New Paradigm for Baryon and Lepton Number Violation

Pavel Fileviez Perez
Max-Planck Institute for Nuclear Physics (MPIK)
Heidelberg, Germany

MAX-PLANCK-GESELLSCHAFT

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

Université Libre de Bruxelles, Feb 14, 2014

Collaborators:

2009 - M. B. Wise (Caltech, USA)

2012 - S.Spinner (UPenn, USA)

2012 - J.M. Arnold (Caltech, USA)

2012 - B. Fornal (Caltech, USA)

2013 - M. Duerr (MPIK, Germany)

2013 - M. Lindner (MPIK, Germany)

2013- H. H. Patel (MPIK, Germany)

References:

P. F. P., M. B. Wise,
Phys. Rev. D 82, 011901 (2010); JHEP 08 (2011) 068

M. Duerr, P. F. P., M. B. Wise,
Physical Review Letters 110,231801 (2013)

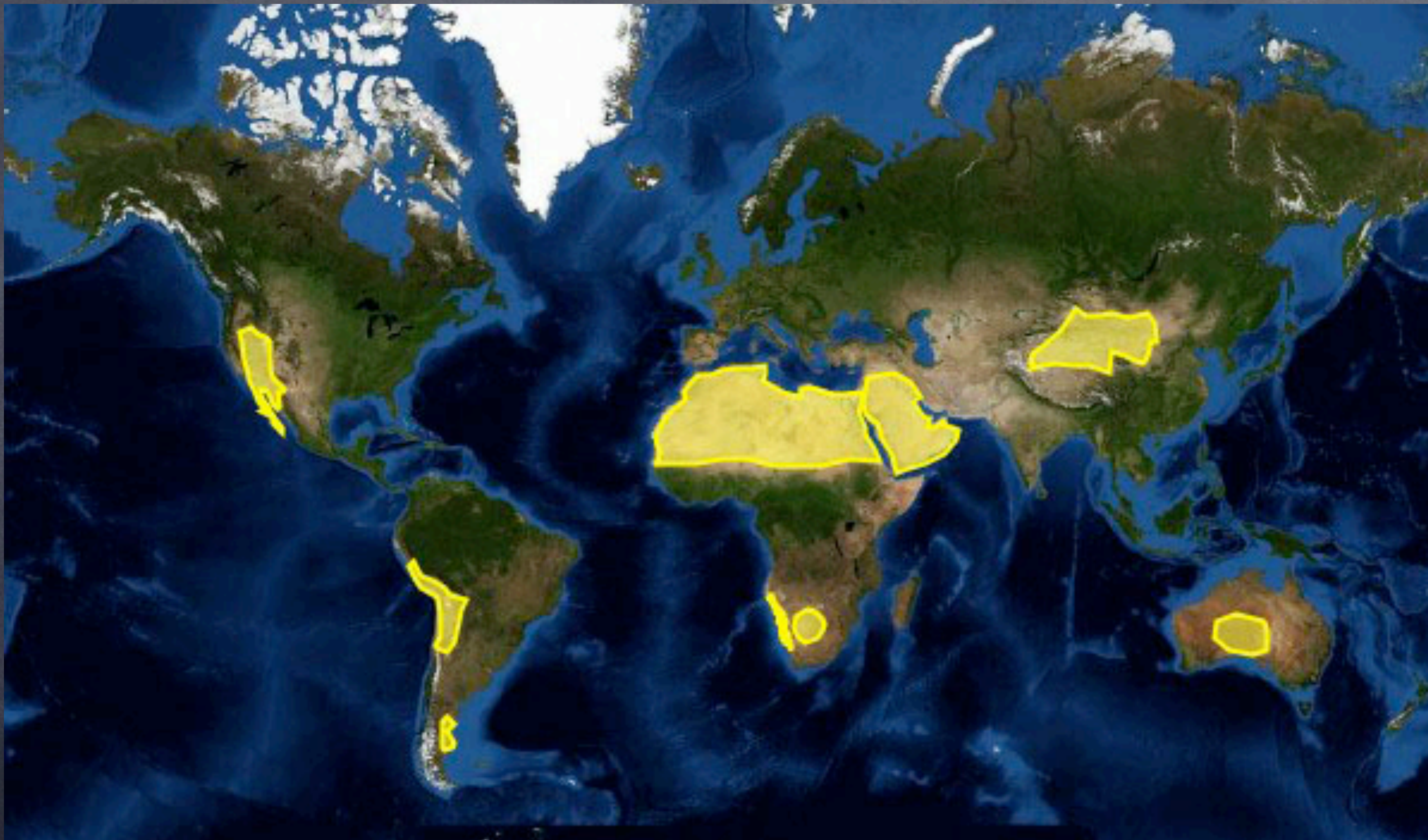
M. Duerr, P. F. P., M. Lindner, Phys.Rev. D88 (2013) 051701

M. Duerr, P. F. P., 1309.3970

P. F. P., H. H. Patel, 1311.6472

J. M. Arnold, P. F. P., B. Fornal, S. Spinner
Phys.Rev. D88 (2013) 115009

The World Largest Deserts

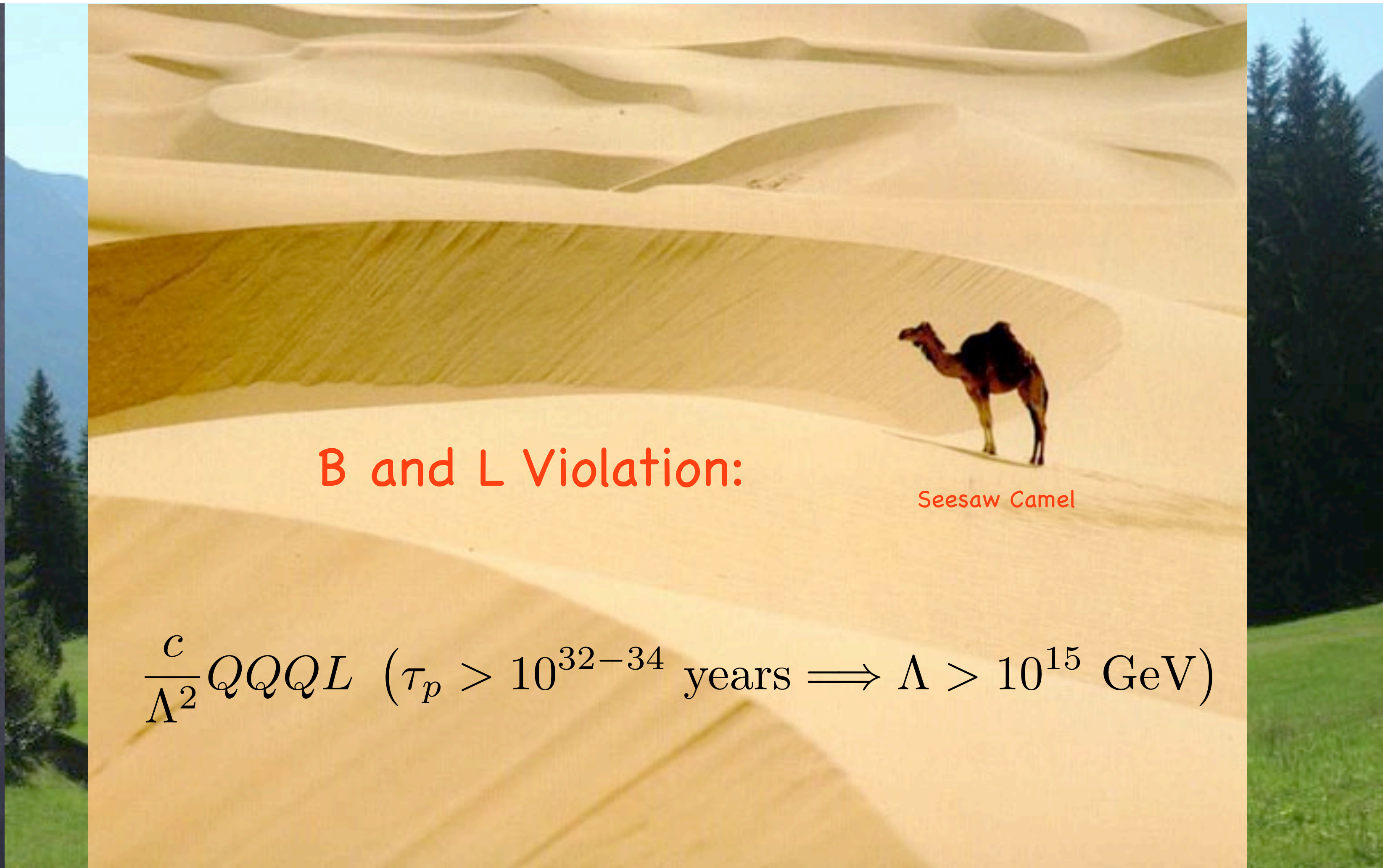


Deserts take up about one third (1/3) of the Earth's land surface

http://en.wikipedia.org/wiki/Desert

The Desert Hypothesis in Particle Physics

LOW SCALE



B and L Violation:

$$\frac{c}{\Lambda^2} QQQL \ \left(\tau_p > 10^{32-34} \text{ years} \Longrightarrow \Lambda > 10^{15} \text{ GeV}\right)$$

HIGH SCALE

Standard Model

$\Lambda_{\text{Weak}} \sim 100$ GeV

GUTs, Strings ?

