

# Testing the Standard Model with the lepton g-2

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# Preamble: today's values

$$a_e = 11596521807.3 \text{ (2.8)} \times 10^{-13}$$

0.24 parts per billion !! (Hanneke et al., PRL100 (2008) 120801)

$$a_\mu = 116592089 \text{ (63)} \times 10^{-11}$$

0.5 parts per million !! (E821 – Final Report: PRD73 (2006) 072003)

$$a_\tau = -0.018 \text{ (17)}$$

Well, not much yet.... (PDG 2013)

# Outline

- ➊ 1. Lepton magnetic moments: the basics
- ➋ 2.  $\mu$ : The muon g-2: a quick update
- ➌ 3.  $e$ : Testing new physics with the electron g-2
- ➍ 4.  $\tau$ : The tau g-2: opportunities & challenges (fantasies?)

# 1. Lepton magnetic moments: the basics

- Uhlenbeck and Goudsmit in 1925 proposed:

$$\vec{\mu} = g \frac{e}{2mc} \vec{s}$$
$$g = \underline{2} \quad (\text{not } 1!)$$

- Dirac 1928:

$$(i\partial_\mu - eA_\mu) \gamma^\mu \psi = m\psi$$

- A Pauli term in Dirac's eq would give a deviation...

$$a \frac{e}{2m} \sigma_{\mu\nu} F^{\mu\nu} \psi \rightarrow g = 2(1 + a)$$

...but there was no need for it! g=2 stood for ~20 yrs.

# Theory of the g-2: Quantum Field Theory

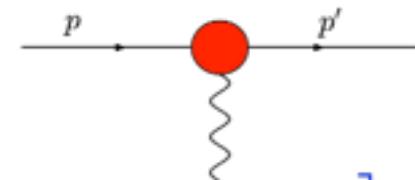
- Kusch and Foley 1948:

$$\mu_e^{\text{exp}} = \frac{e\hbar}{2mc} (1.00119 \pm 0.00005)$$

- Schwinger 1948 (triumph of QED!):

$$\mu_e^{\text{th}} = \frac{e\hbar}{2mc} \left(1 + \frac{\alpha}{2\pi}\right) = \frac{e\hbar}{2mc} \times 1.00116$$

- Keep studying the lepton- $\gamma$  vertex:



$$\bar{u}(p') \Gamma_\mu u(p) = \bar{u}(p') \left[ \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2) + \dots \right] u(p)$$

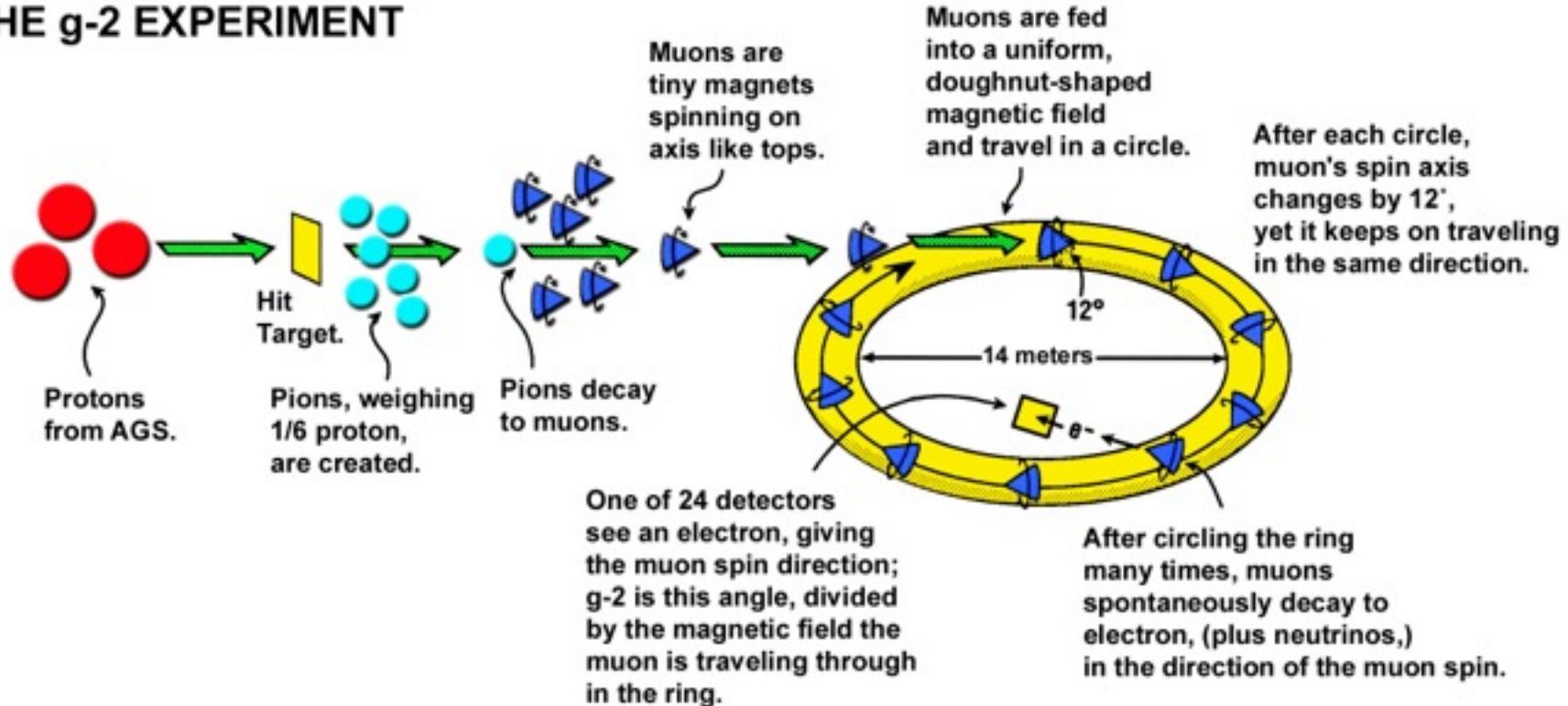
$$F_1(0) = 1 \quad F_2(0) = a_l$$

A pure “quantum correction” effect!

