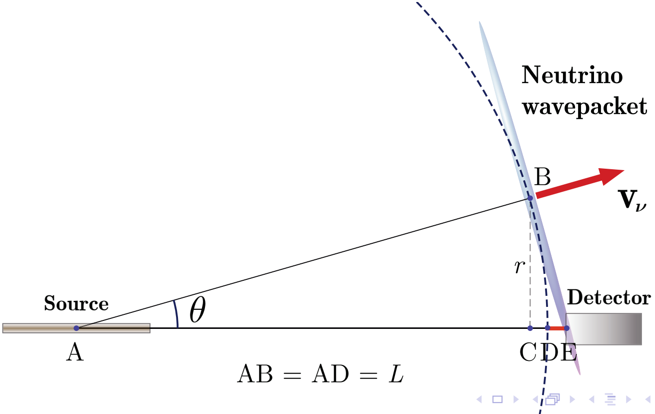
Possible explanations (>140 papers already)

- Superluminal ν. Lorentz symmetry violation.
- Anomalous refraction in Earth interior, dark matter, etc
- Fifth force, new fields, exotics
- GR corrections
- New geometry, several fundamental speed limits
- extra dimensions
- Wave packets, off-shell ν, neutrino oscillations

Our explanation

It is possible that neutrino wave packet (WP) is macroscopically large in the transverse size being microscopic in the direction of motion. This could mimic early arrival of neutrino.

$$d_\perp \gg \left(\frac{0.1\ \text{eV}}{m_\nu}\right)\ \text{km}, \qquad d_\parallel = \frac{d_\perp}{\Gamma_\nu} \gg 10^{-2}\left(\frac{10\ \text{GeV}}{E_\nu}\right)\left(\frac{0.1\ \text{eV}}{m_\nu}\right)\ \mu\text{m}.$$



Relativistic Gauss Wave Packet

We built an explicit example of WP - Relativistic Gauss Wave Packet

$$\psi(\mathbf{p}, x) = \frac{K_1(\zeta m^2/2\sigma^2)}{\zeta K_1(m^2/2\sigma^2)} \stackrel{\text{def}}{=} \psi_G(\mathbf{p}, x), \tag{46}$$

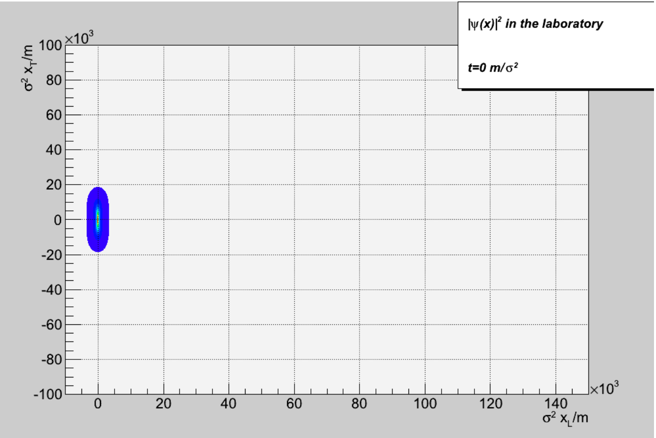
with dimensionless variable:

$$\zeta = |\zeta| e^{i\varphi} = \sqrt{1 - \frac{4\sigma^2}{m^2}\left[\sigma^2 x^2 + i(px)\right]}. \tag{47}$$

- RGP is a solution of Klein-Gordon equation
- RGP is in general a dispersing with time wave packet. However its dispersion can be not so fast as one usually assumes.

Evolution of wave packet

A numerical example: $\sigma^2/m^2 = 10^{-10}, \gamma = 10^5$



Evolution of wave packet

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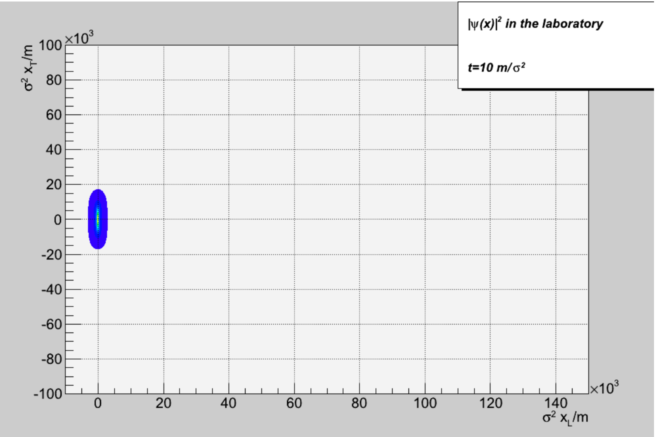