$\Lambda \sim 10^{15-19}$ GeV

Georgi-Glashow Model--SU(5)

VOLUME 32, NUMBER 8 PHYSICAL REVIEW LETTERS 25 FEBRUARY 1974

Unity of All Elementary-Particle Forces

Howard Georgi* and S. L. Glashow
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 10 January 1974)

Strong, electromagnetic, and weak forces are conjectured to arise from a single fundamental interaction based on the gauge group SU(5).

We present a series of hypotheses and speculations leading inescapably to the conclusion that SU(5) is the gauge group of the world—that all elementary particle forces (strong, weak, and electromagnetic) are different manifestations of the same fundamental interaction involving a single coupling strength, the fine-structure constant. Our hypotheses may be wrong and our speculations idle, but the uniqueness and simplicity of our scheme are reasons enough that it be taken seriously.

Our starting point is the assumption that *weak and electromagnetic forces are mediated by the vector bosons of a gauge-invariant theory with spontaneous symmetry breaking*. A model describing the interactions of leptons using the gauge group $SU(2) \otimes U(1)$ was first proposed by Glashow, and was improved by Weinberg and Salam who incorporated spontaneous symmetry breaking.[1] This scheme can also describe hadof the GIM mechanism with the notion of colored quarks[4] keeps the successes of the quark model and gives an important bonus: Lepton and hadron anomalies cancel so that the theory of weak and electromagnetic interactions is renormalizable.[5]

The next step is to include strong interactions. We assume that *strong interactions are mediated by an octet of neutral vector gauge gluons* associated with local color SU(3) symmetry, and that there are no fundamental strongly interacting scalar-meson fields.[6] This insures that parity and hypercharge are conserved to order α,[7] and does not lead to any new anomalies, so that the theory remains renormalizable. The strongest binding forces are in color singlet states which may explain why observed hadrons lie in qqq and $q\bar{q}$ configurations.[8] And, it gives another important bonus: Since the strong interactions are associated with a non-Abelian theory, they may be asymptotically free.[9]

Hierarchy of Interactions in Unified Gauge Theories*

H. Georgi,† H. R. Quinn, and S. Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 15 May 1974)

We present a general formalism for calculating the renormalization effects which make strong interactions strong in simple gauge theories of strong, electromagnetic, and weak interactions. In an SU(5) model the superheavy gauge bosons arising in the spontaneous breakdown to observed interactions have mass perhaps as large as 10^{17} GeV, almost the Planck mass. Mixing-angle predictions are substantially modified.

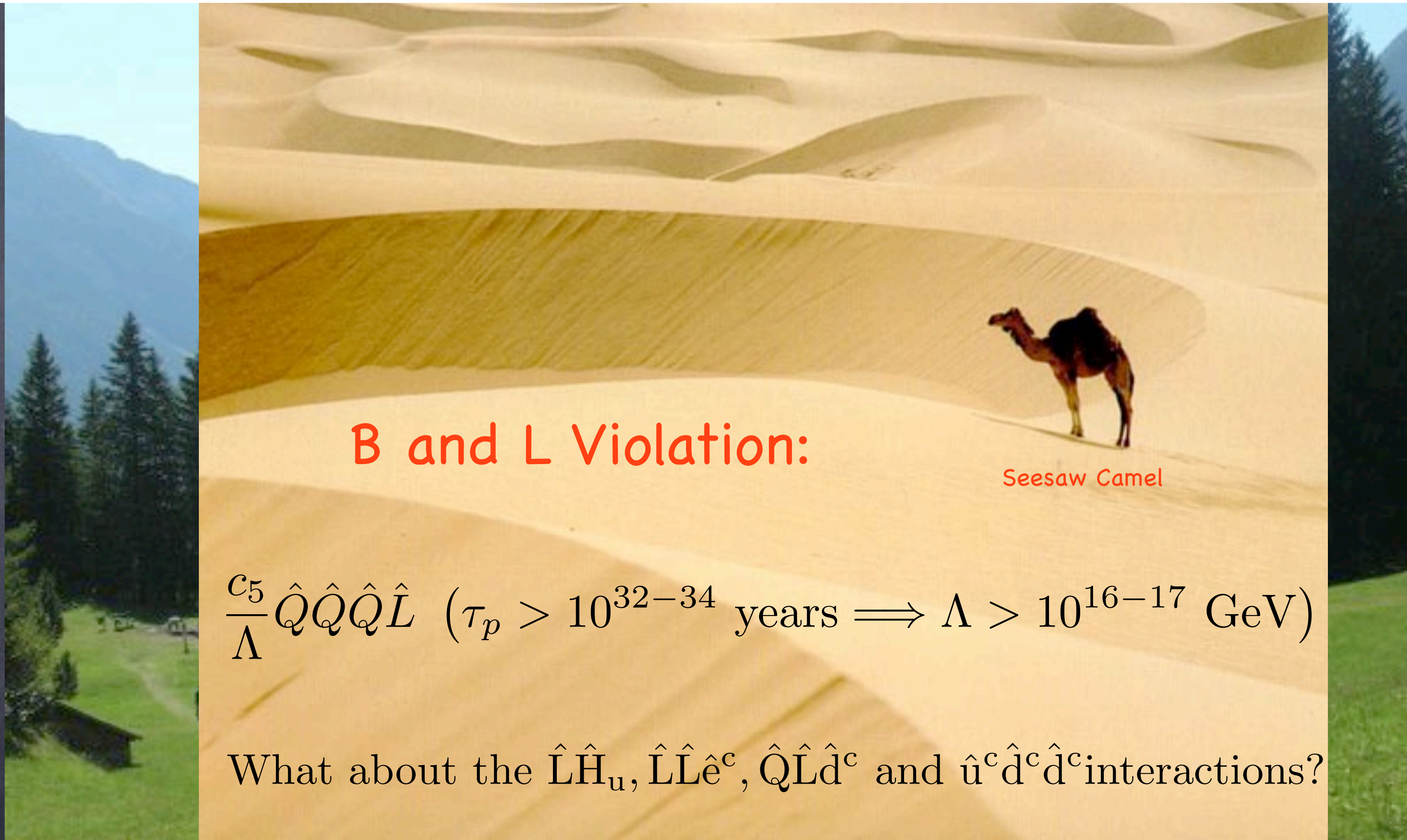
The scaling observed in deep inelastic electron scattering suggests that what are usually called the strong interactions are not so strong at high energies. Asymptotically free gauge theories of the strong interactions[1] provide a possible explanation: The gluon coupling constant $g(\mu)$ (defined as the value of a three-gluon or gluon-fermion-fermion vertex with momenta characterized by a mass μ) is small when μ is several GeV or larger, but becomes large when μ is small, through the piling up of the logarithms encountered in perturbation theory. In one recent calculation[2] a fit was found for a gauge coupling [in a color SU(3) model][3] with $g^2(\mu)/4\pi \simeq 0.1$ when $\mu \simeq 2$ GeV.

electromagnetic[6] interactions. In order to suppress unobserved interactions, Georgi and Glashow made the necessary assumption[7] that some vector bosons are superheavy.

We find the notion of a simple gauge group uniting strong, weak, and electromagnetic interactions extraordinarily attractive. However, as emphasized by Georgi and Glashow, the success of any such scheme hinges on an understanding of the effects which produce the obvious disparity in strength between the strong and the weak and electromagnetic interactions at ordinary energies. We therefore wish to present in this paper a general formalism for the calculation of such effects. This will lead us to an estimate of the

The Desert Hypothesis and Supersymmetry

LOW SCALE



B and L Violation:

$$\frac{c_5}{\Lambda}\hat{Q}\hat{Q}\hat{Q}\hat{L} \;\; \left(\tau_p > 10^{32-34} \text{ years} \Longrightarrow \Lambda > 10^{16-17} \text{ GeV}\right)$$

What about the $\hat{\mathrm{L}}\hat{\mathrm{H}}_{\mathrm{u}}, \hat{\mathrm{L}}\hat{\mathrm{L}}\hat{\mathrm{e}}^{\mathrm{c}}, \hat{\mathrm{Q}}\hat{\mathrm{L}}\hat{\mathrm{d}}^{\mathrm{c}}$ and $\hat{\mathrm{u}}^{\mathrm{c}}\hat{\mathrm{d}}^{\mathrm{c}}\hat{\mathrm{d}}^{\mathrm{c}}$ interactions?