## 2. The muon g-2: theory update

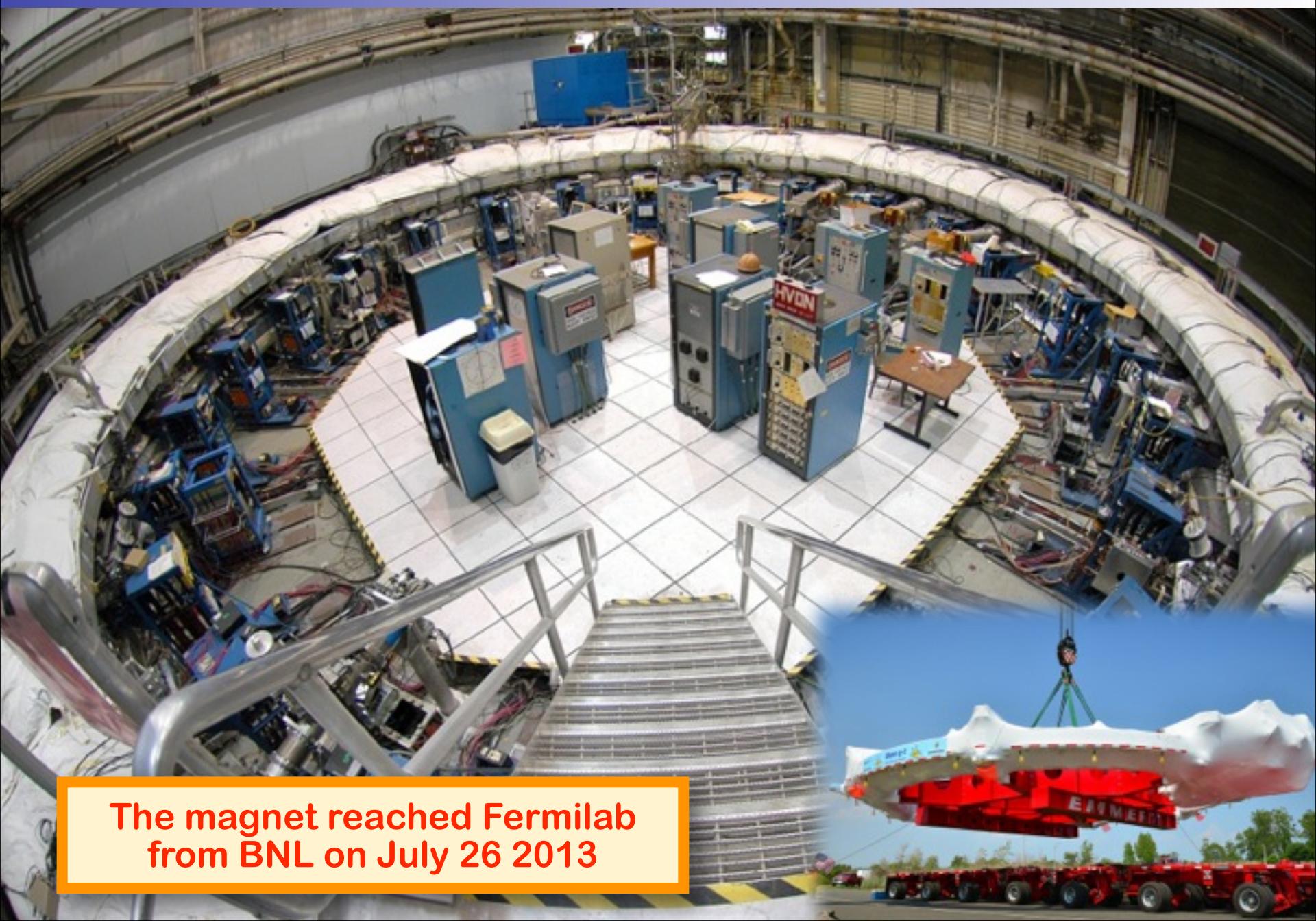
# The old experiment E821

## LIFE OF A MUON: THE g-2 EXPERIMENT



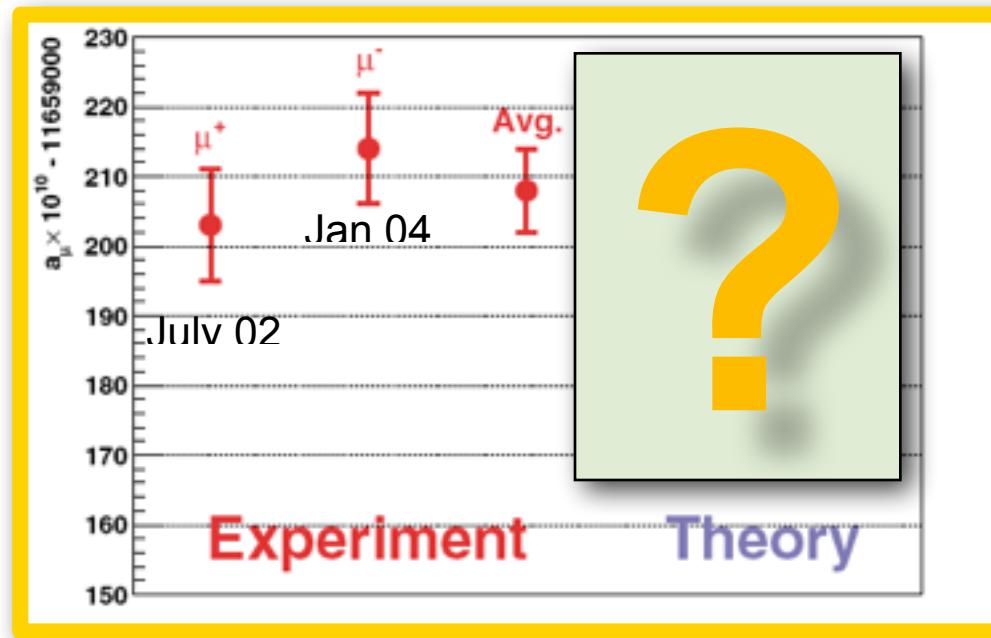
E821 @ BNL

## The old experiment E821 (2)



The magnet reached Fermilab  
from BNL on July 26 2013

# The muon g-2: the experimental result



- Today:  $a_\mu^{\text{EXP}} = (116592089 \pm 54_{\text{stat}} \pm 33_{\text{sys}}) \times 10^{-11}$  [0.5 ppm].
- Future: new muon g-2 experiments proposed at:
  - Fermilab E989, aiming at  $\pm 16 \times 10^{-11}$ , ie 0.14 ppm
  - J-PARC aiming at 0.1 ppm
- Are theorists ready for this (amazing) precision? No(t yet)

Sep 2012:  
CD0 approval!  
Data in (late)  
2016?

# The muon g-2: the QED contribution

$$a_\mu^{\text{QED}} = (1/2)(\alpha/\pi) \quad \text{Schwinger 1948}$$

$$+ 0.765857426 (16) (\alpha/\pi)^2$$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$$+ 24.05050988 (28) (\alpha/\pi)^3$$

Remiddi, Laporta, Barbieri ... ; Czarnecki, Skrzypek; MP '04;  
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$$+ 130.8796 (63) (\alpha/\pi)^4$$

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '04, '05;  
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012,  
Steinhauser et al. 2013 (analytic, in progress).