Evolution of wave packet

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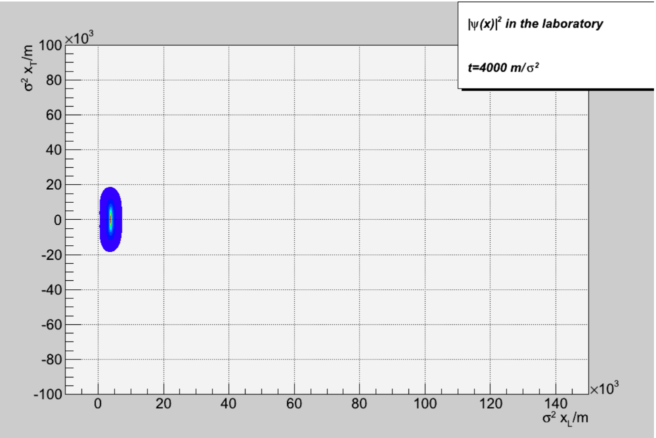

Evolution of wave packet

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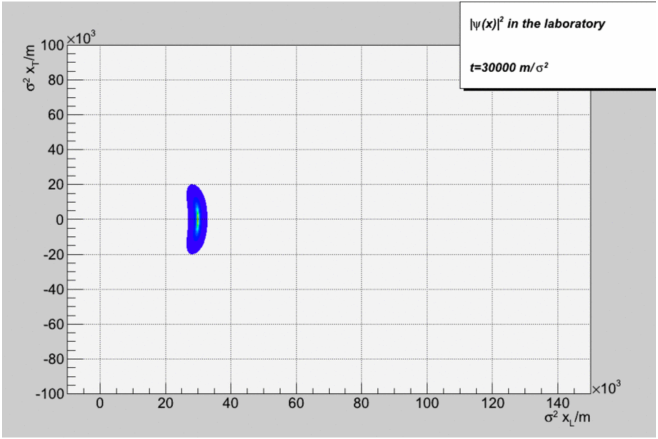

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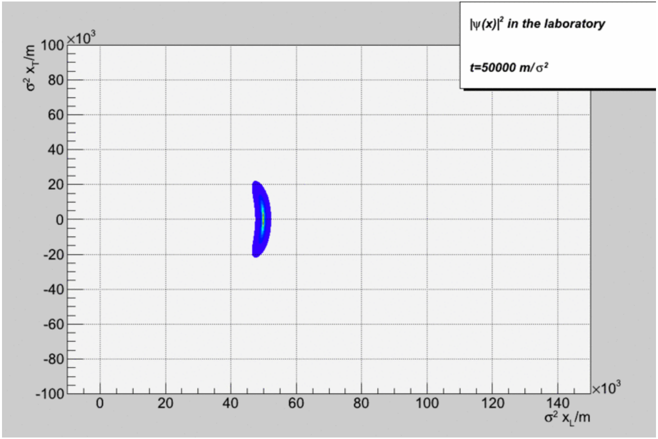
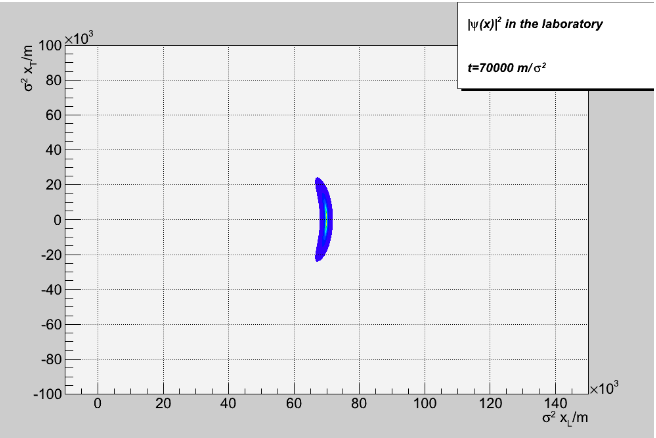


