The Role of Symmetries in Cosmology

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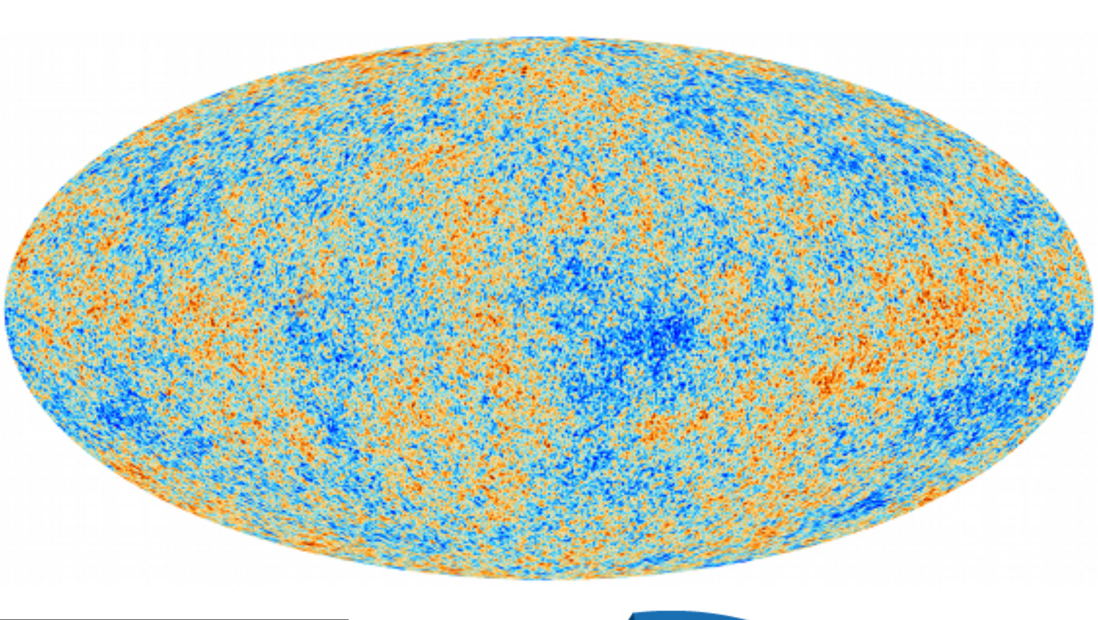
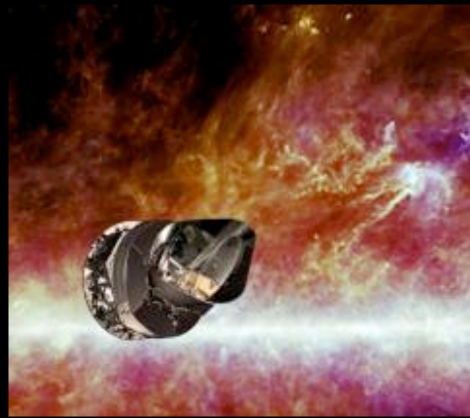
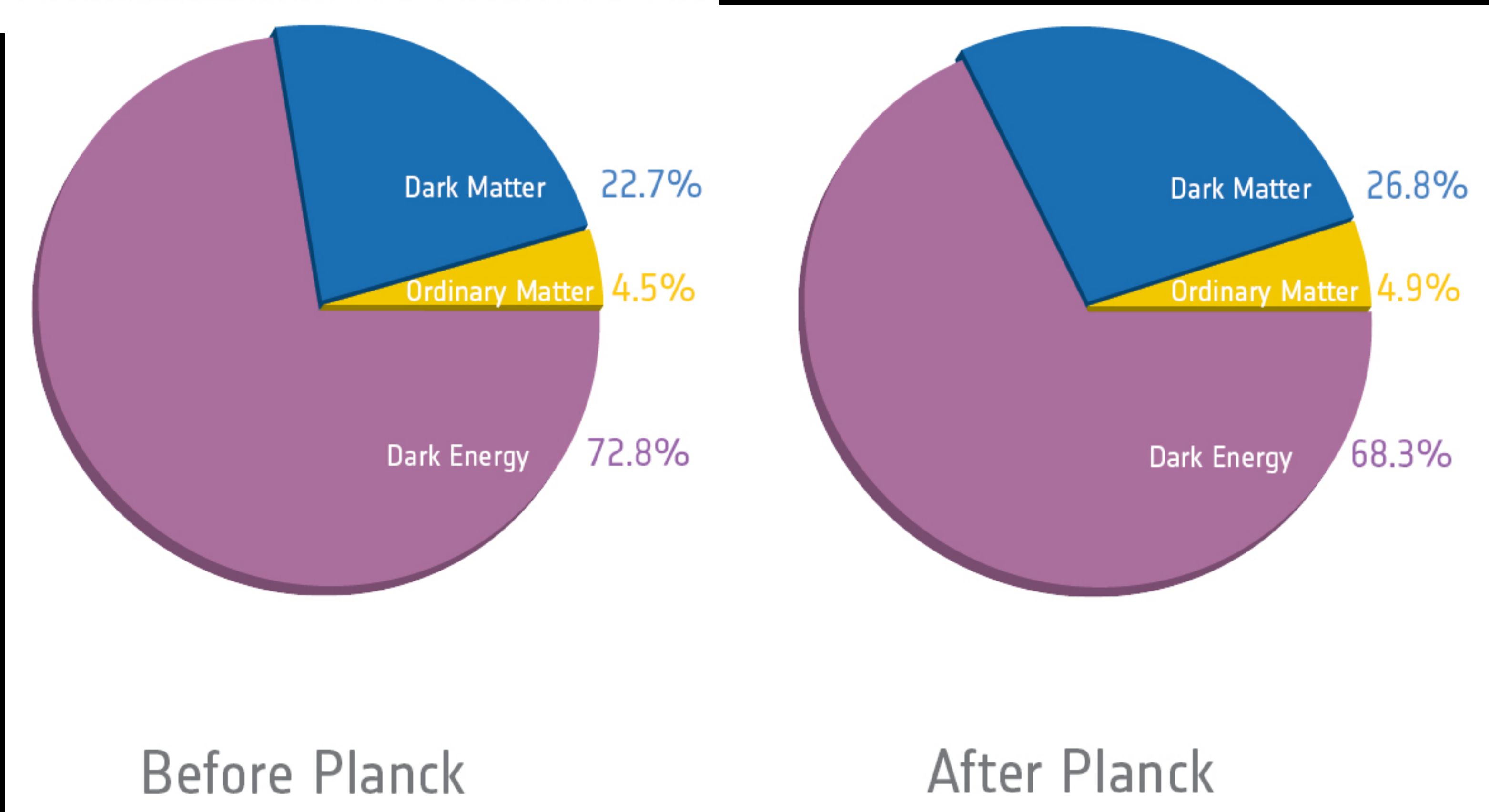
Plan of the talk