$$+ 753.29 (1.04) (\alpha/\pi)^5 \text{ COMPLETED!}$$

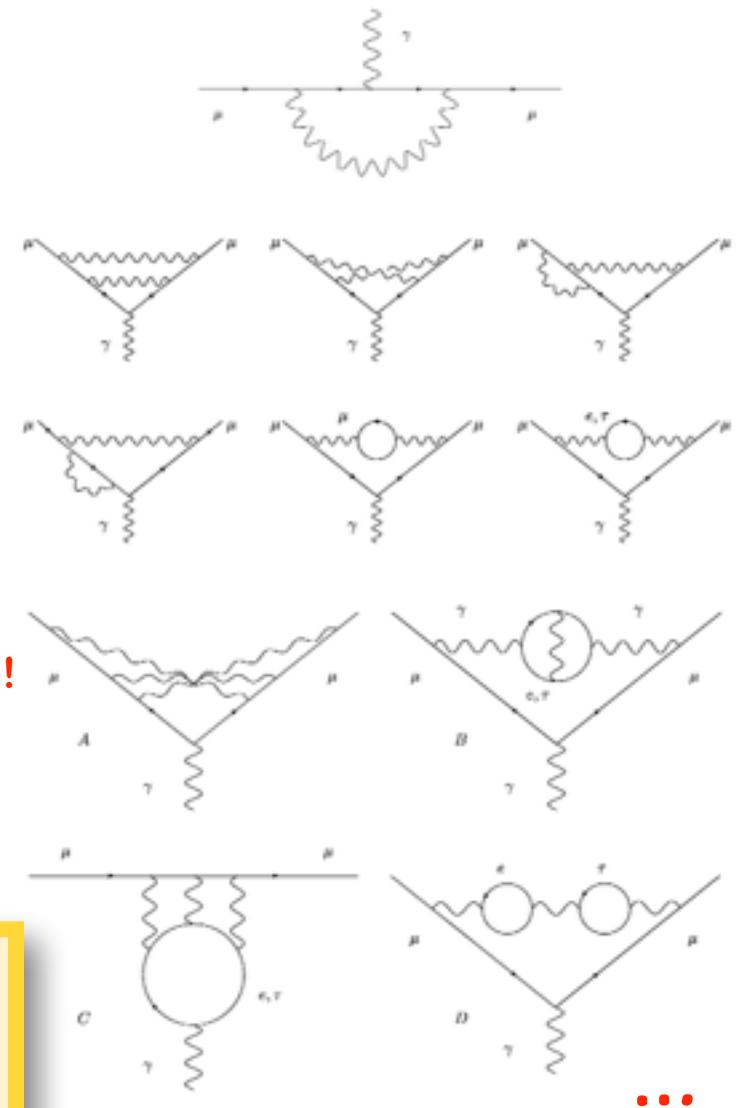
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,  
Karshenboim,..., Kataev, Kinoshita & Nio '06, Kinoshita et al. 2012

**Adding up, we get:**

$$a_\mu^{\text{QED}} = 116584718.951 (22)(77) \times 10^{-11}$$

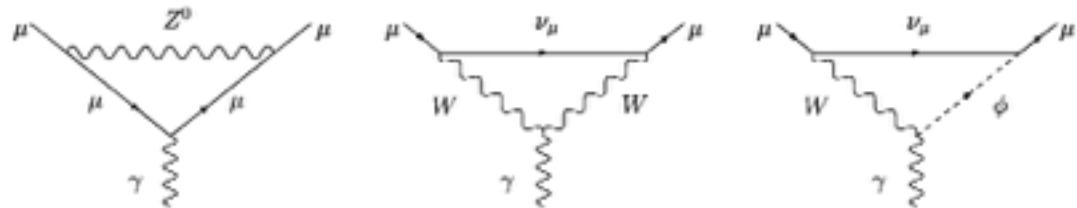
from coeffs, mainly from 4-loop unc from  $\delta\alpha(\text{Rb})$

$$\text{with } \alpha = 1/137.035999049(90) [0.66 \text{ ppb}]$$



# The muon g-2: the electroweak contribution

## ● One-loop term:



$$a_\mu^{\text{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2} \left[ 1 + \frac{1}{5} (1 - 4 \sin^2 \theta_W)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right) \right] \approx 195 \times 10^{-11}$$

1972: Jackiw, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;  
Studenikin et al. '80s

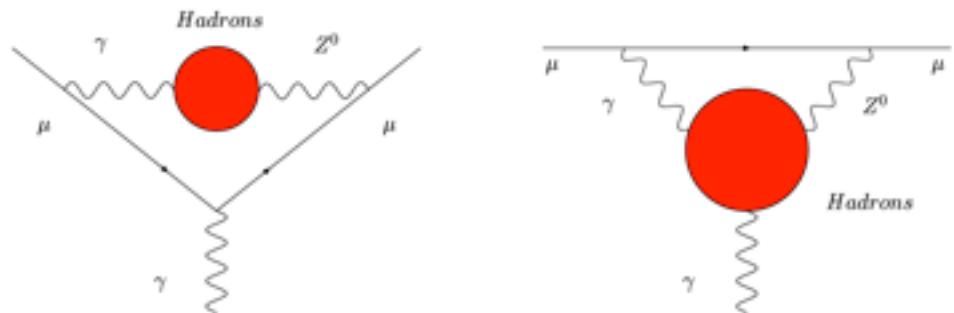
## ● One-loop plus higher-order terms:

$$a_\mu^{\text{EW}} = 153.6 (1) \times 10^{-11}$$

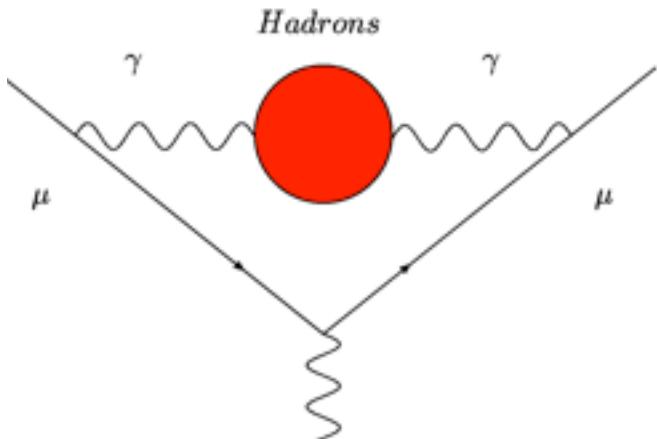
with  $M_{\text{Higgs}} = 125.6 (1.5) \text{ GeV}$

Hadronic loop uncertainties  
and 3-loop nonleading logs.

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.