Evolution of wave packet

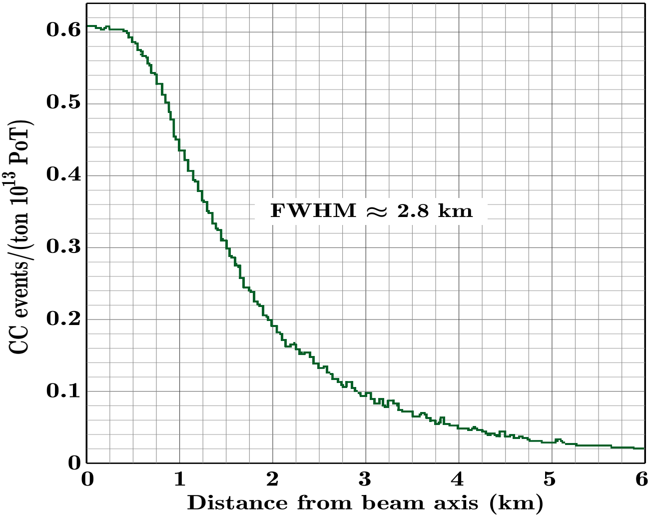
A numerical example: $\sigma^2/m^2 = 10^{-10}, \gamma = 10^5$



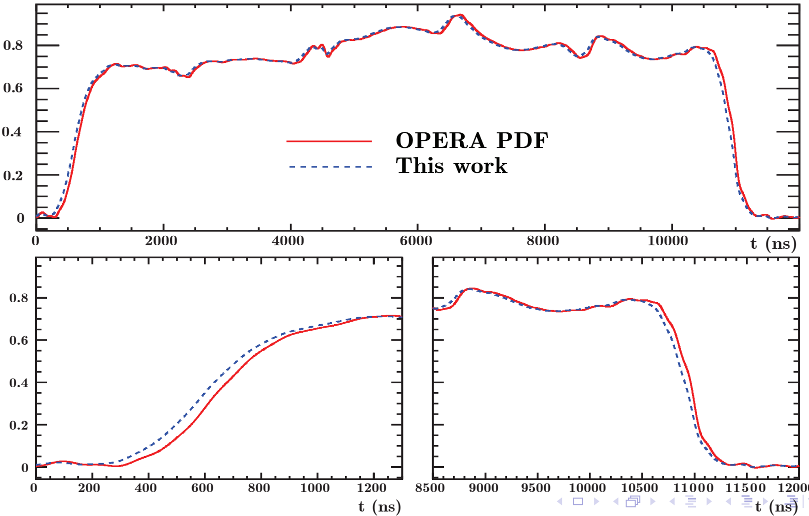
- For the given example RGP becomes a spherical wave like at $t \simeq 10^4 m/\sigma^2$. At shorter distances $\psi_G(x)$ behaves like a stable structure!
- For realistic parameters of neutrino RGP the spherical asymptotics becomes even later
- For CERN-LNGS environment neutrino wave packet can be written as a non -dispersing approximation of $\psi_G(x)$:

$$\psi_\nu^* = e^{iE_\nu (x_0 - \mathbf{v}_\nu \mathbf{x}) - \sigma_\nu^2 \Gamma_\nu^2 \left(\mathbf{x}_\parallel - \mathbf{v}_\nu x_0\right)^2 - \sigma_\nu^2 \mathbf{x}_\perp^2}.$$

Probability of neutrino CC interaction at LNGS as a function of transverse radius r from the beam axis



Calculation of OPERA expected arrival time distribution with and without our effect



Some outlook

- Without any free parameter we find that OPERA should see some 20 ns earlier arrival of neutrino with similar variance
- Left edge of the time distribution is expected to be shifted by some 60 ns, while the right one y some 20-25 ns.
- Similar calculations performed for MINOS give $120ns$ earlier arrival of neutrino with surprizingly good agreement withe MINOS result

$$\delta t = (126 \pm 32_{\text{stat}} \pm 64_{\text{sys}}) \text{ нс } (68\% \text{ C.L.}).$$