HIGH SCALE

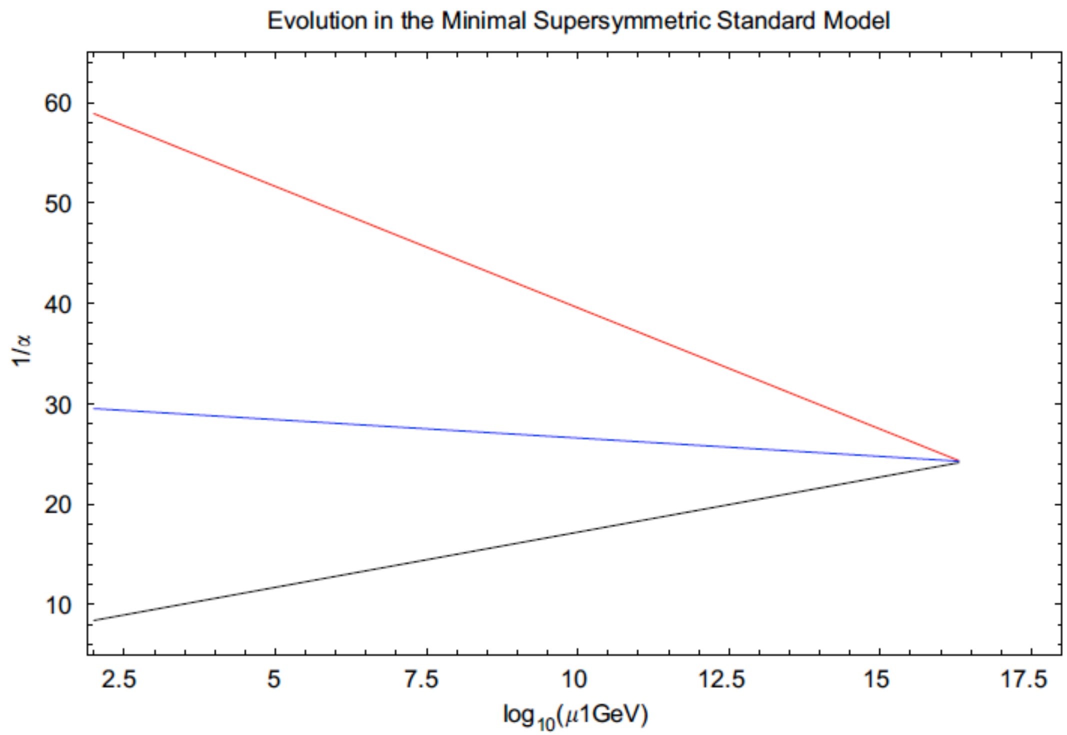
Unification of Gauge Couplings !

MSSM

10 TeV ? 100 TeV ? ...

GUTs, Strings ?

PHYSICAL REVIEW D VOLUME 24, NUMBER 6 15 SEPTEMBER 1981
P. Nath, P. Fileviez Pérez / Physics Reports 441 (2007) 191–317



Unif
soluti
renor
chang
sugge

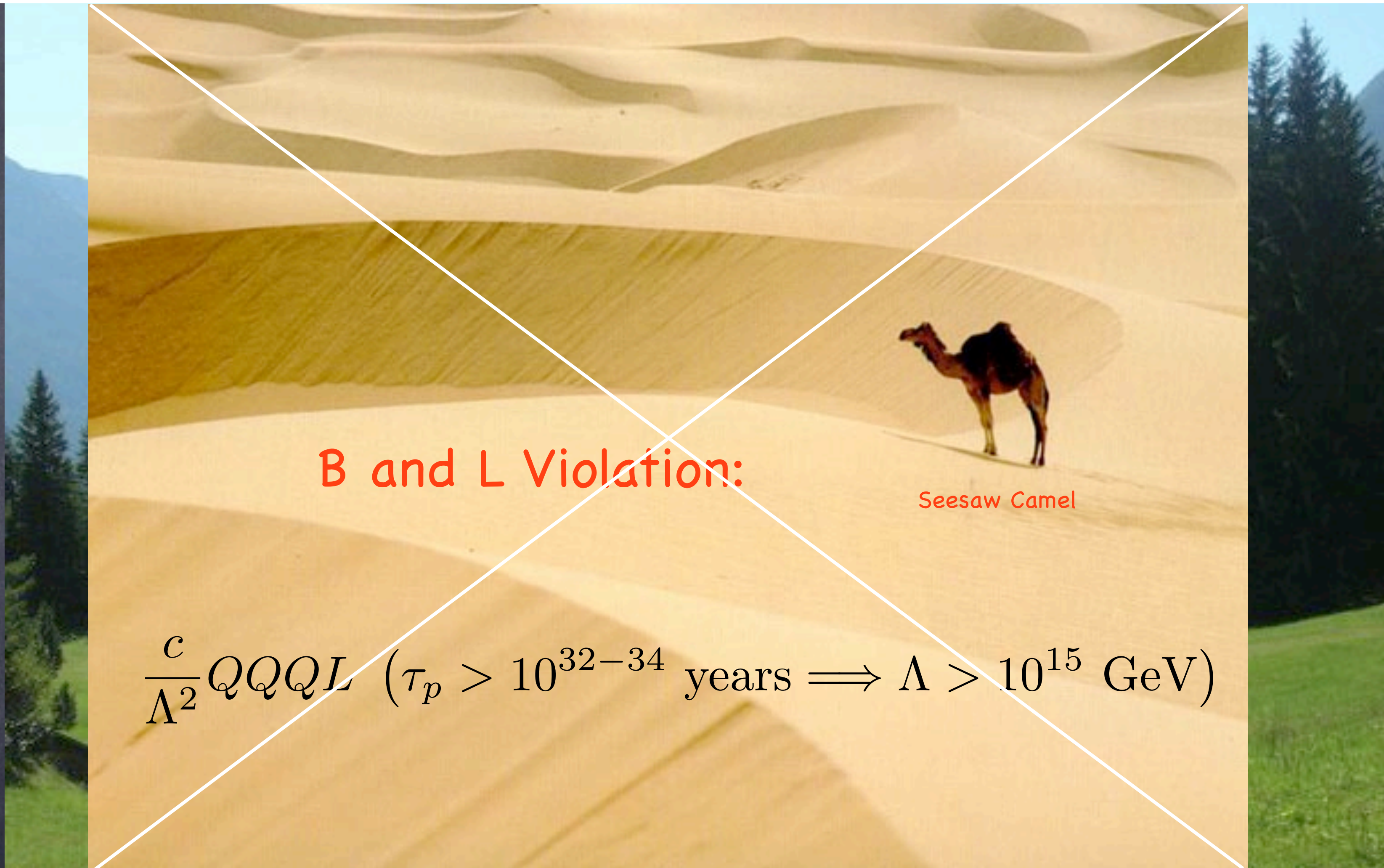
ssible
s on
lightly
nay be
ay.

Apparently successful gauge theories of the strong, electromagnetic, and weak interactions have suggested that all these interactions are manifestations of a larger, encompassing gauge symmetry softly broken at a large mass scale. Models realizing this idea have been constructed

boson system which breaks $SU(2) \times U(1)$ symmetry at $\sim 10^2$ GeV have such a small mass parameter? Ordinarily mass parameters for scalar fields violate no symmetries and their smallness cannot be explained on symmetry grounds. In supersymmetric theories, where there are symmetry

Aim: Can we break B and L at the TeV scale?

LOW SCALE



$$\frac{c}{\Lambda^2} QQQL \ \left(\tau_p > 10^{32-34} \ \text{years} \Longrightarrow \Lambda > 10^{15} \ \text{GeV}\right)$$