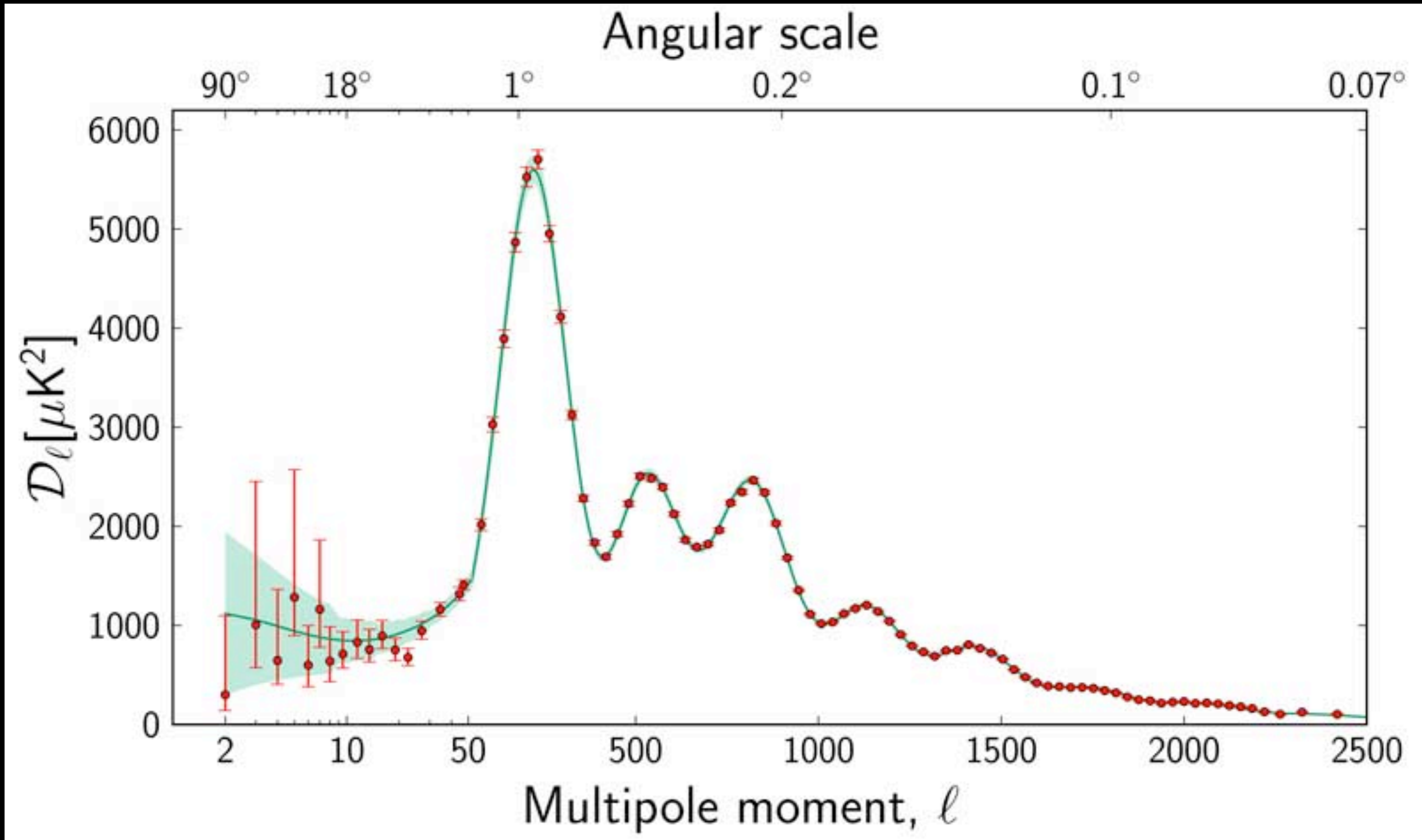
- Short introduction on cosmological perturbations during inflation
- The role of symmetries in the general knowledge we may acquire about the cosmological perturbations



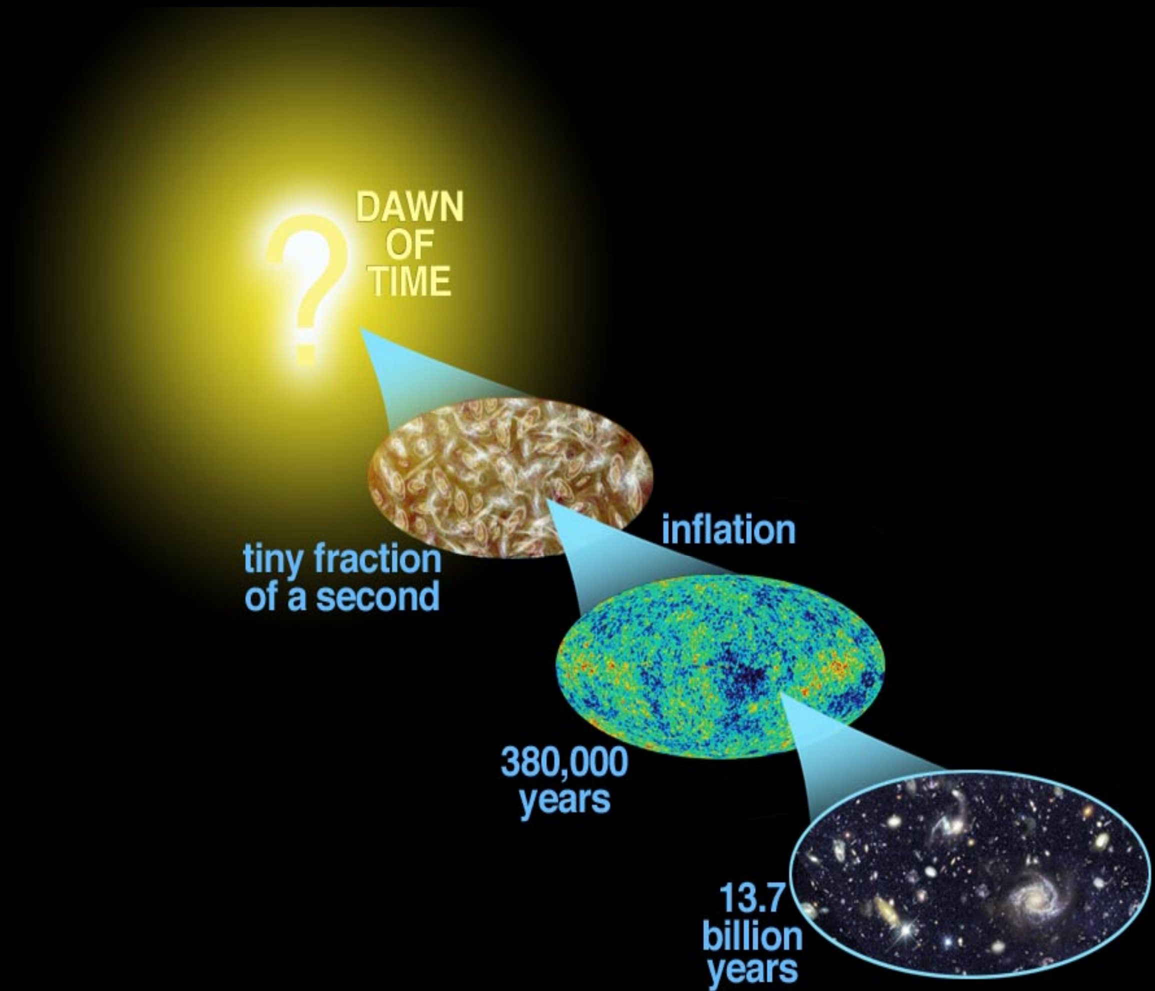



Dark Matter
22.7%
Ordinary Matter
4.5%
Dark Energy
72.8%
Before Planck
Dark Matter
26.8%
Ordinary Matter
4.9%
Dark Energy
68.3%
After Planck

