Stability of the Electroweak Scale and Inflation

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Based on

- Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio and Strumia, JHEP **1312** (2013) 089, arXiv:1307.3536; updated version: September 22, 2014
- Salvio, Phys. Lett. B **727** (2013) 234, arXiv:1308.2244
- Salvio and Strumia, JHEP **1406** (2014) 080, arXiv:1403.4226

Outline

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Results at the Large Hadron Collider (LHC)

- *Discovery of a h boson at CMS and ATLAS in 2012*
  it weights $M_h = 125.15 \pm 0.24$ GeV

  [CMS Collaboration (2013, 2014); ATLAS Collaboration (2013, 2014); naive average from Giardino, Kannike, Masina, Raidal and Strumia (2014)]

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The triumph of simplicity? We do not yet know!

Extra dimensions, extra Z', W', ... , etc are not yet excluded, but ...

a doublet H with the potential $\quad V(H) = \lambda \left(|H|^2 - \frac{v^2}{2}\right)^2 \quad$ fits the data

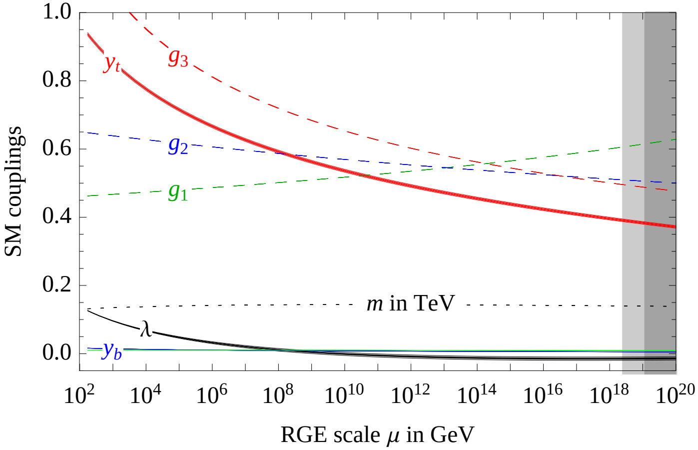
- Measurements of G_μ provides $v = \sqrt{2}\langle|H|\rangle = \left[1/(\sqrt{2}G_\mu)\right]^{1/2}$ (tree level)
- and $m^2 \equiv 2\lambda v^2 = M_h^2$ (tree level) fixes the last parameter of the SM

However, it has also unsatisfactory features:
e.g. does not provide a dynamical explanation of EW symmetry breaking

But now we can use the SM to make predictions up to the Planck scale ...

Consistency: ok (up to the Planck scale)

- The measured M_h implies that the EW vacuum expectation value (VEV) is either stable or metastable with a life-time $>$ than the age of the universe ...
- The Landau pole of λ and $g_1 \equiv \sqrt{5/3}g_Y$ are above the Planck mass $M_{\rm Pl}$



Solutions of the renormalization group equations (RGEs) of the most relevant SM parameters (defined in the $\overline{MS}$ scheme ...)

Still there are unsolved problems

The SM is not the final theory: it does not include gravity and

- **Dark matter**
  well-motivated candidates: axions (also solve the strong CP problem), ...
- **(small) neutrino masses**
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- **Baryon asymmetry**
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Origin of inflation

is it part of this list?

$\rightarrow$ *One possibility is that inflation is generated by h, however,*
it is known that this is possible essentially only if the stability bound is not violated

($\rightarrow$ see $2^{\rm nd}$ part)

Qualitative origin of the stability bound

$$V_{\rm eff} = V + V_1 + V_2 + \dots$$

$$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - v^2\right)^2, \quad V_1(\phi) = \frac{1}{(4\pi)^2}\sum_i c_i m_i(\phi)^4\left(\ln\frac{m_i(\phi)^2}{\mu^2} + d_i\right), \quad \dots$$

where $\phi^2 \equiv 2|H|^2$ and c_i and d_i are ~ 1 constants

Considering the RG-improved effective potential (bare parameters $\to$ running ones) ...

$$\Longrightarrow \frac{\partial V_{\rm eff}}{\partial \mu} = 0 \quad \text{and one is free to choose } \mu \text{ to improve perturbation theory}$$

Since at large fields, $\phi \gg v$, we have $m_i(\phi)^2 \propto \phi^2$, we choose $\mu^2 = \phi^2$, then

$$V_{\rm eff}(\phi) = \frac{\lambda(\phi)}{4}\left(\phi^2 - v(\phi)^2\right)^2 + \dots = -\frac{m(\phi)^2}{2}\phi^2 + \frac{\lambda(\phi)}{4}\phi^4 + \dots$$