HIGH SCALE

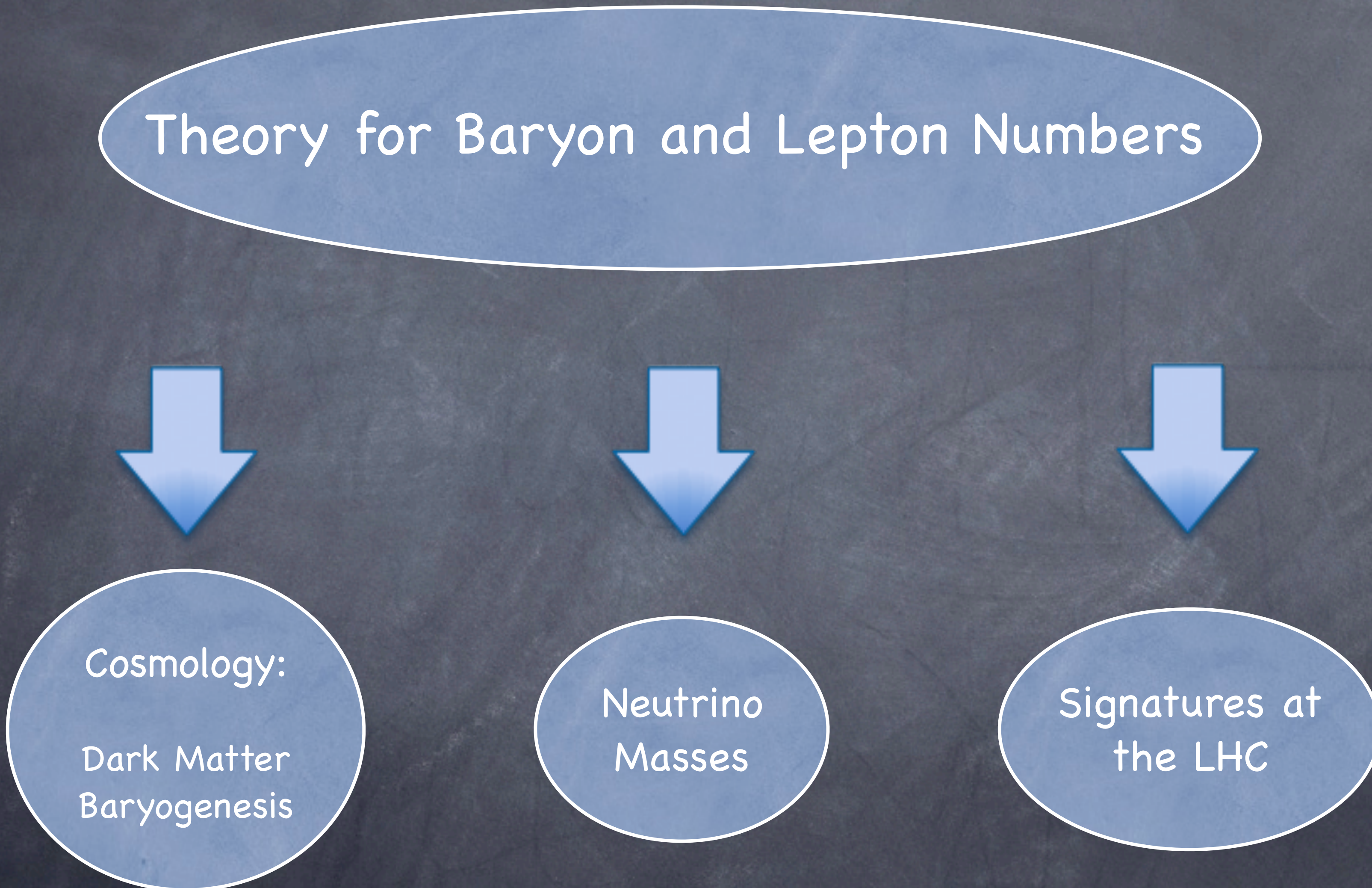
Standard Model

$\Lambda_{\text{Weak}} \sim 100$ GeV

GUTs, Strings ?

$\Lambda \sim 10^{15-19}$ GeV

Aim



P. Fileviez Perez

Outline

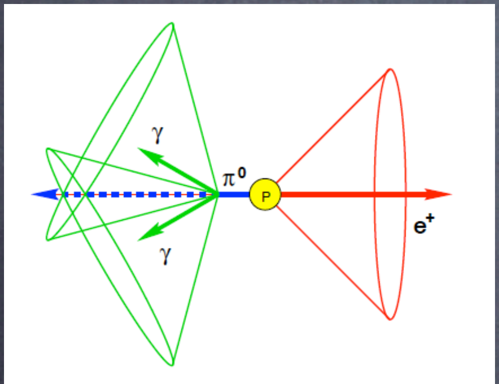
- Introduction
- Living without the Great Desert
- Towards Unification and Neutrino Masses
- Summary

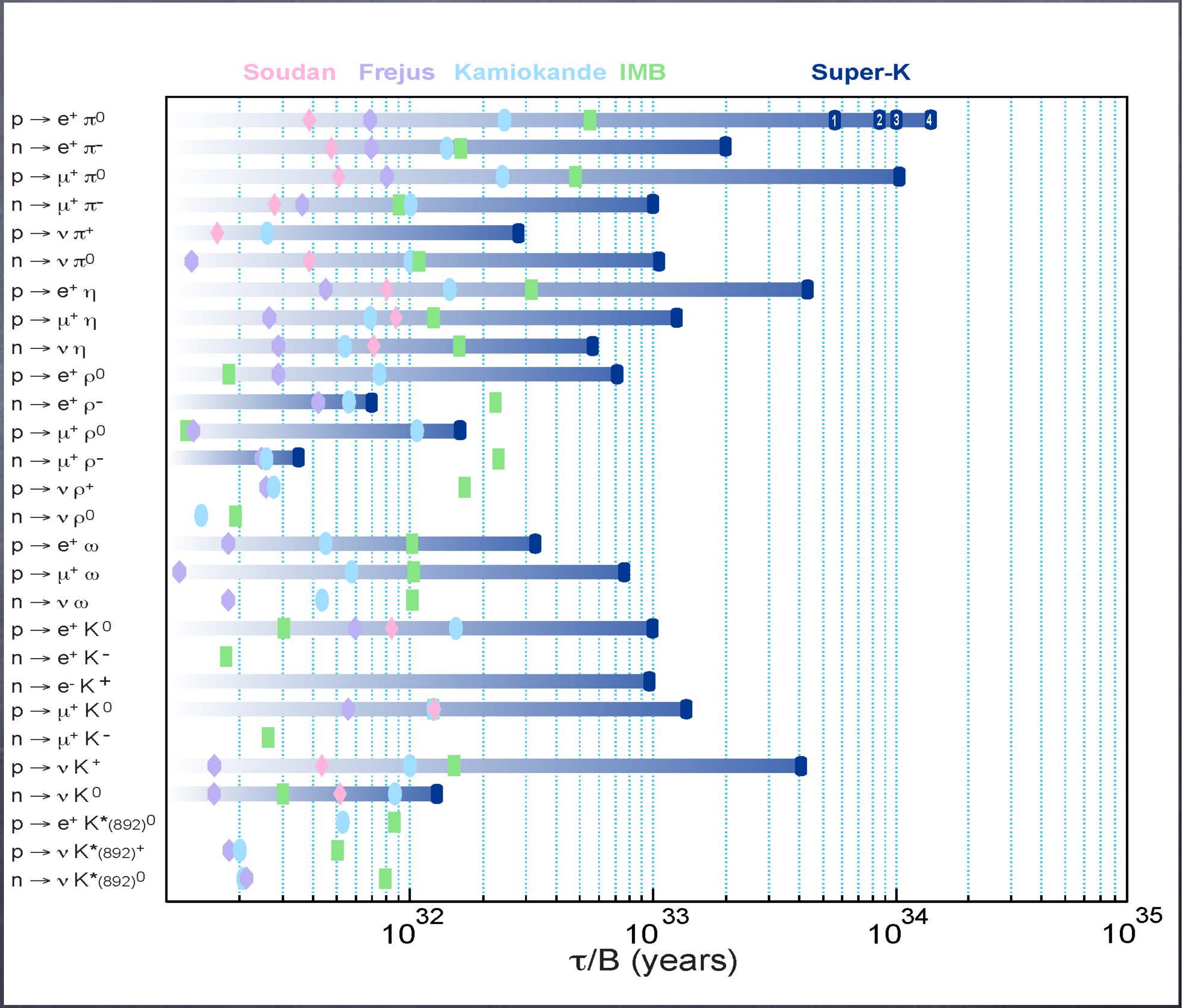
Introduction

Experimental Results

- Proton Decay:

$$\Delta B = 1, \; \Delta L = \text{odd}$$





P. Fileviez Perez

- Neutrino Oscillations: $\Delta L_e \neq 0, \Delta L_\mu \neq 0, \Delta L_\tau \neq 0$

Cosmology

Baryon Asymmetry:

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 10^{-10}$$

→

Baryon Number Violation

$$\Delta B \neq 0$$

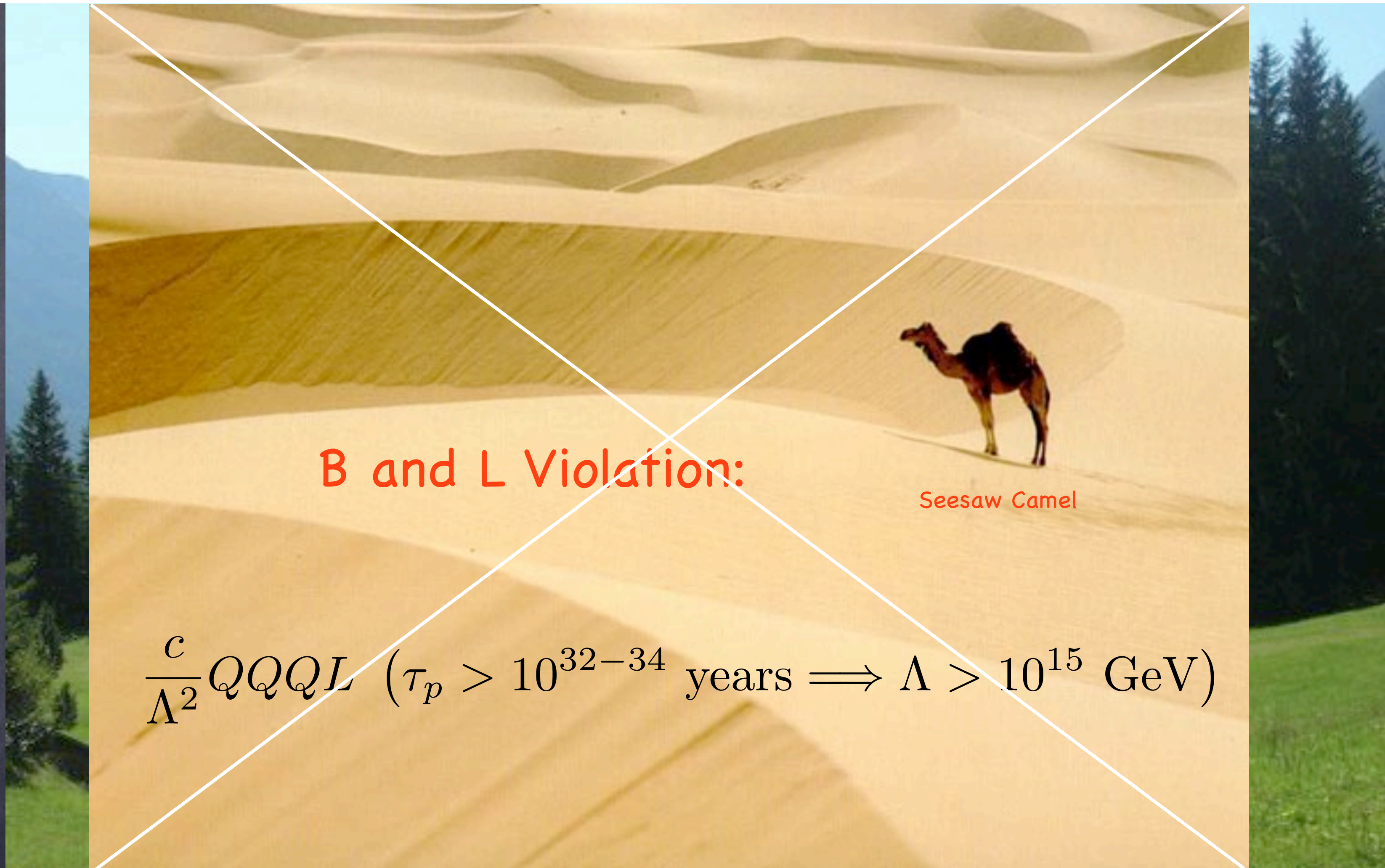
Sakharov's Condition, 1967

P. Fileviez Perez

Living without the Great Desert

Aim: Can we break B and L at the TeV scale?

LOW SCALE



HIGH SCALE

B and L Violation:

$$\frac{c}{\Lambda^2} QQQL \ \left(\tau_p > 10^{32-34} \text{ years} \Longrightarrow \Lambda > 10^{15} \text{ GeV}\right)$$

Standard Model

$\Lambda_{\text{Weak}} \sim 100$ GeV

GUTs, Strings ?

$\Lambda \sim 10^{15-19}$ GeV

B and L as Local Symmetries

Abraham Pais, 1973 (B as a Local Symmetry)

S. Rajpoot, 1987; Foot, Joshi, Lew, 1989

Carone, Murayama, 1995

Breaking Local Baryon and Lepton Numbers at the TeV Scale (NO Desert !!)

P. F. P., M.B. Wise, 2010

P. F. P., M. B. Wise, JHEP 1108(2011) 068

M. Duerr, P. F. P., M. B. Wise, arXiv:1304.0576 (Phys. Rev. Lett. 2013)

P. F. P., M. B. Wise, PRD82 (2010)011901; JHEP1108(2011)068

Breaking B and L at the TeV scale !

$$\Rightarrow \quad SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B \otimes U(1)_L$$

where: $U(1)_B$ and $\mathrm{U(1)_L}$ can be broken at the TeV Scale !

$$Q_L \sim (3, 2, 1/6, 1/3, 0), \;\; u_R \sim (3, 1, 2/3, 1/3, 0), \;\; d_R \sim (3, 1, -1/3, 1/3, 0)$$

$$\ell_L \sim (1, 2, -1/2, 0, 1), \;\; e_R \sim (1, 1, -1, 0, 1), \nu_R \sim (1, 1, 0, 0, 1)$$

How to define an anomaly free theory ?

P. Fileviez Perez

Anomalies Cancellation

Baryonic Anomalies: $\mathcal{A}_1\left(SU(3)^2 \otimes U(1)_B\right)$, $\mathcal{A}_2\left(SU(2)^2 \otimes U(1)_B\right)$,

$\mathcal{A}_3\left(U(1)_Y^2 \otimes U(1)_B\right)$, $\mathcal{A}_4\left(U(1)_Y \otimes U(1)_B^2\right)$,

$\mathcal{A}_5\left(U(1)_B\right)$, $\mathcal{A}_6\left(U(1)_B^3\right)$,

Leptonic Anomalies: $\mathcal{A}_7\left(SU(3)^2 \otimes U(1)_L\right)$, $\mathcal{A}_8\left(SU(2)^2 \otimes U(1)_L\right)$,

$\mathcal{A}_9\left(U(1)_Y^2 \otimes U(1)_L\right)$, $\mathcal{A}_{10}\left(U(1)_Y \otimes U(1)_L^2\right)$,

$\mathcal{A}_{11}\left(U(1)_L\right)$, $\mathcal{A}_{12}\left(U(1)_L^3\right)$,

Mixed: $\mathcal{A}_{13}\left(U(1)_B^2 \otimes U(1)_L\right)$, $\mathcal{A}_{14}\left(U(1)_L^2 \otimes U(1)_B\right)$,

$\mathcal{A}_{15}\left(U(1)_Y \otimes U(1)_L \otimes U(1)_B\right)$,

In the SM: $\mathcal{A}_2 = -\mathcal{A}_3 = 3/2 \qquad \mathcal{A}_8 = -\mathcal{A}_9 = 3/2$

P. Fileviez Perez

P. F. P., M. B. Wise, PRD82 (2010)011901; JHEP1108(2011)068

Possible Solutions

- Sequential Family (B=-1, L=-3)
- Mirror Family (B=1, L=3)
- Vector-like Family with Seesaw

Now they are in disagreement with LHC Constraints !

What about Fermionic Leptoquarks ?

P. Fileviez Perez

M. Duerr, P. F. P., M. B. Wise, arXiv:1304.0576 (Phys. Rev. Lett. 2013)

One can define an anomaly free theory using the Fermionic Lepto-quarks:

$$\Psi_L \sim (1, 2, -1/2, B_1, L_1), \quad \Psi_R \sim (1, 2, -1/2, B_2, L_2)$$

$$\eta_R \sim (1, 1, -1, B_1, L_1), \quad \eta_L \sim (1, 1, -1, B_2, L_2)$$

$$\chi_R \sim (1, 1, 0, B_1, L_1), \quad \chi_L \sim (1, 1, 0, B_2, L_2)$$

$$B_1 - B_2 = -3, \quad L_1 - L_2 = -3$$

They can have vector-like masses and cancel all anomalies !

P. Fileviez Perez

M. Duerr, P. F. P., M. B. Wise, arXiv:1304.0576 (Phys. Rev. Lett. 2013)

Interactions:

$$-\mathcal{L} \supset h_1 \overline{\Psi}_L H \eta_R + h_2 \overline{\Psi}_L \tilde{H} \chi_R + h_3 \overline{\Psi}_R H \eta_L + h_4 \overline{\Psi}_R \tilde{H} \chi_L$$
$$+ \lambda_1 \overline{\Psi}_L \Psi_R S_{BL} + \lambda_2 \overline{\eta}_R \eta_L S_{BL} + \lambda_3 \overline{\chi}_R \chi_L S_{BL}$$
$$+ a_1 \chi_L \chi_L S_{BL} + a_2 \chi_R \chi_R S_{BL}^\dagger + \text{h.c.}$$

$$B_1 = -B_2 = -3/2$$
$$L_1 = -L_2 = -3/2$$

$$-\mathcal{L}_\nu = Y_\nu \overline{\ell}_L \tilde{H} \nu_R + \frac{\lambda_R}{2} \nu_R \nu_R S_L + \text{h.c.}$$

Higgses: $S_{BL} \sim (1, 1, 0, -3, -3), \quad S_L \sim (1, 1, 0, 0, -2)$

P. Fileviez Perez

M. Duerr, P. F. P., M. B. Wise, arXiv:1304.0576 (Phys. Rev. Lett. 2013)

Some Features:

Symmetry Breaking: $S_{BL} \sim (1, 1, 0, -3, -3), \quad S_L \sim (1, 1, 0, 0, -2)$

$$\Delta B = \pm 3, \Delta L = \pm 2, \Delta L = \pm 3$$

NO Proton Decay !