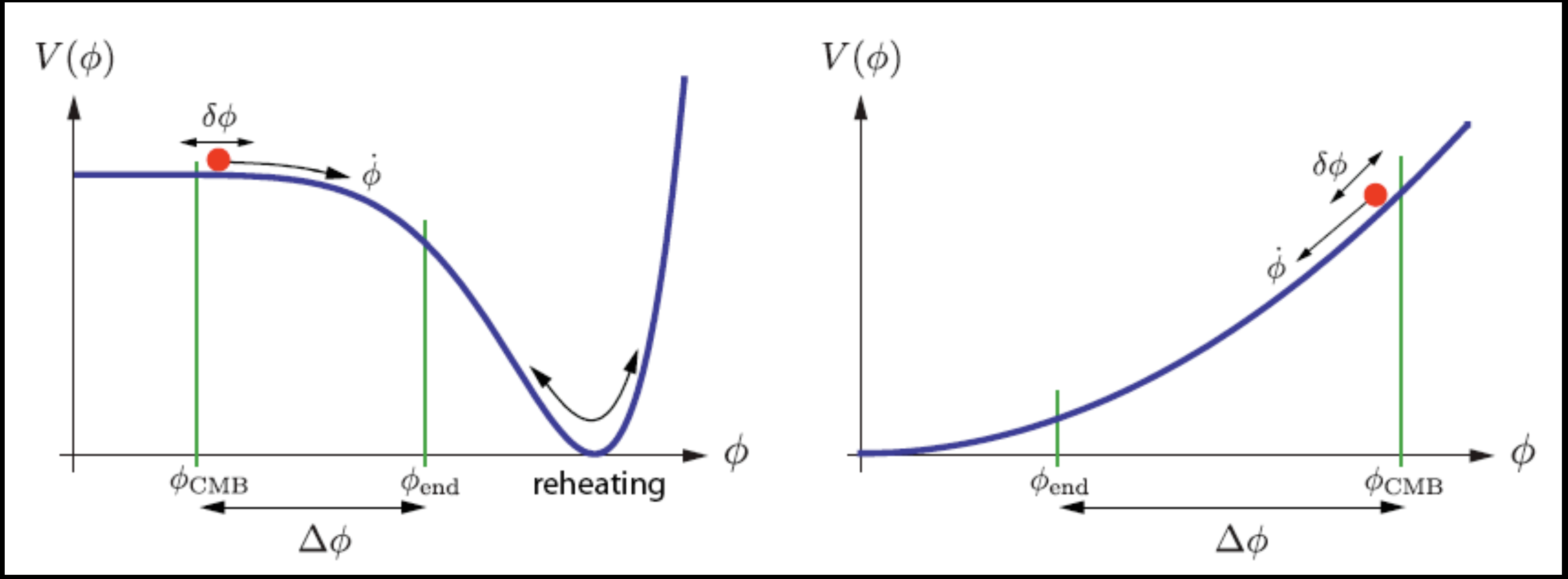
The ΛCDM model matches Planck data



The Inflationary Cosmology



The Inflationary Cosmology



$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\mathrm{d}\vec{x}^2 \quad \Rightarrow \quad a(t) \sim e^{Ht}$$

$$H = \dot{a}/a$$

$$-\dot{H} \ll H^2$$

All massless scalar fields are quantum-mechanically excited during Inflation

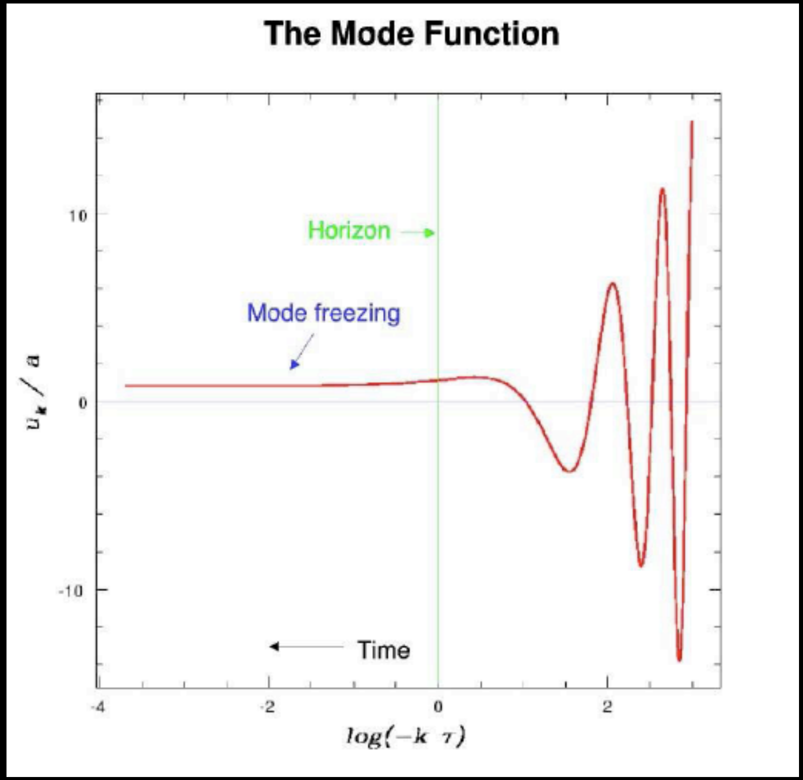
$$\begin{aligned}\sigma(\mathbf{x},\tau) &= \sigma_0(\tau)+\delta\sigma(\mathbf{x},\tau),\\ u_k(\tau) &= a(\tau)\delta\sigma_k(\tau),\\ \mathrm{d}\tau &= \frac{\mathrm{d}t}{a}\end{aligned}$$