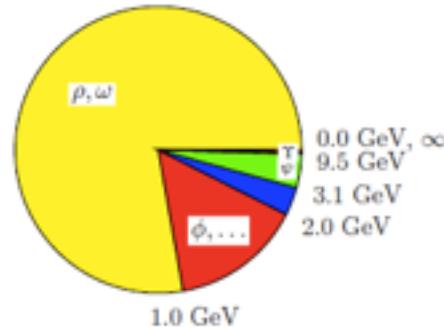
# The muon g-2: the hadronic LO contribution (HLO)



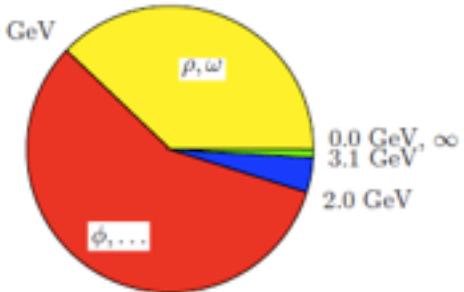
$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty ds K(s) \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} K(s) R(s)$$

Central values



Errors<sup>2</sup>



F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$a_\mu^{\text{HLO}} = 6903 (53)_{\text{tot}} \times 10^{-11}$$

F. Jegerlehner, A. Nyffeler, Phys. Rept. 477 (2009) 1

$$= 6923 (42)_{\text{tot}} \times 10^{-11}$$

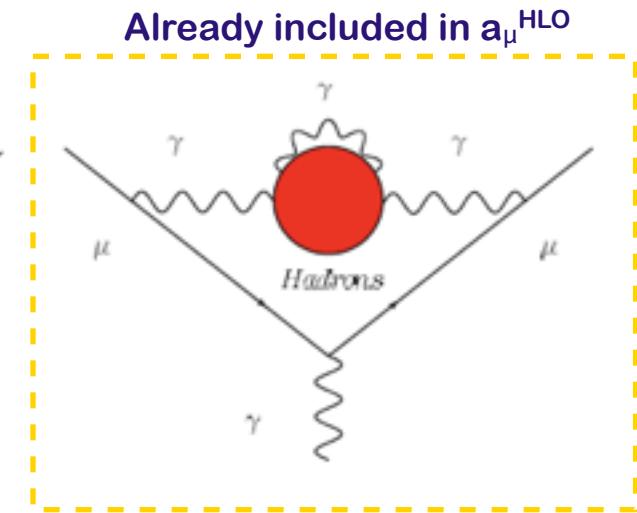
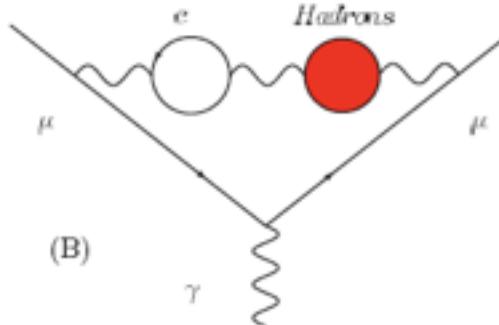
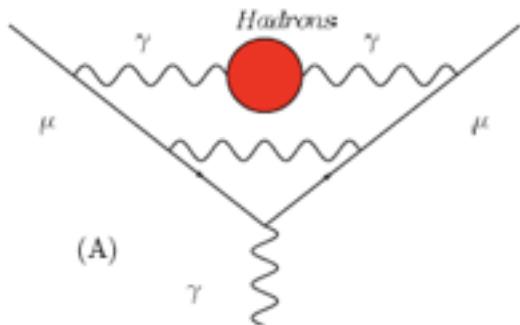
Davier et al, EPJ C71 (2011) 1515 (incl. BaBar & KLOE10  $2\pi$ )

$$= 6949 (37)_{\text{exp}} (21)_{\text{rad}} \times 10^{-11}$$

Hagiwara et al, JPG 38 (2011) 085003



- HHO: Vacuum Polarization**



$\mathcal{O}(\alpha^3)$  contributions of diagrams containing hadronic vacuum polarization insertions:

$$a_\mu^{\text{HHO(vp)}} = -98(1) \times 10^{-11}$$

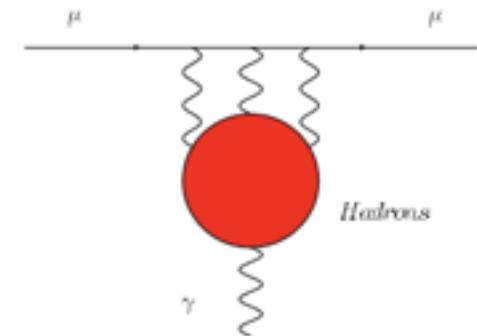
Krause '96, Alemany et al. '98, Hagiwara et al. 2011

Only tiny shifts if  $\tau$  data are used instead of the  $e^+e^-$  ones

Davier & Marciano '04.

- HHO: Light-by-light contribution**

- 📌 Unlike the HLO term, for the hadronic I-b-I term we must rely on theoretical approaches.
- 📌 This term had a troubled life! Latest values:



$$a_\mu^{\text{HHO}(\text{lbl})} = +80(40) \times 10^{-11} \quad \text{Knecht \& Nyffeler '02}$$

$$a_\mu^{\text{HHO}(\text{lbl})} = +136(25) \times 10^{-11} \quad \text{Melnikov \& Vainshtein '03}$$

$$a_\mu^{\text{HHO}(\text{lbl})} = +105(26) \times 10^{-11} \quad \text{Prades, de Rafael, Vainshtein '09}$$

$$a_\mu^{\text{HHO}(\text{lbl})} = +116(39) \times 10^{-11} \quad \text{Jegerlehner \& Nyffeler '09}$$

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

- 📌 “Bound”  $a_\mu^{\text{HHO}(\text{lbl})} < \sim 160 \times 10^{-11}$  Erler, Sanchez '06, Pivovarov '02; also Boughezal, Melnikov '11
- 📌 **Lattice? Very hard... in progress.** M. Golterman @ PhiPsi 2013; T. Blum @ Lattice 2012
- 📌 **Pion exch. in holographic QCD agrees.** D.K.Hong, D.Kim '09; Cappiello, Catà, D'Ambrosio '11
- 📌 “By far not complete” calculation:  $188 \times 10^{-11}$  Fischer et al, PRD87(2013)034013
- 📌 **Had Ibl is likely to become the ultimate limitation of the SM prediction.**

# The muon g-2: SM vs. Experiment

Adding up all contributions, we get the following SM predictions and comparisons with the measured value:

$$a_\mu^{\text{EXP}} = 116592089 (63) \times 10^{-11}$$

E821 – Final Report: PRD73  
(2006) 072 with latest value  
of  $\lambda = \mu_\mu / \mu_p$  from CODATA'06

$a_\mu^{\text{SM}} \times 10^{11}$	$\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}}$	$\sigma$
116 591 793 (66)	$296 (91) \times 10^{-11}$	3.2 [1]
116 591 813 (57)	$276 (85) \times 10^{-11}$	3.2 [2]
116 591 839 (58)	$250 (86) \times 10^{-11}$	2.9 [3]

with the “conservative”  $a_\mu^{\text{HHO}}(|\vec{b}|) = 116 (39) \times 10^{-11}$  and the LO hadronic from:

[1] Jegerlehner & Nyffeler, Phys. Rept. 477 (2009) 1

[2] Davier et al, EPJ C71 (2011) 1515 (includes BaBar & KLOE10  $2\pi$ )

[3] Hagiwara et al, JPG38 (2011) 085003 (includes BaBar & KLOE10  $2\pi$ )