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So for $\phi \gg v$

$$V_{\text{eff}}(\phi) \simeq \frac{\lambda(\phi)}{4}\phi^4$$

- M_h contributes positively to $\lambda \rightarrow$ lower bound on M_h
- y_t contributes negatively to the running of $\lambda \rightarrow$ upper bound on M_t

Procedure to extract the stability bound

Steps of the procedure:

- $V_{\rm eff}$, including relevant parameters
- RGEs of the relevant couplings
- Values of the relevant parameters (also called *threshold corrections* or *matching conditions*) at the EW scale (e.g. at M_t) ...

Finally impose that $V_{\rm eff}$ at the EW vacuum is the absolute minimum!

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State-of-the-art loop calculation:

- Two loop $V_{\rm eff}$ including the leading couplings $= \{\lambda, y_t, g_3, g_2, g_1\}$
- Three loop RGEs for $\{\lambda, y_t, g_3, g_2, g_1\}$ and one loop RGE for $\{y_b, y_\tau\}$ ...
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Previous calculations: *[Cabibbo, Maiani, Parisi, Petronzio (1979); Casas, Espinosa, Quiros (1994, 1996); Bezrukov, Kalmykov, Kniehl, Shaposhnikov (2012); Degrassi, Di Vita, Elias-Miró, Espinosa, Giudice, Isidori, Strumia (2012)]*

Input values of the SM observables

(used to fix relevant parameters: λ, y_t, g_1, g_2)

$$
\begin{array}{rcll}
M_W & = & 80.384 \pm 0.014 \ \mathrm{GeV} & \text{Mass of the } W \text{ boson [1]} \\
M_Z & = & 91.1876 \pm 0.0021 \ \mathrm{GeV} & \text{Mass of the } Z \text{ boson [2]} \\
M_h & = & 125.15 \pm 0.24 \ \ \mathrm{GeV} & \text{(source already quoted)} \\
M_t & = & 173.34 \pm 0.76 \pm 0.3 \ \mathrm{GeV} & \text{Mass of the top quark [3]} \\
V \equiv (\sqrt{2} G_\mu)^{-1/2} & = & 246.21971 \pm 0.00006 \, \mathrm{GeV} & \text{Fermi constant [4]} \\
\alpha_3(M_Z) & = & 0.1184 \pm 0.0007 & \mathrm{SU}(3)_c \text{ coupling (5 flavors) [5]}
\end{array}
$$

[1] TeVatron average: FERMILAB-TM-2532-E. LEP average: CERN-PH-EP/2006-042

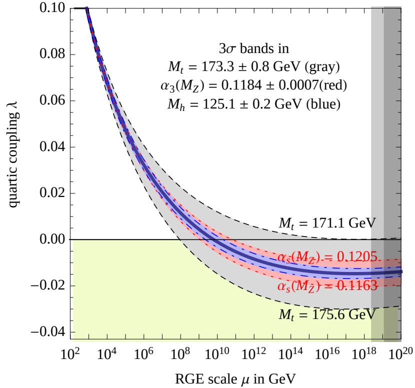
[2] 2012 Particle Data Group average, pdg.lbl.gov

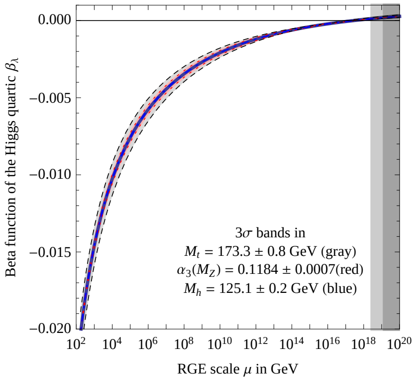
[3] ATLAS, CDF, CMS, D0 Collaborations, arXiv:1403.4427. Plus an uncertainty $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ because of non-perturbative effects [Alekhin, Djouadi, Moch (2013)]

[4] MuLan Collaboration, arXiv:1211.0960

[5] S. Bethke, arXiv:1210.0325

Precise running of λ and its β-function





RGE evolution of λ and its β-function varying M_t, $\alpha_3(M_Z)$, M_h by $\pm 3\sigma$.