$$\mathrm{d}s^2 = \frac{1}{H^2\tau^2}(\mathrm{d}\tau^2 - \mathrm{d}\vec{x}^2)$$

$$u_k'' + \left(k^2 - \frac{a''}{a}\right)u_k = 0$$

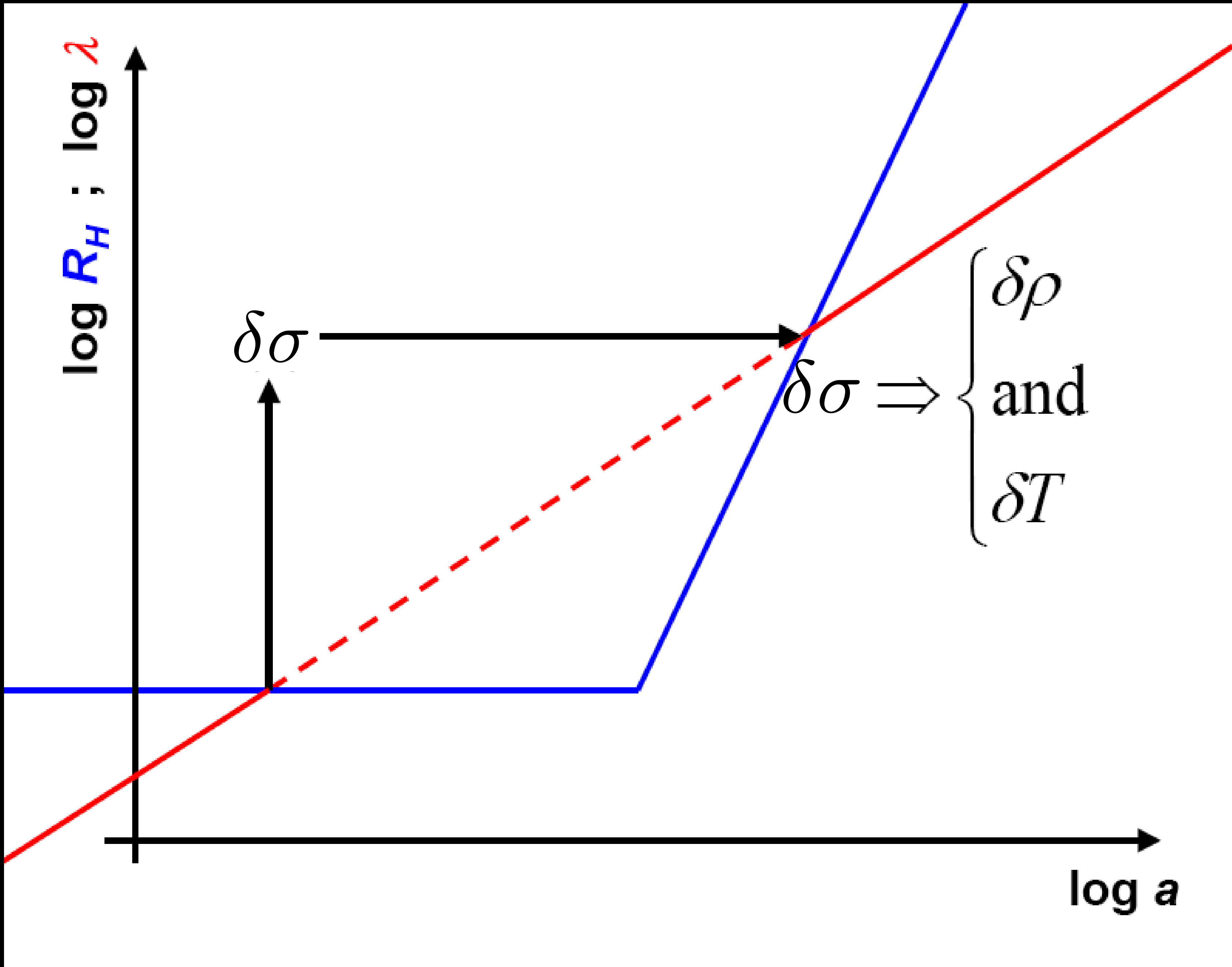
Oscillator with time-dependent frequency

Any light scalar field is quantum mechanically excited during inflation with a scale-invariant power spectrum



$$\mathcal{P}_\sigma = \frac{k^3}{2\pi^2}\left|\delta\sigma_k\right|^2 = \left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{n-1}$$

$$n \simeq 1 + \mathcal{O}(10^{-2})$$


log R_H ; log λ
$\delta\sigma$
$\delta\sigma \Rightarrow \begin{cases} \delta\rho \\ \text{and} \\ \delta T \end{cases}$
log a

What do we really know about the inflationary pertubations?

- Perturbations are of adiabatic/ curvature type
- They are nearly Gaussian

Pertubations are of the adiabatic/curvature type

After inflation all components have the same gauge-invariant comoving curvature perturbation