Note that the th. error is now about the same as the exp. one

## The muon g-2: connection with the SM scalar mass

- $\Delta a_\mu$  can be explained in many ways: errors in LBL, QED, EW, HHO-VP, g-2 EXP, HLO; or, we hope, New Physics!
- Can  $\Delta a_\mu$  be due to hypothetical mistakes in the hadronic  $\sigma(s)$ ?
- An upward shift of  $\sigma(s)$  also induces an increase of  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ .
- Consider:

$$\begin{aligned} a_\mu^{\text{HLO}} &\rightarrow a = \int_{4m_\pi^2}^{s_u} ds f(s) \sigma(s), \quad f(s) = \frac{K(s)}{4\pi^3}, \quad s_u < M_Z^2, \\ \Delta\alpha_{\text{had}}^{(5)} &\rightarrow b = \int_{4m_\pi^2}^{s_u} ds g(s) \sigma(s), \quad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)}, \end{aligned}$$

and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

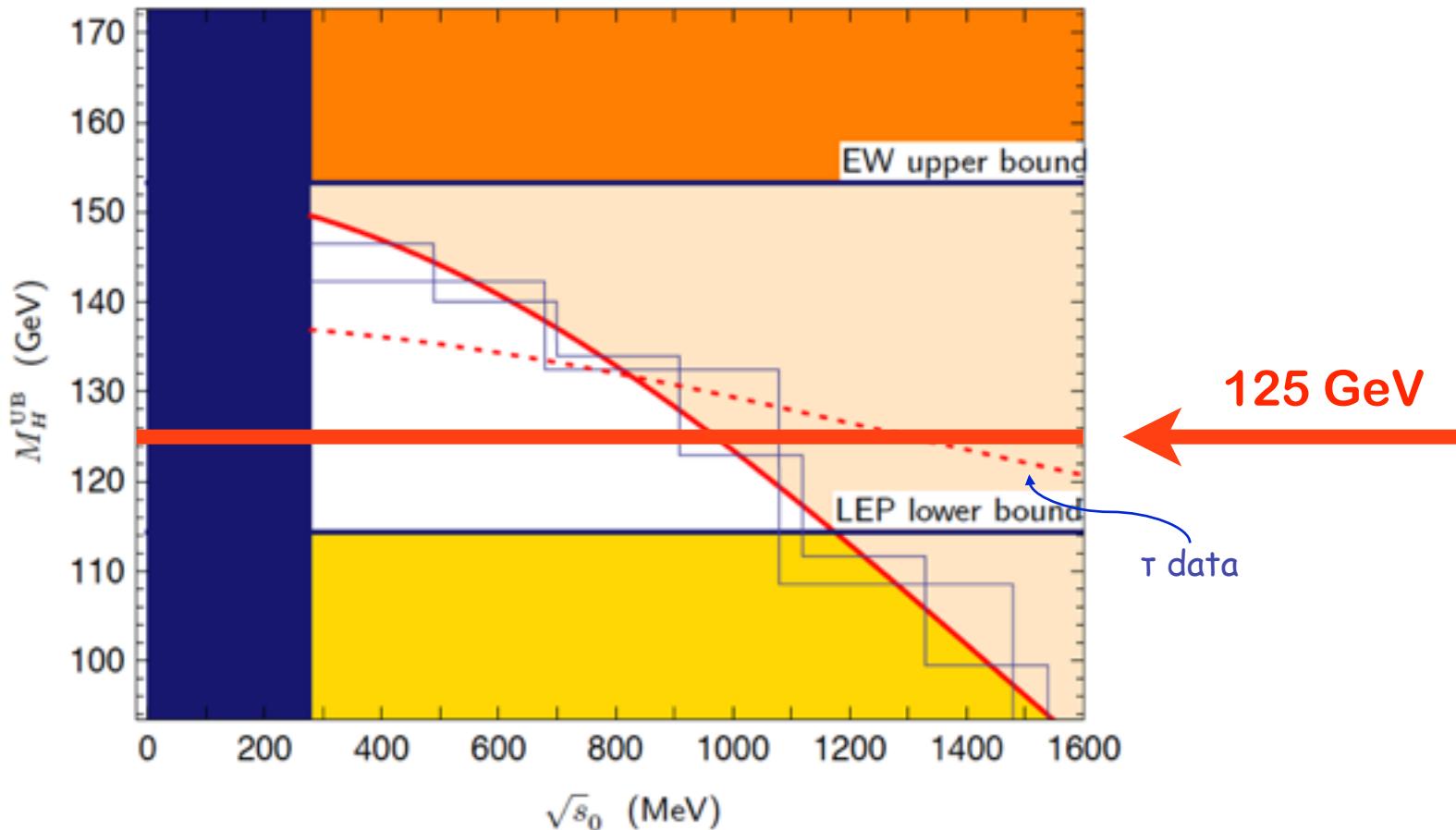
( $\epsilon > 0$ ), in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$



## The muon g-2: connection with the SM scalar mass (2)

- How much does the  $M_H$  upper bound from the EW fit change when we shift  $\sigma(s)$  by  $\Delta\sigma(s)$  [and thus  $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$ ] to accommodate  $\Delta a_\mu$  ?



W.J. Marciano, A. Sirlin, MP, 2008 & 2010

## The muon g-2: connection with the SM scalar mass (3)

- Given the quoted exp. uncertainty of  $\sigma(s)$ , the possibility to explain the muon g-2 with these very large shifts  $\Delta\sigma(s)$  appears to be very unlikely.
- Also, given a 125 GeV SM scalar, these hypothetical shifts  $\Delta\sigma(s)$  could only occur at very low energy (below  $\sim 1$  GeV).
- Vice versa, assuming we now have a SM scalar with  $M = 125$  GeV, if we bridge the  $M$  discrepancy in the EW fit via changes in the low-energy hadronic cross section, the muon g-2 discrepancy increases.

W.J. Marciano, A. Sirlin, MP, 2008 & 2010 (and work in progress)

### **3. Testing new physics with the electron g-2**

G.F. Giudice, P. Paradisi, MP

JHEP 1211 (2012) 113

# The QED prediction of the electron g-2

$$a_e^{\text{QED}} = + (1/2)(\alpha/\pi) - 0.328\ 478\ 444\ 002\ 55(33) (\alpha/\pi)^2$$