Result for the stability bound

$$M_h > 129.6\,\mathrm{GeV} + 2.0(M_t - 173.34\,\mathrm{GeV}) - 0.5\,\mathrm{GeV}\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.3_{\mathrm{th}}\,\mathrm{GeV}$$

Combining in quadrature the experimental and theoretical uncertainties we obtain

$$M_h > (129.6 \pm 1.5)\,\mathrm{GeV}$$

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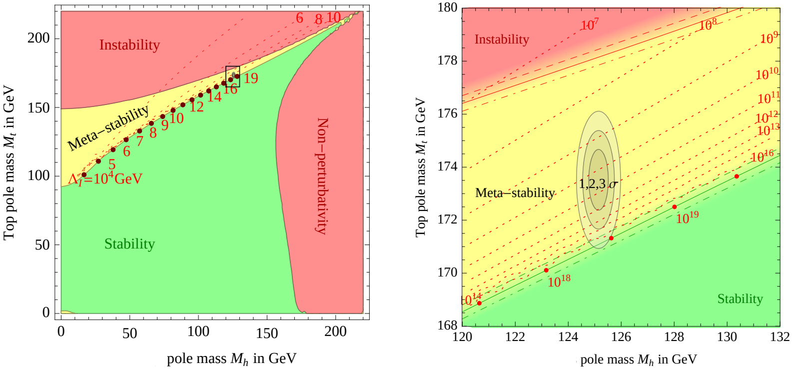
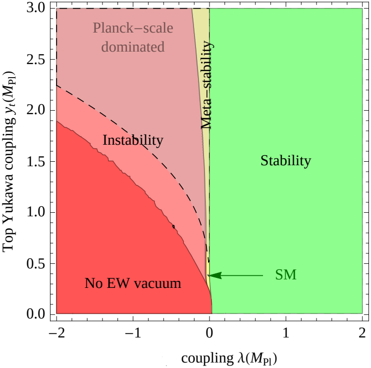
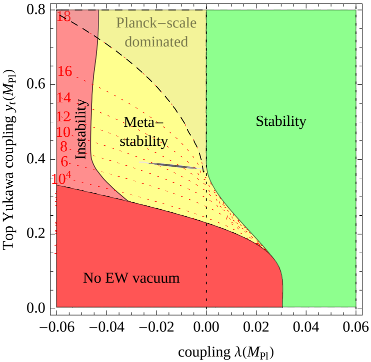


Λ_I = scale (field value) at which $V_{\rm eff}$ becomes smaller than its value at the EW scale

The SM phase diagram in terms of Planck scale couplings

$y_t(M_{\rm Pl})$ versus $\lambda(M_{\rm Pl})$



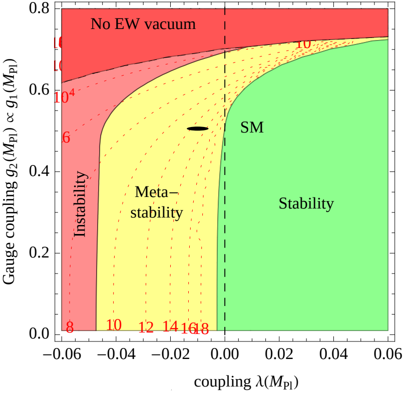


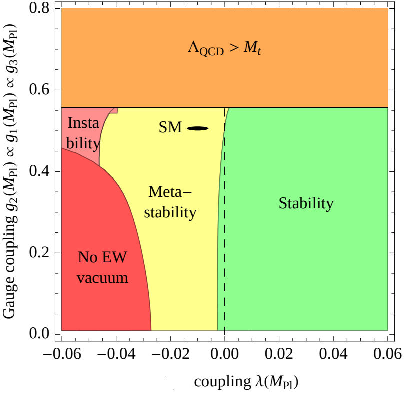
"Planck-scale dominated" corresponds to $\Lambda_I > 10^{18}$ GeV

"No EW vacuum" corresponds to a situation in which λ is negative at the EW scale

The SM phase diagram in terms of Planck scale couplings

Gauge coupling g_2 at $M_{\rm Pl}$ versus $\lambda(M_{\rm Pl})$

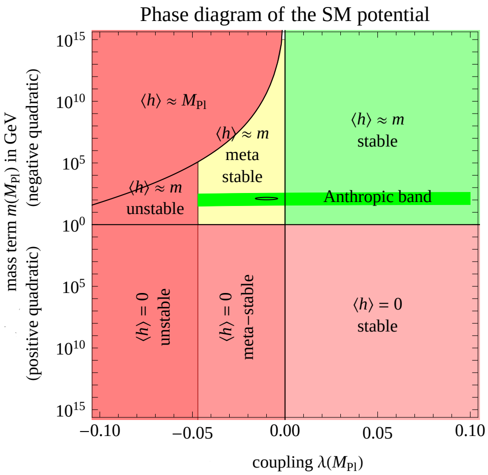




Left: $g_1(M_{\rm Pl})/g_2(M_{\rm Pl}) = 1.22$, $y_t(M_{\rm Pl})$ *and* $g_3(M_{\rm Pl})$ *are kept to the SM value*