$$\zeta_i = \psi + H\frac{\delta\rho_i}{\dot{\rho}_i}$$

$$\zeta_m = \zeta_\gamma = \cdots = \zeta_\nu$$

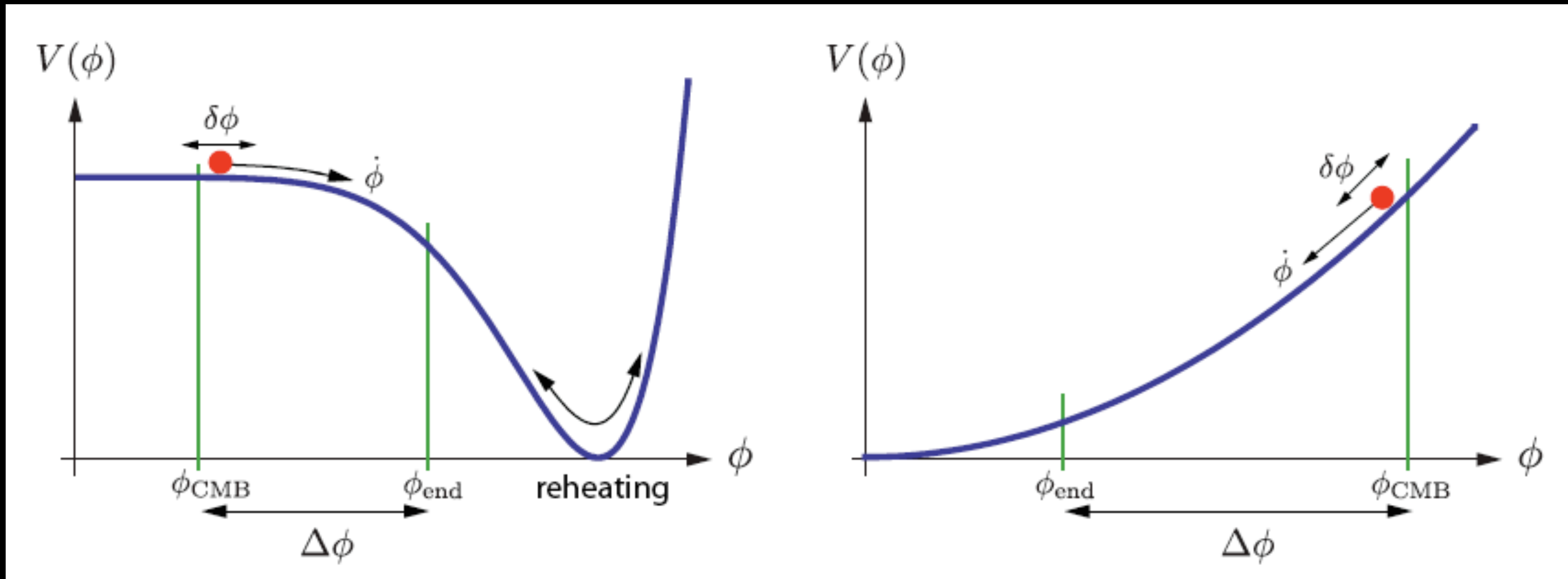
Total curvature perturbation

$$\zeta = \psi + H\frac{\delta\rho}{\dot{\rho}}$$

$$\dot{\zeta} = -\frac{H}{(\rho + P)}\delta P_{\rm nonad}$$

$$\frac{\Delta T}{T} = \frac{1}{5}\zeta(\vec{x})$$ on large scales

Single-field models of inflation



$$\mathcal{P}_\zeta = \frac{1}{2}\left(\frac{H}{2\pi M_{\rm Pl}\epsilon^{1/2}}\right)^2\left(\frac{k}{aH}\right)^{n_\zeta - 1}$$

$$n_\zeta = 1 + 2\eta - 6\epsilon$$

$$\epsilon = -\frac{\dot{H}}{H^2}\,,\eta = \frac{\dot{\epsilon}}{H\epsilon}$$

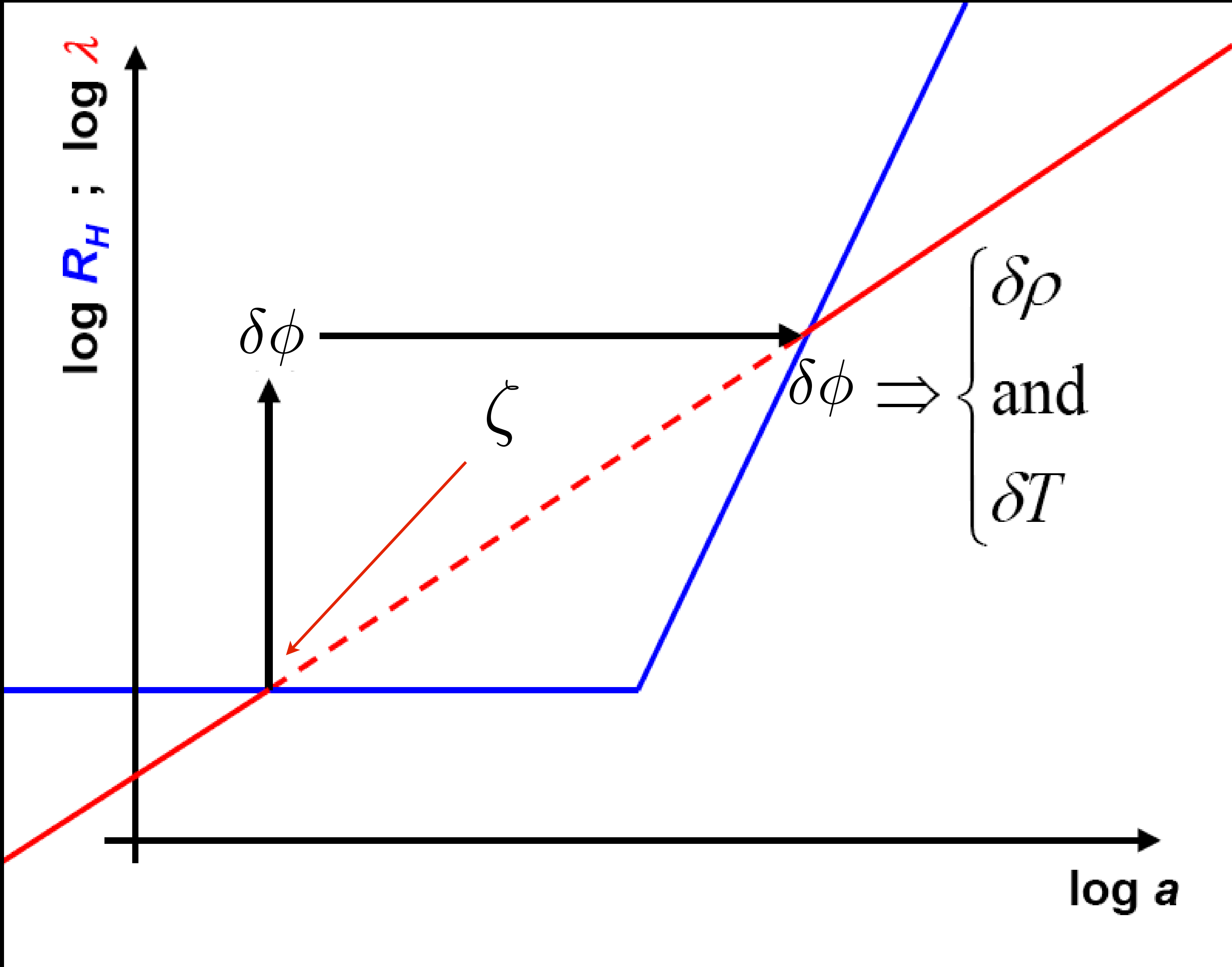
Single-field models of inflation

$$\zeta = \psi + H\frac{\delta\rho}{\dot{\rho}}$$

$$\boxed{\dot{\zeta} = 0}$$

$$P = P(\rho) \text{ and } \delta P_{\rm nonad} = 0$$

The fluid is adiabatic during inflation: the adiabatic perturbation is generated at Hubble crossing during inflation



log R_H ; log λ
$\delta\phi$
ζ
$\delta\phi \Rightarrow \begin{cases} \delta\rho \\ \text{and} \\ \delta T \end{cases}$
log a

Multi-field models of inflation

$$\zeta = \psi + H\frac{\delta\rho}{\dot{\rho}}$$

$$\dot{\zeta} \neq 0$$

$$P \neq P(\rho) \text{ and } \delta P_{\mathrm{nonad}} \neq 0$$

The fluid is not adiabatic during inflation: the adiabatic perturbation is generated after the end of inflation when the extra degree of freedom finally decays into radiation