Schwinger 1948   Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$$A_1^{(4)} = -0.328\ 478\ 965\ 579\ 193\ 78\dots$$

$$A_2^{(4)} (m_e/m_\mu) = 5.197\ 386\ 68 (26) \times 10^{-7}$$

$$A_2^{(4)} (m_e/m_\tau) = 1.837\ 98 (33) \times 10^{-9}$$

$$+ 1.181\ 234\ 016\ 816 (11) (\alpha/\pi)^3$$

Kinoshita; Barbieri; Laporta, Remiddi; ... , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$$A_1^{(6)} = 1.181\ 241\ 456\ 587\dots$$

$$A_2^{(6)} (m_e/m_\mu) = -7.373\ 941\ 62 (27) \times 10^{-6}$$

$$A_2^{(6)} (m_e/m_\tau) = -6.5830 (11) \times 10^{-8}$$

$$A_3^{(6)} (m_e/m_\mu, m_e/m_\tau) = 1.909\ 82 (34) \times 10^{-13}$$

$$- 1.9097 (20) (\alpha/\pi)^4$$

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2012

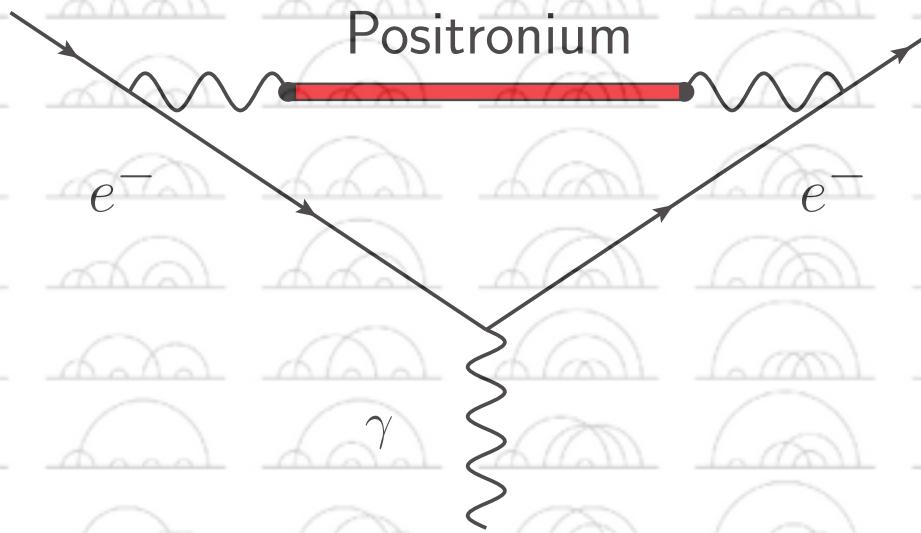
$$+ 9.16 (58) (\alpha/\pi)^5 \quad \text{COMPLETED! (12672 mass independent diagrams!)}$$

Aoyama, Hayakawa, Kinoshita, Nio, PRL 109 (2012) 111807.

# What is the positronium contribution to the electron g-2?

The leading contribution of positronium to  $a_e$  comes from:

Go Mishima arXiv:1311.7109; M. Fael & MP arXiv:1402.xxxx



An electron-positron bound state will appear as a pole singularity in  $\Pi(q^2)$  below the  $q^2 = (2m)^2$  branch-point. In fact, there is an infinite number of such poles.

## What is the positronium contribution to the electron g-2? (2)

In any of its  $n$  discrete states ( $n = 1, 2, 3, \dots$  is the principal quantum number), positronium may be regarded as an (unstable) particle with mass  $M_n = 2m - E_n$ , where  $E_n > 0$  is the binding energy.

$$a_e^P = \frac{\alpha^5}{4\pi} \zeta(3) \left( 8 \ln 2 - \frac{11}{2} \right) = 8.94 \times 10^{-14} = 1.32 \left( \frac{\alpha}{\pi} \right)^5$$

This result is of the same magnitude of the experimental uncertainty of  $a_e$  and of the same order of  $\alpha$  as the five-loop QED contribution!

# The SM prediction of the electron g-2

The SM prediction is:

$$a_e^{\text{SM}}(\alpha) = a_e^{\text{QED}}(\alpha) + a_e^{\text{EW}} + a_e^{\text{HAD}}$$

The EW (1&2 loop) term is: Czarnecki, Krause, Marciano '96 [Codata 2012]

$$a_e^{\text{EW}} = 0.2973(52) \times 10^{-13}$$

The Hadronic contribution is: Nomura & Teubner '12, Jegerlehner & Nyffeler '09; Krause '97

$$a_e^{\text{HAD}} = 16.82(16) \times 10^{-13}$$

Which value of  $\alpha$  should we use to compute  $a_e^{\text{SM}}$  and compare it with  $a_e^{\text{EXP}}$  ?? Not the PDG/Codata one (obtained equating  $a_e^{\text{SM}}(\alpha) = a_e^{\text{EXP}}$ )! Use atomic-physics measurements of alpha.

## The electron g-2 gives the best determination of alpha

- The 2008 measurement of the electron g-2 is:

$$a_e^{\text{EXP}} = 11596521807.3 \text{ (2.8)} \times 10^{-13} \quad \text{Hanneke et al, PRL100 (2008) 120801}$$

vs. old (factor of 15 improvement,  $1.8\sigma$  difference):

$$a_e^{\text{EXP}} = 11596521883 \text{ (42)} \times 10^{-13} \quad \text{Van Dyck et al, PRL59 (1987) 26}$$

- Equate  $a_e^{\text{SM}}(\alpha) = a_e^{\text{EXP}} \rightarrow$  best determination of alpha (2014):

$$\alpha^{-1} = 137.035\ 999\ 184 \text{ (35)} \quad [0.25 \text{ ppb}]$$

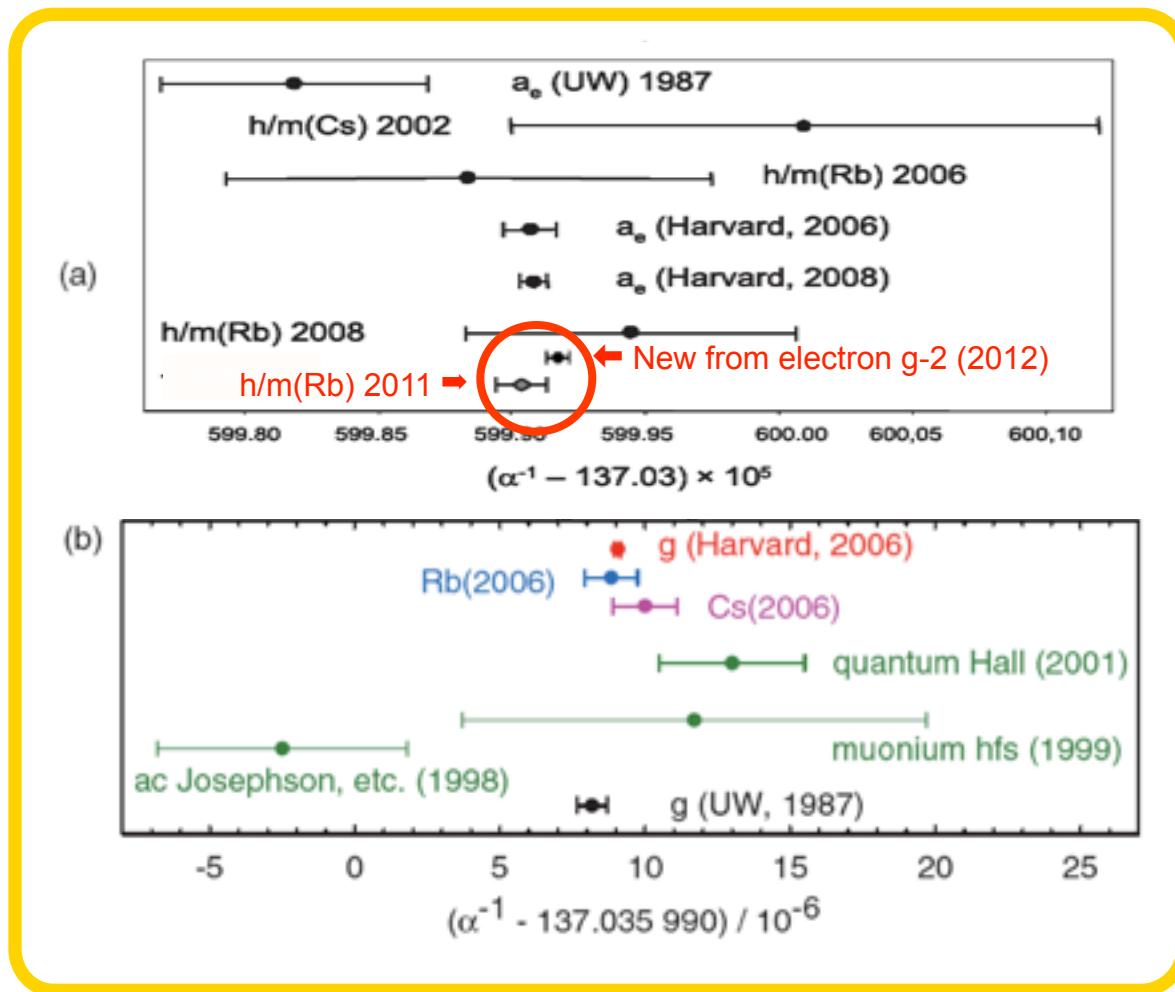
- Compare it with other determinations (independent of  $a_e$ ):