Dark Matter: Ψ^0_{LF} can be a cold dark matter candidate !

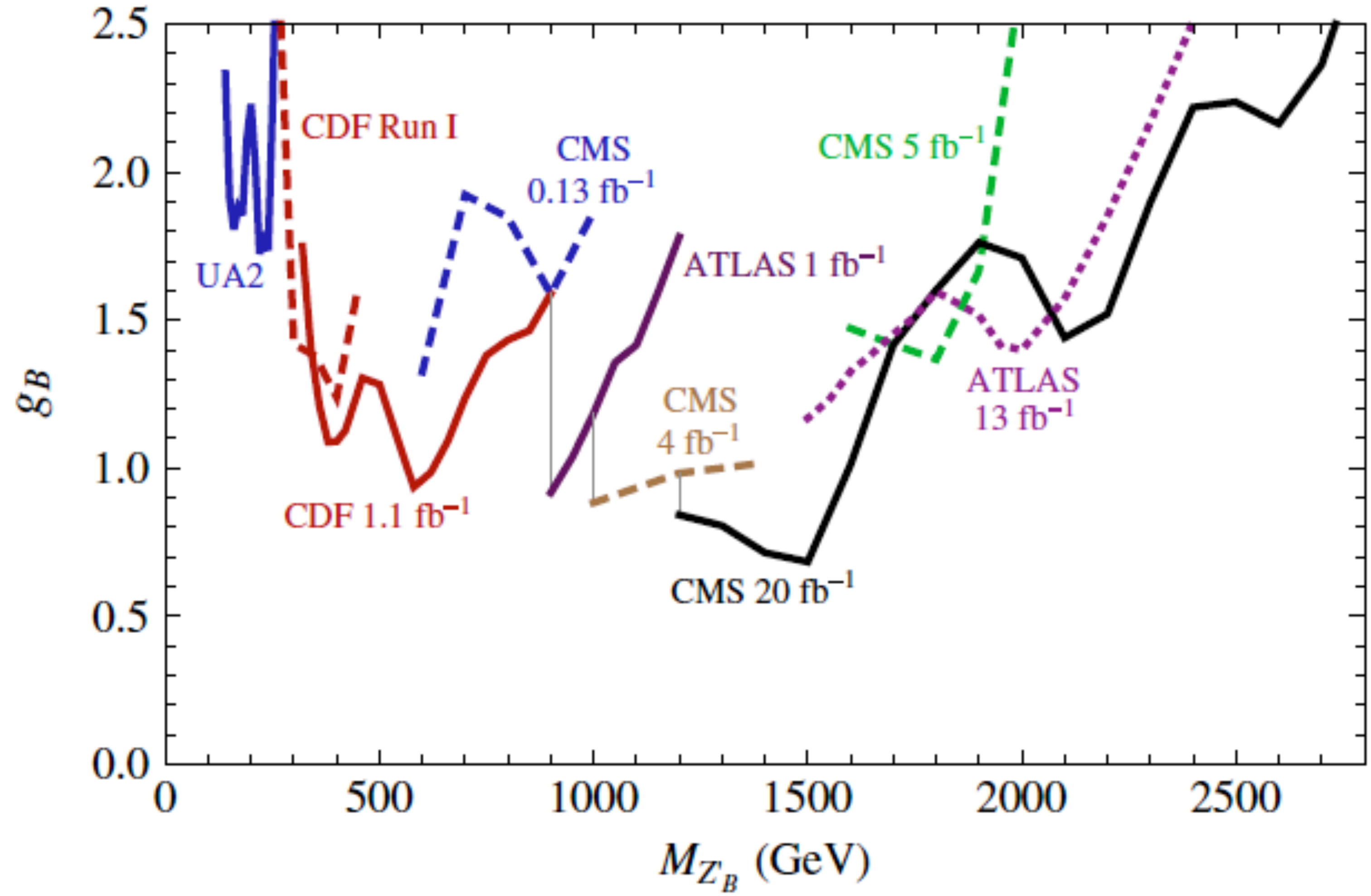
NO extra Flavour violation !

New Gauge Bosons: Z_L, Z_B

Dobrescu, Yu, PRD 88, 035021 (2013)

An, Hou, Wang, DU 2 (2013) 50

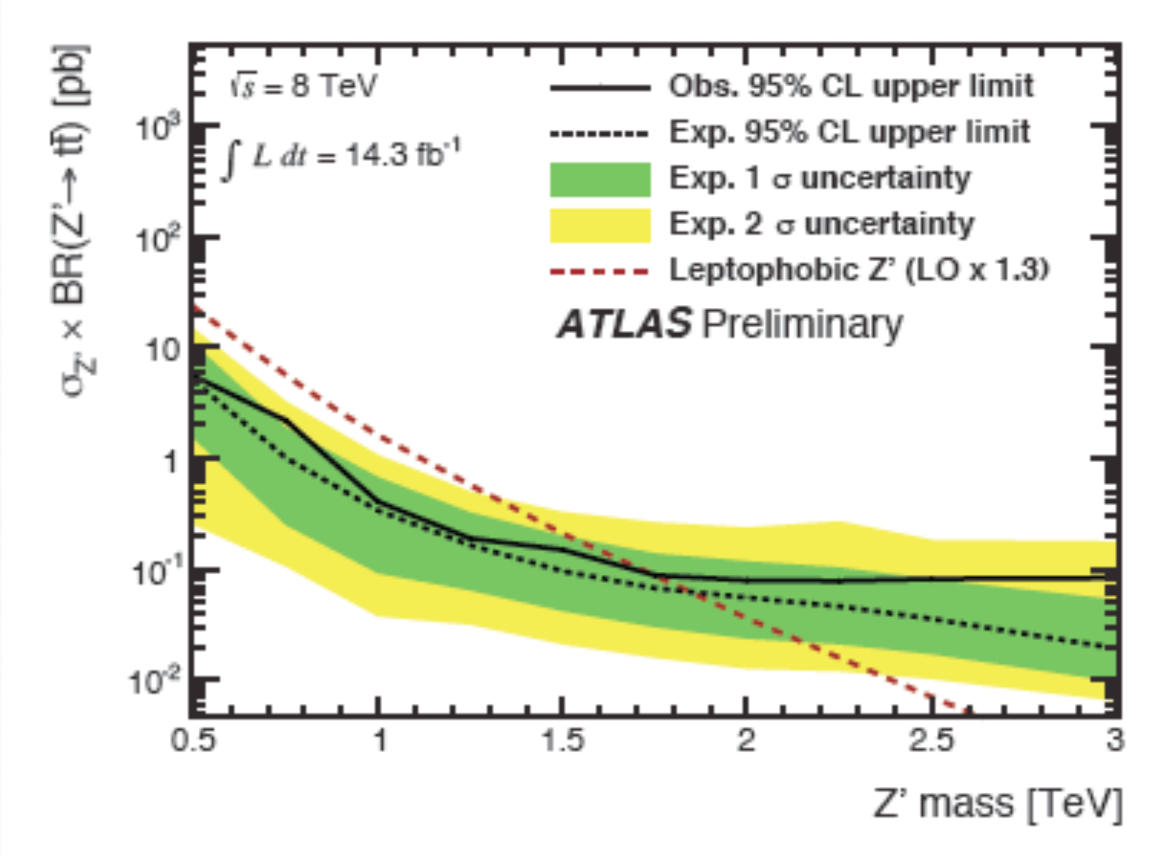
Bounds on the Baryonic Breaking Scale !



P. Fileviez Perez

ATLAS-CONF-2013-052

May 13, 2013



(a) Z' upper cross section limits.