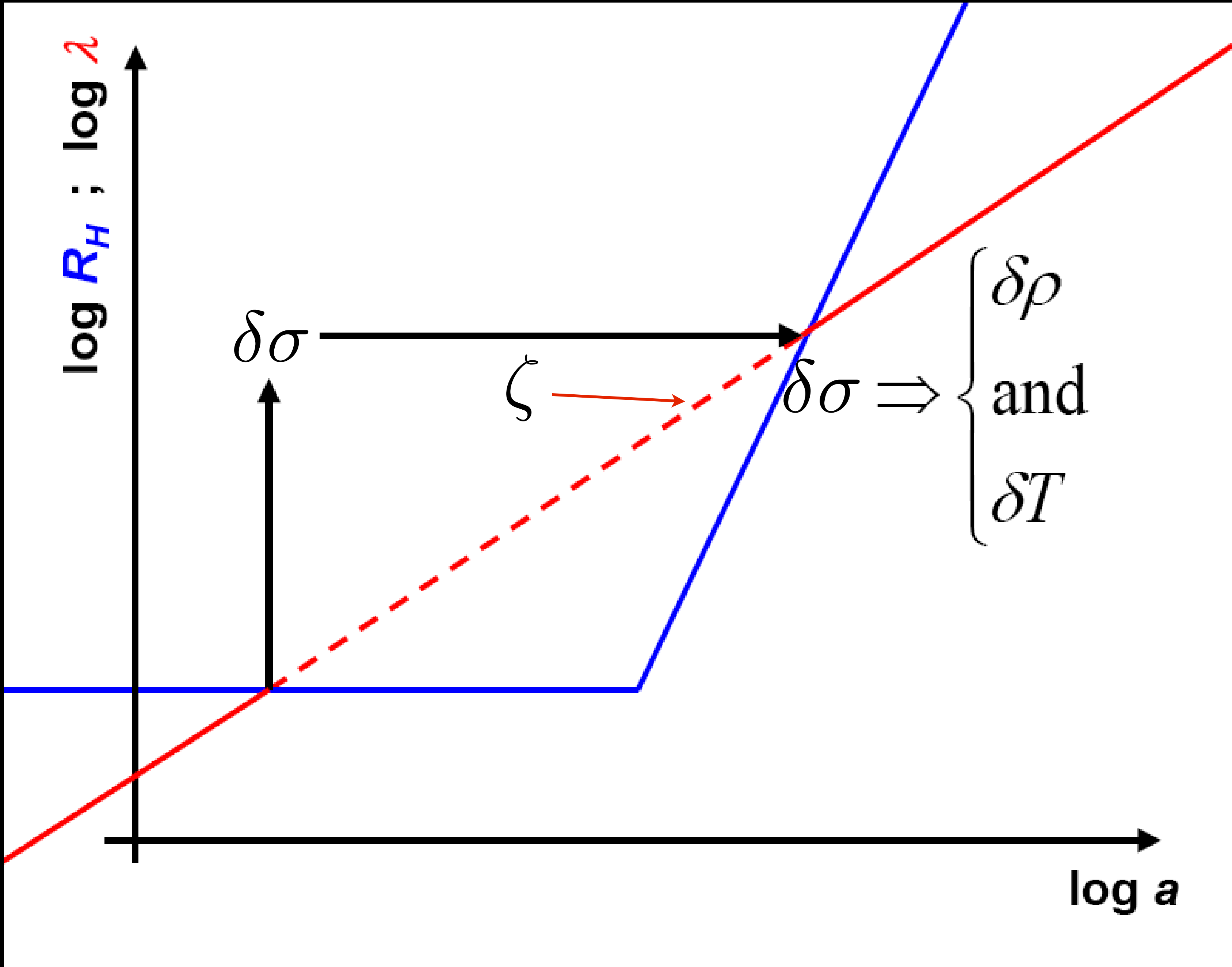
Example: modulated decay rate

The decay rate of the inflaton may depend on a light field: its large scale fluctuations lead to fluctuations of the reheating temperature

$$t \sim H^{-1} \sim \Gamma^{-1} \Rightarrow H \sim \Gamma \sim T_{\rm RH}^2/M_{\rm Pl}$$

$$\text{if } \Gamma = \Gamma(\sigma), \text{ then}$$

$$\frac{\delta T_{\rm RH}}{T_{\rm RH}} \sim \frac{\delta\Gamma}{\Gamma} \sim \left(\frac{{\rm d}\ln\Gamma}{{\rm d}\ln\sigma}\right)\left(\frac{\delta\sigma}{\sigma}\right)$$


log R_H ; log λ
$\delta\sigma$
ζ
$\delta\sigma \Rightarrow \begin{cases} \delta\rho \\ \text{and} \\ \delta T \end{cases}$
log a

How can we distinguish among these two basic scenarios?

Non-Gaussianity
is the key ingredient

Gaussian ⟺ free (i.e. non-interacting) field, linear theory

- Collection of independent harmonic oscillators (no mode-mode coupling)

- NG requires more than linear theory

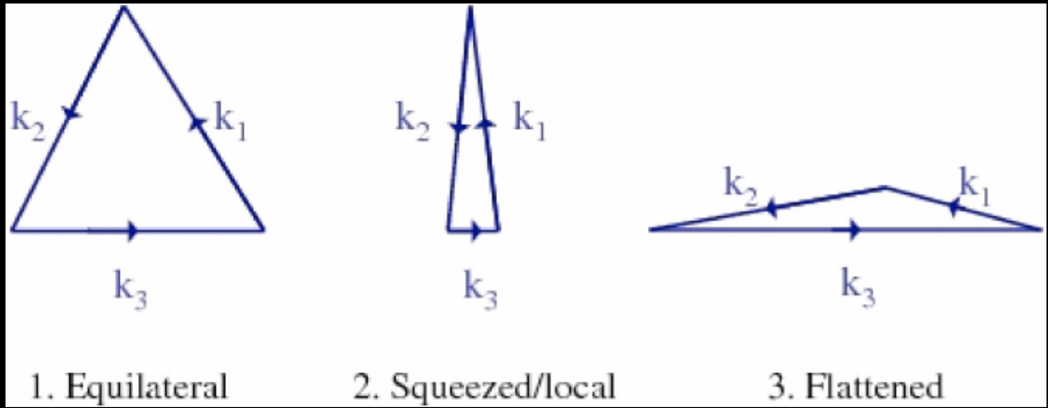
"*... the linear perturbations are so surprisingly simple that a perturbation analysis accurate to second order may be feasible ...*" (Sachs & Wolfe 1967)

Non-Linearities in the perturbations

$$\zeta(x) = \zeta_g(x) + f_{\rm NL}\left(\zeta_g^2(x) - \langle\zeta_g^2\rangle\right)$$

- The expanding parameter is $f_{\rm NL}\zeta_g \ll 1$
- The non-linear parameter is usually momentum-dependent

The primordial NG from single-field models of inflation is suppressed in the local squeezed limit



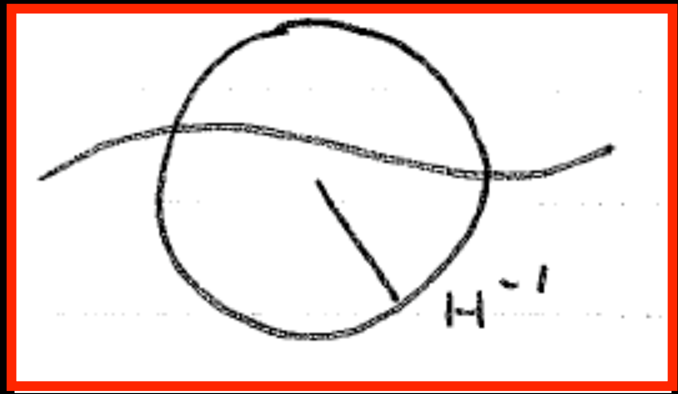
$$f_{\rm NL}(k_1, k_2, k_3) \propto (n_\zeta - 1)$$

$$\text{in the squeezed limit } k_3 \ll k_1, k_2$$

J. Maldacena, 2003

If NG observed in this configuration, ALL single-field models would have been ruled out

A local observer sees the long wavelength mode as a scaling of coordinates



$$\mathrm{d}s^2 = \frac{1}{H^2\tau^2}(\mathrm{d}\tau^2 - e^{-2\zeta_L}\mathrm{d}\vec{x}^2)$$

The transformation

$$\vec{x} \to e^{\zeta_L}\vec{x}, \text{ or } \vec{k} \to e^{-\zeta_L}\vec{k}$$

gets rid of the long wavelength mode if the system is exactly scale-invariant

What is the shape of non-Gaussianity in multi-field inflation ?