Right: *a common rescaling factor is applied to* g_1, g_2, g_3. $y_t(M_{\rm Pl})$ *is kept to the SM value*

The SM phase diagram in terms of potential parameters



If $\lambda(M_{\rm Pl}) < 0$ there is an upper bound on m requiring a VEV at the EW scale.

This bound is, however, much weaker than the anthropic bound of
[Agrawal, Barr, Donoghue, Seckel (1997); Schellekens (2014)]

Inflation *[Brout, Englert and Gunzig (1978); Brout, Englert, Spindel (1979); Brout, Englert, Frère, Gunzig, Nardone, Truffin, Spindel (1980); Guth (1981); Linde (1982); Albrecht and Steinhardt (1982)]*

What it can solve: horizon, flatness, monopole problems

To solve these problems inflation should last enough $\rightarrow$ lower bounds on

$$N \equiv \ln\left(\frac{a(t_{\mathrm{end}})}{a(t_{\mathrm{in}})}\right) \equiv \text{number of } e\text{-foldings}$$

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How it is implemented (slow-roll inflation):

- we assume a scalar field φ (the inflaton)
- at some early time the potential $U(\varphi)$ is large, but quite flat ...
- $\rightarrow$ the Hubble constant changes slowly $\rightarrow$ nearly exponential expansion

The inflaton rolls slowly when ...

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{1}{U}\frac{dU}{d\varphi}\right)^2 \ll 1, \quad \eta \equiv \frac{M_P^2}{U}\frac{d^2U}{d\varphi^2} \ll 1, \quad \text{where } M_P \simeq 2.4\times 10^{18}\text{GeV}$$

... from which we can compute observable inflationary parameters:
the scalar amplitude A_s, its spectral index n_s and the tensor-to-scalar ratio $r = \frac{A_t}{A_s}$

$$A_s = \frac{U/\epsilon}{24\pi^2 M_P^4}, \qquad n_s = 1 - 6\epsilon + 2\eta, \qquad r = 16\epsilon \qquad \text{computed at } \varphi = \varphi_{in}$$

h inflation: definition

In the h inflation model the role of the inflaton is played by h

The model: *[Bezrukov, Shaposhnikov (2008)]*

$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_{SM} + \xi |H|^2 R$$

h inflation: classical analysis

The part of S that depends on $g_{\mu\nu}$ and H *only* $\longrightarrow$

$$S_{gH} = \int d^4x\sqrt{-g}\left[\left(\frac{M_P^2}{2} + \xi|H|^2\right) R + |D_\mu H|^2 - V(H)\right]$$

The non-minimal coupling can be eliminated through a *conformal* transformation ...

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} \equiv \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{2\xi|H|^2}{M_P^2}$$

In the unitary gauge, where the only scalar field is the radial mode $\phi \equiv \sqrt{2|H|^2}$

$$S_{gH} = \int d^4x\sqrt{-\hat{g}}\left[\frac{M_P^2}{2}\hat{R} + K\frac{(\partial\phi)^2}{2} - \frac{V}{\Omega^4}\right]$$

where $K \equiv (\Omega^2 + 6\xi^2\phi^2/M_P^2)/\Omega^4$ and we set the gauge fields to zero.

The ϕ kinetic term can be made canonical through $\phi = \phi(\chi)$ defined by

$$\frac{d\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2\phi^2/M_P^2}{\Omega^4}}$$

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This is what we want in order to have slow-roll ...

Thus, χ feels a potential $\quad U \equiv \frac{V}{\Omega^4} = \frac{\lambda(\phi(\chi)^2 - v^2)^2}{4(1+\xi\phi(\chi)^2/M_P^2)^2} \overset{\phi > M_P/\sqrt{\xi}}{\simeq} \frac{\lambda}{4\xi^2}M_P^4$

h inflation: classical analysis

All parameters can be fixed through experiments and observations ...