P. Fileviez Perez

B and L Violation

$$S_{BL} \sim (1,1,0,-3,-3), \quad S_L \sim (1,1,0,0,-2)$$

$$\mathcal{L} \supset \frac{c}{\Lambda^{15}} (Q_L Q_L Q_L \ell_L)^3 S_{BL}$$

$\Delta B = \Delta L = \pm 3$

$$p + p + p \;\to\; e^+ \; e^+ \; e^+$$

$$n + n + n \;\to\; \bar{\nu}\bar{\nu}\bar{\nu}$$

$$p + p + n \;\to\; e^+ \; e^+ \; \bar{\nu}$$

$$p + n + n \;\to\; e^+ \; \bar{\nu} \; \bar{\nu}$$

P. Fileviez Perez

P. F. P., H. H. Patel, 1311.6472

Baryon Asymmetry and Dark Matter

Simple Scenario: $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_B$

Interactions:

$$-\mathcal{L} \supset Y_1 \bar{\Psi}_L H \eta_R + Y_2 \bar{\Psi}_R H \eta_L + Y_3 \bar{\Psi}_L \tilde{H} \chi_R + Y_4 \bar{\Psi}_R \tilde{H} \chi_L + \lambda_1 \bar{\Psi}_L \Psi_R S_B + \lambda_2 \bar{\eta}_R \eta_L S_B + \lambda_3 \bar{\chi}_R \chi_L S_B + \text{h.c.}$$

Symmetries: $(B-L)_{SM}$

$$\Psi_{L,R} \to e^{i\eta} \Psi_{L,R}, \quad \eta_{L,R} \to e^{i\eta} \eta_{L,R}, \quad \chi_{L,R} \to e^{i\eta} \chi_{L,R}.$$

Sphalerons:

$$(QQQL)^3 \bar{\psi}_R \psi_L .$$

$$3(3\mu_{u_L} + \mu_{e_L}) + \mu_{\Psi_L} - \mu_{\Psi_R} = 0.$$

P. F. P., H. H. Patel, 1311.6472

$$\Delta(B-L)_{\rm SM} = \frac{15}{4\pi^2 g_* T}(12\mu_{u_L} - 9\mu_{e_L} + 3\mu_0),$$
$$\Delta\eta = \frac{15}{4\pi^2 g_* T}(4\mu_{\Psi_L} + 4\mu_{\Psi_R}),$$
$$0 = 6\mu_{u_L} + (2B_1 - 3)\mu_{\Psi_L} + (2B_2 + 3)\mu_{\Psi_R},$$

$$0 = 3\mu_{u_L} + 8\mu_0 - 3\mu_{e_L} - \mu_{\Psi_L} - \mu_{\Psi_R},$$
$$0 = 9\mu_{u_L} + 3\mu_{e_L} + \mu_{\Psi_L} - \mu_{\Psi_R}.$$

$$B_T = 0$$

$$Q_{em} = 0$$

$$B_f^{SM} = \frac{15}{4\pi^2 g_* T}(12\mu_{u_L}) = C_1\Delta(B-L)_{SM} + C_2\Delta\eta,$$

$$C_1 = \frac{32}{99} \quad \text{and} \quad C_2 = \frac{(15-14B_2)}{198}.$$

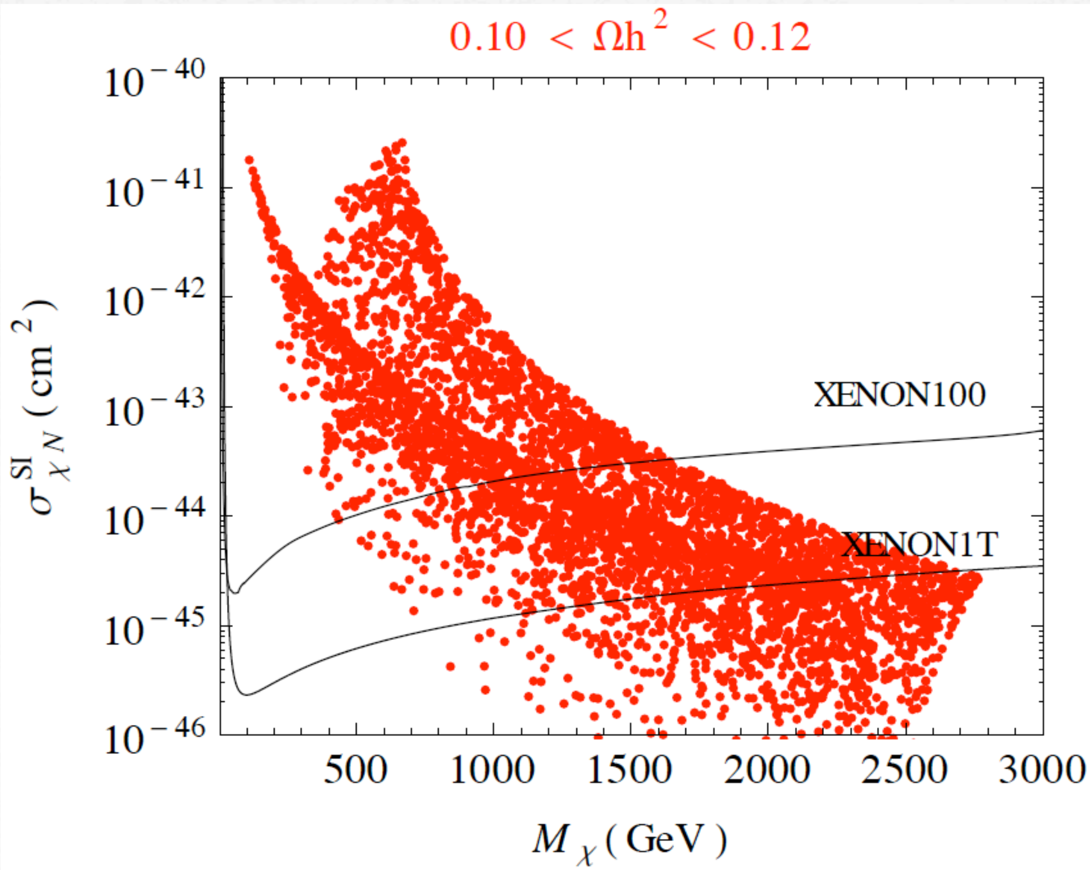
$$|n_\chi - n_{\bar\chi}| \leq n_{DM}$$

$$\Rightarrow$$

$$M_\chi \leq \frac{\Omega_{DM} C_2 M_p}{|\Omega_B - C_1 M_p \Delta n_{(B-L)_{SM}}|}.$$

M. Duerr, P. F. P., arXiv: 1309.3970

Baryonic Dark Matter



$g_B, \ M_{Z_B}, \ M_\chi \ \& \ B$

Annihilation:

$\bar{\chi}\chi \to Z_B \to \bar{q}q$

Direct Detection:

$\chi N \to Z_B \to \chi N$

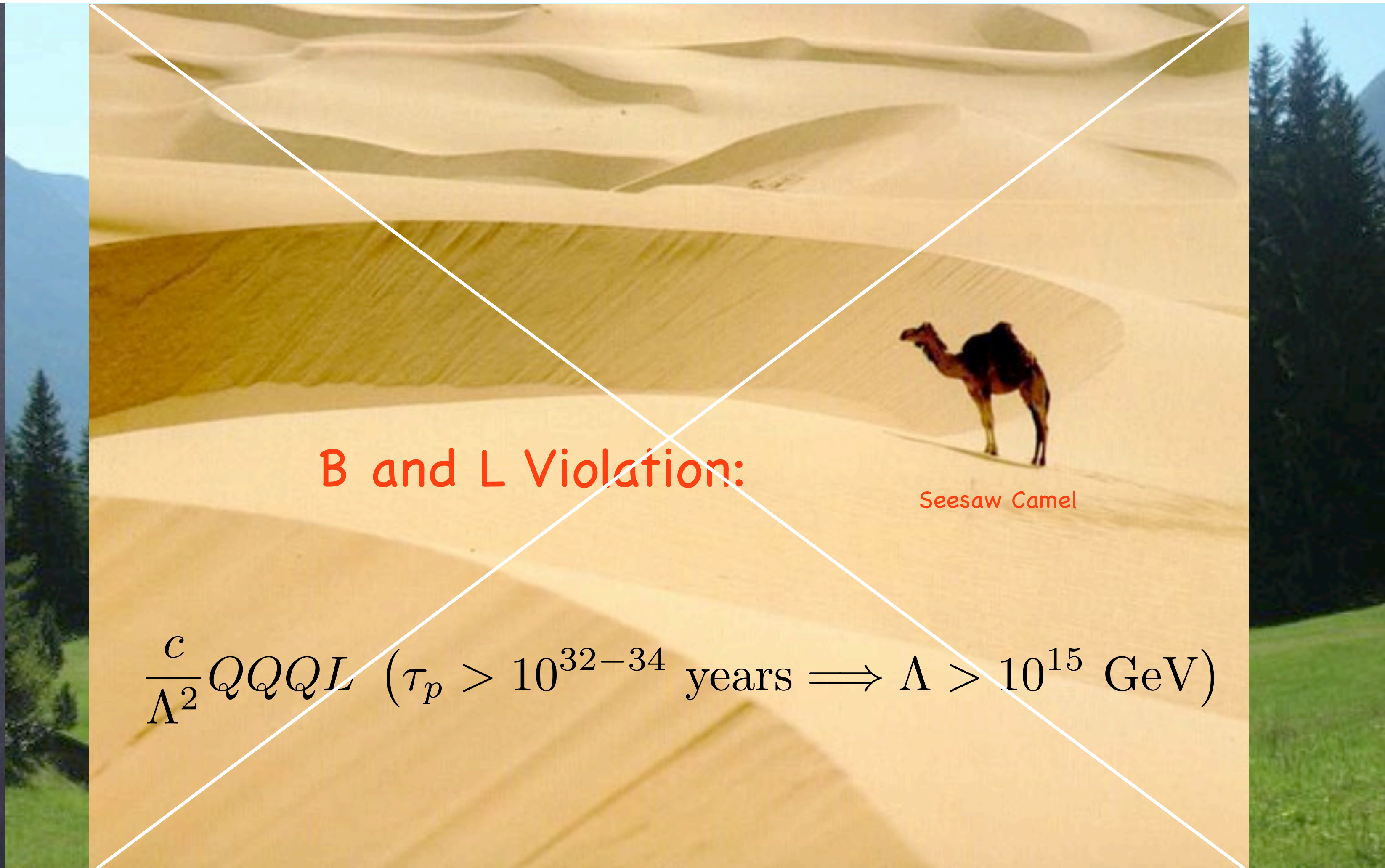
LHC Signatures:

$pp \to Z_B \to \bar{\chi}\chi, \ \bar{q}q, \ \bar{\chi}\chi j, ..$

Towards Unification and Neutrino Masses

Aim: Can we break B and L at the TeV scale?