During inflation the extra light fields do not contribute to the energy density driving inflation and inflation takes place in a geometry very close to de Sitter

Use the symmetries of de Sitter

I. Antoniadis et al., 2011; P. Creminelli, 2011; A. Kehagias and A.R., 2012 & 2013

Isometries of de Sitter

$$\mathrm{d}s^2 = \frac{1}{H^2\tau^2}(\mathrm{d}\tau^2 - \mathrm{d}\vec{x}^2)$$

- Translations plus rotations in the spatial coordinates
- Dilations $\tau \to \lambda\tau$ and $\vec{x} \to \lambda\vec{x}$
- Special conformal transformations

  $\tau \to \tau - 2\tau(\vec{b} \cdot \vec{x}),$

  $x^i \to x^i + b^i(-\tau^2 + \vec{x}^2) - 2x^i(\vec{b} \cdot \vec{x})$

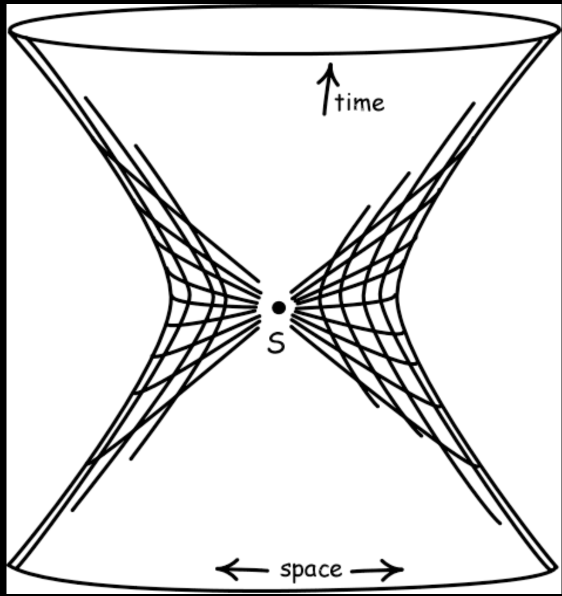
The SO(1,4) de Sitter isometry group acts
on constant time hypersurfaces
as a conformal group on $\mathbb{R}^3$
when the fluctuations are on super-Hubble scales

$$x'_i = a_i + M_i^j x_j$$

$$x'_i = \lambda x_i$$

$$x'_i = \frac{x_i + b_i \vec{x}^2}{1 + 2\vec{b} \cdot \vec{x} + b^2 \vec{x}^2}$$

The de Sitter metric is the induced metric on the hyperboloid from the five-dimensional Minkowski space-time



$$\eta_{AB}X^A X^B = -X_0^2 + X_i^2 + X_5^2 = \frac{1}{H^2}$$

$$\mathrm{d}s_5^2 = \eta_{AB}\mathrm{d}X^A\mathrm{d}X^B$$

$$\begin{aligned} X^0 &= \frac{1}{2H}\left(H\tau - \frac{1}{H\tau}\right) - \frac{1}{2}\frac{\vec{x}^2}{\tau}, \\ X^i &= \frac{x^i}{H\tau}, \\ X^5 &= -\frac{1}{2H}\left(H\tau + \frac{1}{H\tau}\right) + \frac{1}{2}\frac{\vec{x}^2}{\tau} \end{aligned}$$

SO(1,4) isometry group manifest

For perturbations on super-Hubble scales

$$X^0 = -\frac{1}{2H^2\tau} - \frac{1}{2}\frac{\vec{x}^2}{\tau},$$
$$X^i = \frac{x^i}{H\tau},$$
$$X^5 = -\frac{1}{2H^2\tau} + \frac{1}{2}\frac{\vec{x}^2}{\tau}$$

$$\eta_{AB}X^A X^B = -X_0^2 + X_i^2 + X_5^2 = 0$$

The conformal group SO(1,4) acts linearly on X_A but it induces the non-linear conformal transformations

$$x'_i = a_i + M_i^j x_j$$
$$x'_i = \lambda x_i$$
$$x'_i = \frac{x_i + b_i\vec{x}^2}{1 + 2\vec{b}\cdot\vec{x} + b^2\vec{x}^2}$$

CFT dictates the correlators

$$\vec{x} \to \vec{x}' \text{ implies } \sigma^I(\vec{x}) \to \left|\frac{\partial x_i'}{\partial x_j}\right|^{-w_I/3} \sigma^I(\vec{x})$$

$w_I = \frac{1}{3}\frac{m_I^2}{H^2} \ll 1$ is the conformal dimension of the field

$$\langle \sigma^I(\vec{x}_1)\sigma^J(\vec{x}_2)\rangle = \begin{cases} \frac{c_{IJ}}{|\vec{x}_1 - \vec{x}_2|^{2w_I}} & w_I = w_J, \\ 0 & w_I \neq w_J, \end{cases}$$

$$\begin{aligned} \langle \sigma^I(\vec{x}_1)\sigma^J(\vec{x}_2)\sigma^K(\vec{x}_3)\rangle &= \frac{c_{IJK}}{x_{12}^{w_I+w_J-w_K}\, x_{23}^{w_J+w_K-w_I}\, x_{13}^{w_I+w_K-w_J}}, \\ x_{ij} &= |\vec{x}_i - \vec{x}_j| \end{aligned}$$

The three-point correlator is enhanced in the squeezed limit

$$\langle \sigma^I_{\vec{k}_1}\,\sigma^J_{\vec{k}_2}\,\sigma^K_{\vec{k}_3}\rangle' \sim \frac{c_{IJK}}{k_1^{3-2w}\,k_2^{3-w}} + \text{cyclic} \qquad (k_1 \ll k_2 \sim k_3)$$

A detection of non-Gaussianity in the squeezed limit would have indicated that the cosmological perturbations are generated by a field different from the inflaton