$$\alpha^{-1} = 137.036\ 000\ 0 \text{ (11)} \quad [7.7 \text{ ppb}] \quad \text{PRA73 (2006) 032504 (Cs)}$$

$$\alpha^{-1} = 137.035\ 999\ 049 \text{ (90)} \quad [0.66 \text{ ppb}] \quad \text{PRL106 (2011) 080801 (Rb)}$$

Excellent agreement  $\rightarrow$  beautiful test of QED at 4-loop level!

# Old and new determinations of alpha



Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL99 (2007) 039902  
Hanneke, Fogwell & Gabrielse, PRL100 (2008) 120801  
Bouchendira et al, PRL106 (2011) 080801

## The electron g-2: SM vs. Experiment

- Using  $\alpha = 1/137.035\ 999\ 049\ (90)$  [ $^{87}\text{Rb}$ , 2011], the SM prediction for the electron g-2 is

$$a_e^{\text{SM}} = 115\ 965\ 218\ 18.7 (0.6) (0.4) (0.2) (7.6) (0.1) \times 10^{-13}$$

$\delta C_4^{\text{qed}}$     $\delta C_5^{\text{qed}}$     $\delta a_e^{\text{had}}$    from  $\delta \alpha$   
from positronium

- The EXP-SM difference is:

$$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -11.4 (8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment ( $1.4\sigma$ ).

NB: The 4-loop contrib. to  $a_e^{\text{QED}}$  is  $-5.56 \times 10^{-11} \sim 70 \Delta a_e$ !  
(the 5-loop one is  $6.2 \times 10^{-13}$ )

# The electron g-2 sensitivity and NP tests

- The present sensitivity is  $\delta\Delta a_e = 8.1 \times 10^{-13}$ , ie (10<sup>-13</sup> units):

$$\underbrace{(0.6)_{\text{QED4}}, (0.4)_{\text{QED5}}, (0.2)_{\text{HAD}}, (0.1)_{\text{Pos}}, (7.6)_{\delta\alpha}, (2.8)_{\delta a_e^{\text{EXP}}}}_{(0.7)_{\text{TH}} \leftarrow \text{may drop to 0.2 or 0.3}}$$

- The  $(g-2)_e$  exp. error may soon drop below 10<sup>-13</sup> and work is in progress for a significant reduction of that induced by  $\delta\alpha$ .

→ sensitivity of 10<sup>-13</sup> may be reached with ongoing exp. work

F. Terranova & G.M. Tino, arXiv:1312.2346

- In a broad class of BSM theories, contributions to  $a_1$  scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left( \frac{m_{\ell_i}}{m_{\ell_j}} \right)^2 \quad \text{This Naive Scaling leads to:}$$

$$\Delta a_e = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.7 \times 10^{-13}; \quad \Delta a_\tau = \left( \frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.8 \times 10^{-6}$$

## The electron g-2 sensitivity and NP tests (II)

- The experimental sensitivity in  $\Delta a_e$  is not far from what is needed to test if the discrepancy in  $(g-2)_\mu$  also manifests itself in  $(g-2)_e$  under the naive scaling hypothesis.
- BSM scenarios exist which violate Naive Scaling. They can lead to larger effects in  $\Delta a_e$  (&  $\Delta a_\tau$ ) and contributions to EDMs, LFV or lepton universality breaking observables.
- Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles),  $\Delta a_e$  can reach  $10^{-12}$  (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

## 4. The tau g-2: opportunities & challenges

Work in progress in collaboration with  
S. Eidelman, D. Epifanov, M. Fael, L. Mercolli

arXiv:1301.5302

arXiv:1310.1081

# The SM prediction of the tau g-2

The Standard Model prediction of the tau g-2 is:

$$\begin{aligned} a_{\tau}^{\text{SM}} = & 117324 (2) \times 10^{-8} \text{ QED} \\ & + 47.4 (0.5) \times 10^{-8} \text{ EW} \\ & + 337.5 (3.7) \times 10^{-8} \text{ HLO} \\ & + 7.6 (0.2) \times 10^{-8} \text{ HHO (vac)} \\ & + 5 (3) \times 10^{-8} \text{ HHO (lbl)} \end{aligned}$$

$$a_{\tau}^{\text{SM}} = 117721 (5) \times 10^{-8}$$

Eidelman & MP  
2007

$(m_{\tau}/m_{\mu})^2 \sim 280$ : great opportunity to look for New Physics,  
and a “clean” NP test too...

	Muon	Tau
$a_{\text{EW}}$	1/45	1/7
$a_{\text{EW}}$	3	10

... if only we could measure it!!