LOW SCALE



B and L Violation:

$$\frac{c}{\Lambda^2} QQQL \;\; \left(\tau_p > 10^{32-34} \text{ years} \Longrightarrow \Lambda > 10^{15} \text{ GeV}\right)$$

HIGH SCALE

Left-Right Symmetry

GUTs, Strings ?

$\Lambda \sim 10^{15-19}$ GeV

Pati, Salam; Pati, Mohapatra; Senjanovic, Mohapatra

Left-Right Symmetry

$$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$$

- Connection between Neutrino Masses and the Scale of Parity Violation

- Minimal Model has Type I and Type II Seesaw Mechanisms

- Doorway to SO(10) Unification

- If the scale is low one has 'exotic' signals at the LHC

He, Rajpoot, 1990

M. Duerr, P. F. P., M. Lindner, 1306.0568 (PRD)

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_B \otimes U(1)_L$$

SM Fermions:

$$Q_L \sim (2,1,1/3,0),\ Q_R \sim (1,2,1/3,0),\ \ell_L \sim (2,1,0,1),\ \ell_R \sim (1,2,,0,1)$$

Anomalies:

$$\mathcal{A}_1 \left(SU(2)_L^2 \otimes U(1)_B\right) = 3/2,$$
$$\mathcal{A}_2 \left(SU(2)_L^2 \otimes U(1)_L\right) = 3/2,$$
$$\mathcal{A}_3 \left(SU(2)_R^2 \otimes U(1)_B\right) = -3/2,$$
$$\mathcal{A}_4 \left(SU(2)_R^2 \otimes U(1)_L\right) = -3/2.$$

The Simplest Solution is ....

P. Fileviez Perez

P. F. P., JHEP 03 (2009) 142

M. Duerr, P. F. P., M. Lindner, 1306.0568 (PRD)

Type III Seesaw Fields

$$\rho_L \sim (3, 1, -3/4, -3/4), \ \& \ \rho_R \sim (1, 3, -3/4, -3/4),$$

The theory is anomaly free !

Relevant Interactions:

$$\begin{aligned} -\mathcal{L} \supset \ & \bar{\ell}_L \left(Y_3 \Phi + Y_4 \tilde{\Phi} \right) \ell_R \\ + \ & \lambda_D \left(\ell_L^T C i \sigma_2 \rho_L H_L \ + \ \ell_R^T C i \sigma_2 \rho_R H_R \right) \\ + \ & \lambda_\rho \ \mathrm{Tr} \left(\rho_L^T C \rho_L + \rho_R^T C \rho_R \right) S_{BL} \ + \ \mathrm{h.c.}, \end{aligned}$$

P. Fileviez Perez

M. Duerr, P. F. P., M. Lindner, 1306.0568 (PRD)

Type III Seesaw and Left-Right Symmetry

Parity Violation !

$$v_L \ll v_R$$

$$M^{III}_{\nu_L} \ll M^{III}_{\nu_R}.$$

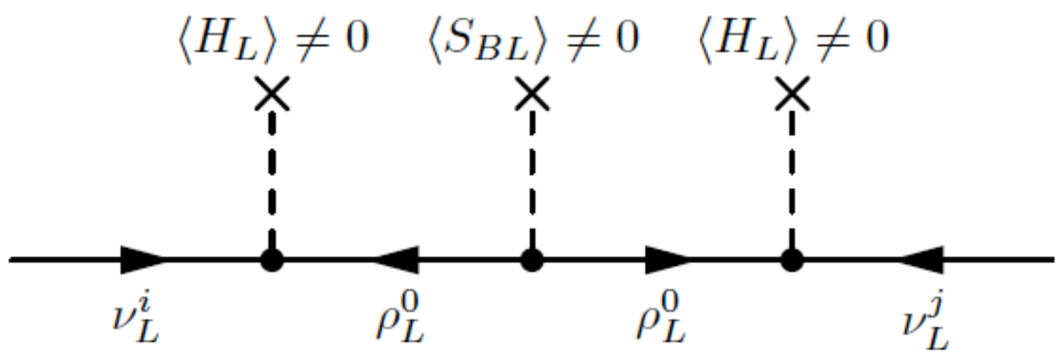


FIG. 1: Type III seesaw for the left-handed neutrinos.

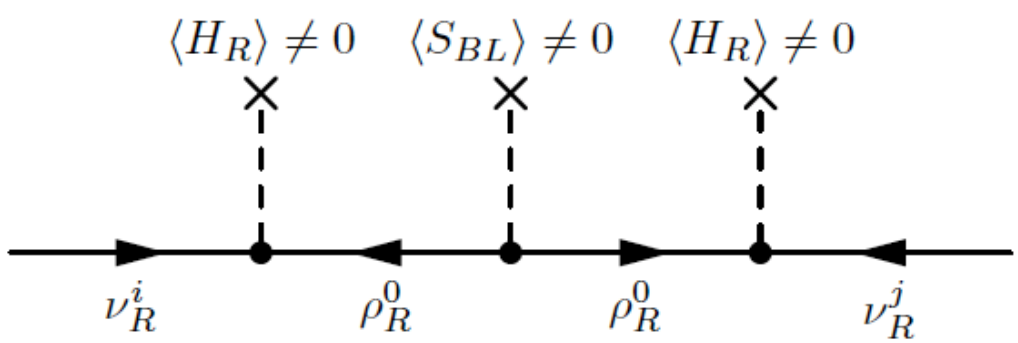


FIG. 2: Type III seesaw for the right-handed neutrinos.

M. Duerr, P. F. P., M. Lindner, 1306.0568 (PRD)

Neutrino Masses

$$-\mathcal{L}_\nu = \tilde{M}_\nu^D \overline{\nu}_L \nu_R - \frac{1}{2} M_{\nu_L}^{III} \nu_L^T C \nu_L - \frac{1}{2} M_R \nu_R^{3T} C \nu_R^3 + \text{h.c.},$$

3 + 2 System

$$-\mathcal{L}_\nu = -\frac{1}{2} M_{\nu_L}^{LL} \nu_L^T C \nu_L + \left(\tilde{M}_\nu^D\right)^{i\alpha} \overline{\nu}_L^i \nu_R^\alpha + \text{h.c.},$$

$$\mathcal{M}_\nu^{3+2} = \begin{pmatrix} 0 & 0 & 0 & m_D^1 & m_D^2 \\ 0 & m_1 & 0 & m_D^3 & m_D^4 \\ 0 & 0 & m_2 & m_D^5 & m_D^6 \\ m_D^1 & m_D^3 & m_D^5 & 0 & 0 \\ m_D^2 & m_D^4 & m_D^6 & 0 & 0 \end{pmatrix}.$$

The theory predicts two light sterile neutrinos !!

P. Fileviez Perez

Summary

- The Desert Hypothesis plays a major role in our view of the relation between the physics at the low and high scales. However, this picture can be WRONG !

- One can define a consistent theory where B and L are local symmetries broken at the low scale in agreement with the experiments and there is no need to postulate the Great Desert. One has a simple theory for dark matter (and baryogenesis) which can be tested at the LHC.

- Local B and L Symmetries together with Left-Right Symmetry requires Type III Seesaw. The Minimal Model predicts light sterile neutrinos.

$$\hat{L}\hat{H}_u, \ \hat{Q}\hat{L}\hat{d}^c, \ \hat{L}\hat{L}\hat{e}^c, \text{ and } \hat{u}^c\hat{d}^c\hat{d}^c,$$

For the application to Supersymmetry see:

P. F. P., M. B. Wise, JHEP 08 (2011) 068

J. M. Arnold, P. F. P., B. Fornal, S. Spinner
1310.7052

MERCI !