The four-point correlator

Use the CFT techniques developed by Gatto, Grillo & Ferrara in the 70's on conformal boostrap to solve for the CFT without Lagrangian

$$\langle \sigma^I(\vec{x}_1)\sigma^J(\vec{x}_2)\sigma^K(\vec{x}_3)\sigma^L(\vec{x}_4)\rangle = \left(\frac{x_{12}}{x_{14}}\right)^{w_I - w_J} \left(\frac{x_{14}}{x_{13}}\right)^{w_K - w_L} \frac{g(u,v)}{x_{12}^{w_I+w_J} x_{34}^{w_K+w_L}}$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2},\ v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

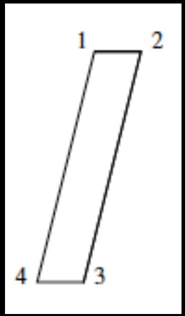
OPE: $$\sigma^I(\vec{x})\sigma^J(\vec{0}) \approx \sum_{\mathcal{O}} f_{IJ\mathcal{O}} \left\{ C^{(m)}(\vec{x})\mathcal{O}_{(m)}(\vec{0}) + \ldots \right\}$$

$$C^{(m)}(\vec{x}) = \frac{x^{i_1}\ldots x^{i_m}}{|\vec{x}|^{\ell}}, \qquad \ell = w^I + w^J - w_{\mathcal{O}} + m.$$

$$g(u,v) = \sum_{\mathcal{O}} f_{12\mathcal{O}} f_{34\mathcal{O}}\, G_{w_{\mathcal{O}},l}(u,v)$$

the sum is over primaries and $G_{w,l}(u,v)$ are the so-called conformal blocks

The four-point correlator is enhanced in the collapsed limit



$$u = z\bar{z}\,, \qquad v = (1-z)(1-\bar{z})$$

$$G_{w,l} = c_{w,l}\, z^w + \cdots\,.$$

$$c_{w,l} = \left(\frac{3}{2w-1}\right)^{l/2} \frac{\Gamma(\frac{w+l+1}{2})}{\Gamma(\frac{l+2}{2})\Gamma(\frac{w+1}{2})}\Big(1 + c_0\big((-)^l - 1\big)\Big)$$

$$c_0 = \frac{1}{12}\left\{6 - \frac{(3\pi)^{1/2}(2w-1)^{1/2}\Gamma(\frac{1+w}{2})}{\Gamma(1+\frac{w}{2})}\right\}$$

$$\langle \sigma^I_{\vec{k}_1}\sigma^J_{\vec{k}_2}\sigma^K_{\vec{k}_3}\sigma^L_{\vec{k}_4}\rangle' \sim P_{|\vec{k}_{12}|}P_{\vec{k}_2}P_{\vec{k}_4} + \text{ permuts } \ (|\vec{k}_{12}| \to 0)$$

However

Planck has seen no signature
of non-Gaussianity

$$f_{\rm NL} = 2.7 \pm 5.8$$

Other methods?
Will we ever know?

Can we at least say that during inflation the geometry was (nearly) de Sitter?

Common lore: the perturbations have a scale invariant spectrum because of scale invariance

$$\mathrm{d}s^2 = \frac{1}{H^2\tau^2}(\mathrm{d}\tau^2 - \mathrm{d}\vec{x}^2)$$

The de Sitter metric is invariant under

$\tau \to \lambda\tau$ and $\vec{x} \to \lambda\vec{x}$

However

All accelerating FRW cosmologies with equation of state

$$w = P/\rho < -1/3$$

exhibits three-dimensional conformal symmetry
on future constant-time hypersurfaces.

A. Kehagias and A.R., 2013

A generic FRW can also be embedded in a five-dimensional Minkowski space-time

$$X_0^2 + X_1^2 + X_2^2 + X_3^3 + X_5^2 = (X_5 + X_0) f(t)$$

$$f(t) = \int^t \frac{\mathrm{d}t'}{\dot{a}(t')} \sim a^{2+3w} \quad \text{and} \quad a(t) \sim t^{2/3(1+w)}$$

$$\begin{aligned} X^0 &= \frac{1}{2\tau_0} a(t) \left(\vec{x}^2 + \tau_0^2\right) + \frac{1}{\tau_0} \int^t \frac{\mathrm{d}t'}{2\dot{a}(t')}, \\ X^i &= a(t) x^i, \\ X^5 &= \frac{1}{2\tau_0} a(t) \left(\vec{x}^2 - \tau_0^2\right) + \frac{1}{\tau_0} \int^t \frac{\mathrm{d}t'}{2\dot{a}(t')} \end{aligned}$$

For perturbations on super-Hubble scales and accelerating universes

$$\begin{aligned} X^0 &= -\frac{1}{2}\frac{\tau_0}{H\tau} - \frac{1}{2}\frac{q}{H\tau}\frac{\vec{x}^2}{\tau_0}, \\ X^i &= -q\frac{x^i}{H\tau}, \\ X^5 &= \frac{1}{2}\frac{\tau_0}{H\tau} - \frac{1}{2}\frac{q}{H\tau}\frac{\vec{x}^2}{\tau_0} \end{aligned}$$

$$\begin{aligned} a &= (\tau/\tau_0)^{-q}, \\ q &= -\frac{2}{1+3w} \end{aligned}$$

$$\eta_{AB}X^A X^B = -X_0^2 + X_i^2 + X_5^2 = 0$$

Accelerating FRW cosmologies exibit three-dimensional conformal symmetry on the future constant-time hypersurfaces

$$\begin{aligned} \delta_D x_i &= \lambda x_i, \quad \delta_D \tau = z\lambda\tau, \quad \delta_{D,K_i}\tau_0 = 0, \\ \delta_K x_i &= -2x_i(\vec{b}\cdot\vec{x}) + b_i(-\tau^2 + \vec{x}^2), \delta_K\tau = -2q\tau(\vec{b}\cdot\vec{x}) \end{aligned}$$

Perturbations on super-Hubble scales
in any accelerating universes
may have a flat spectrum

$$\frac{1}{2} I(\phi) g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma = \frac{1}{2} g^{\mu\nu}_{\rm dS} \partial_\mu \sigma \partial_\nu \sigma$$

Couple appropriately the fluctuating field
with the inflaton field in such a way to get
an induced de Sitter-like metric

How can we know that during inflation the geometry was (nearly) de Sitter?

Tensor perturbations

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(\delta_{ij} + h_{ij})\mathrm{d}x^i\mathrm{d}x^j$$

$$v_k = \frac{aM_{\mathrm{Pl}}}{\sqrt{2}} h_k$$

$$v_k'' + \left(k^2 - \frac{a''}{a}\right) v_k = 0$$

$$\mathcal{P}_T(k) \simeq \frac{8}{M_{\mathrm{Pl}}^2}\left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{n_T}$$

$$n_T = -3(1+w) \Rightarrow n_T > -2$$

Conclusions

- Symmetries are a powerful tool to characterize the cosmological perturbations
- Observationally, we will probably never know the true origin of the inflationary perturbations
- Observationally, we might know the geometry during inflation only by measuring tensor modes