## The tau g-2: experimental bounds

- The very short lifetime of the tau makes it very difficult to determine  $a_\tau$  measuring its spin precession in a magnetic field.
- DELPHI's result, from  $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$  total cross-section measurements at LEP 2 (the PDG value):

$$a_\tau = -0.018 (17)$$

PDG 2012

- With an effective Lagrangian approach, using data on tau lepton production at LEP1, SLC, and LEP2:

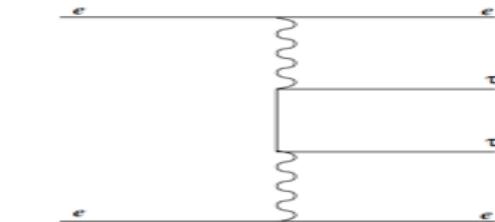
$$-0.004 < a_\tau^{\text{NP}} < 0.006 \quad (95\% \text{ CL})$$

Escribano & Massó 1997

$$-0.007 < a_\tau^{\text{NP}} < 0.005 \quad (95\% \text{ CL})$$

González-Sprinberg et al 2000

- Bernabéu et al, propose the measurement of  $F_2(q^2=M_Y^2)$  from  $e^+e^- \rightarrow \tau^+\tau^-$  production at B factories. NPB 790 (2008) 160



# The tau g-2 via its radiative leptonic decays: a proposal

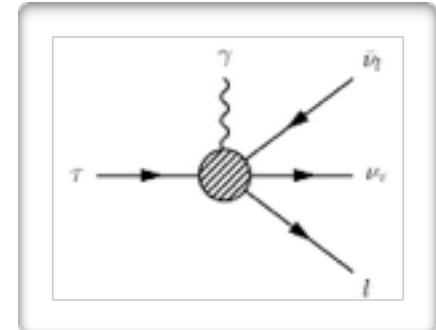
- Tau radiative leptonic decays at LO:

$$\frac{d^3\Gamma}{dx dy d \cos \theta} = \frac{\alpha M_\tau^5 G_F^2 y \sqrt{x^2 - 4r^2}}{2\pi(4\pi)^6} G_0(x, y, \cos \theta, r)$$

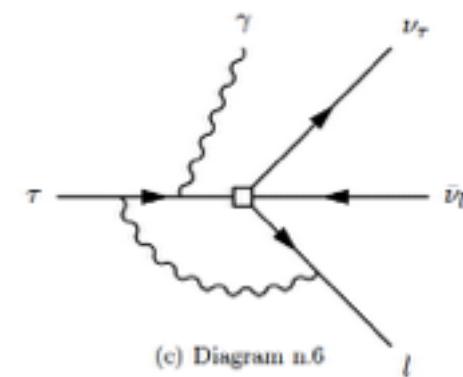
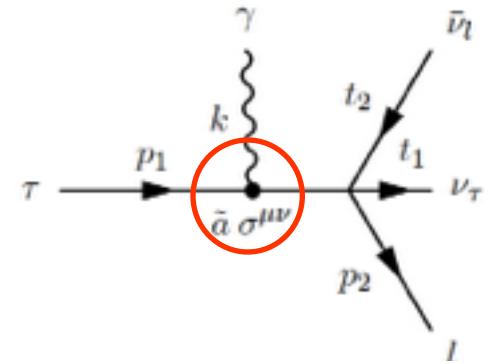
Kinoshita & Sirlin PRL2(1959)177; Kuno & Okada, RMP73(2001)151

$$\left. \frac{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau \gamma)}{\Gamma_{\text{total}}} \right|_{E_\gamma > 10 \text{ MeV}} = \boxed{1.836\% \quad \text{vs} \quad 1.75(18)\% \atop \text{CLEO 2000}}$$

$$\left. \frac{\Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau \gamma)}{\Gamma_{\text{total}}} \right|_{E_\gamma > 10 \text{ MeV}} = \boxed{0.367\% \quad \text{vs} \quad 0.361(38)\%}$$



$$x = \frac{2E_l}{M_\tau}, \quad y = \frac{2E_\gamma}{M_\tau}, \quad r = \frac{m_l}{M_\tau}$$



$$G_0 \rightarrow G_0 + \tilde{a}_\tau G_a + \frac{\alpha}{\pi} G_{\text{RC}}$$

- Measure  $d^3\Gamma$  precisely and get  $\tilde{a}_\tau$ !

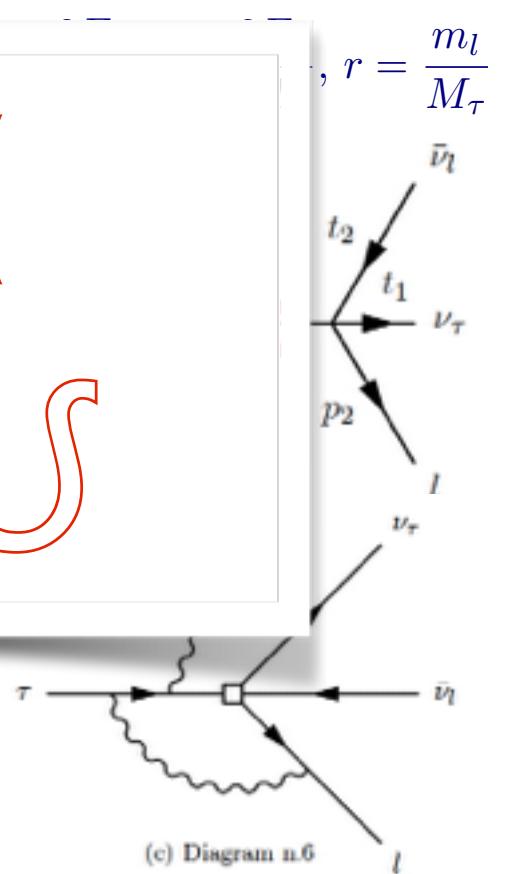
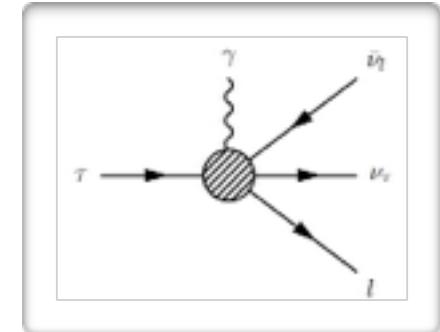
[see also Laursen, Samuel, Sen, PRD29 (1984) 2652]

# The tau g-2 via its radiative leptonic decays: a proposal

- Tau radiative leptonic decays at LO:

$$\frac{d^3\Gamma}{dx dy d \cos \theta} = \frac{\alpha M_\tau^5 G_F^2 y \sqrt{x^2 - 4r^2}}{2\pi(4\pi)^6} G_0(x, y, \cos \theta, r)$$

Kinoshita & Sirlin PRL2(1959)177; Kuno & Okada, RMP73(2001)151



# Conclusions

- The lepton g-2 provide beautiful examples of interplay between theory and experiment.
- The discrepancy is  $\Delta a_\mu \sim 3 \div 3.5 \sigma$ . Is it NP? New g-2 experiment, ring now in Fermilab! QED & EW terms ready for the challenge; How about the hadronic one? Future of LBL??
- Could  $\Delta a_\mu$  be due to mistakes in the hadronic  $\sigma(s)$ ? Very unlikely. Also, given a 125 GeV SM scalar, these hypothetical shifts  $\Delta\sigma(s)$  could only occur at very low energies (below  $\sim 1$ GeV).
- The sensitivity of the electron g-2 has improved. The positronium contribution has recently been included. It may soon be possible to test if  $\Delta a_\mu$  manifests itself also in the electron g-2! A robust and ambitious exp program is needed to improve  $\alpha$  &  $a_e$ .
- The tau g-2 is essentially unknown: we propose to measure it at Belle II via its radiative leptonic decays.

**The End**