ξ can be fixed requiring the WMAP normalization *[WMAP Collaboration (2013)]*

$$\frac{U(\phi = \phi_{WMAP})}{\epsilon(\phi = \phi_{WMAP})} \simeq (0.02746 M_P)^4$$

ϕ_{WMAP} is fixed by requiring $\quad N = \int_{\phi_{\rm end}}^{\phi_{WMAP}} \frac{U}{M_P^2} \left(\frac{dU}{d\phi}\right)^{-1} \left(\frac{d\chi}{d\phi}\right)^2 d\phi \simeq 59$

[Bezrukov, Gorbunov, Shaposhnikov (2009); Garcia-Bellido, Figueroa, Rubio (2009)]

and $\phi_{\rm end}$ is the field value at the end of inflation: $\quad \epsilon(\phi_{\rm end}) \simeq 1$

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This leads to $\xi \simeq 4.7 \times 10^4 \sqrt{\lambda}$ and indicates that xi has to be large ...

h inflation: quantum analysis

Two regimes *[Bezrukov, Shaposhnikov, (2009)]*:

- small fields: $\phi \ll M_P/\xi$ (the SM is recovered)
- large fields: $\phi \gg M_P/\xi$ (chiral EW action with VEV set to $\phi/\Omega \simeq M_P/\sqrt{\xi}$) $\to$ decoupling of ϕ in the inflationary regime

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State-of-the-art calculation of the bound on M_h to have inflation:

- Two loop effective potential U_{eff} in the inflationary regime including the effect of ξ and the leading SM couplings $= \{\lambda, y_t, g_3, g_2, g_1\}$
- Three loop SM RGE from the EW scale up to M_P/ξ for $\{\lambda, y_t, g_3, g_2, g_1\}$ ...
- Two loop RGE for the same SM couplings and one loop RGE for ξ in the chiral EW theory
- Two loop threshold corrections at the top mass, for these SM couplings

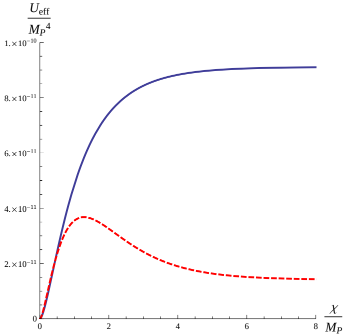
Previous calculations: *[Bezrukov, Magnin, Shaposhnikov (2009); Bezrukov, Shaposhnikov (2009); Allison (2013)]*

Bound on M_h to have h inflation

Derivation

1. We fix ξ as in the classical case, but with U replaced by $U_{\rm eff}$.
   ... this already gives $\xi_{\rm inf} \equiv \xi(M_P/\sqrt{\xi_t})$, where conventionally $\xi_t = \xi(M_t)$

2. If M_h is too small (or M_t is too large) we go from the blue behavior to the red one! When the slope is negative h cannot roll towards the EW vacuum



We set the th. errors to zero and the input parameters to the central values, except M_t:

- *Solid line:* $M_t = 171.43\,GeV$ *(ξ fixed as described above)*
- *Dashed line:* $M_t = 171.437\,GeV$ *($\xi_t = 300$)*

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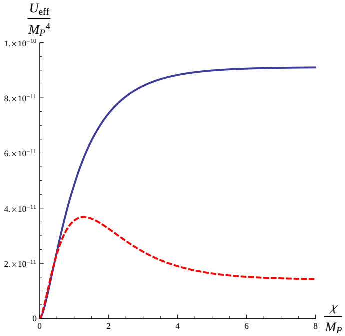

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The rate of tunnelling is the probability of nucleating a bubble of true VEV in $dV\,dt$
[Kobzarev, Okun, Voloshin (1975); Coleman (1977); Callan, Coleman (1977)]

$$d\wp = dt\,dV\ \Lambda_B^4\,e^{-S(\Lambda_B)}$$

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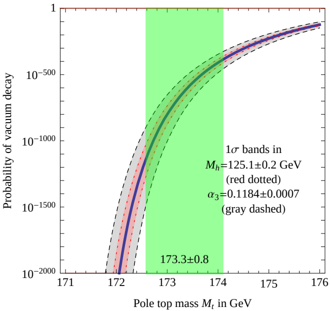
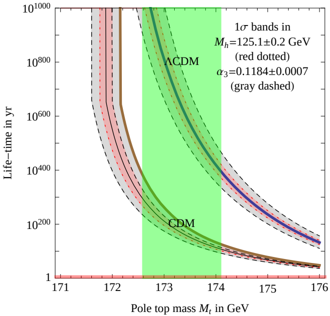




Left: *The probability that EW vacuum decay happened in our past light-cone, taking into account the expansion of the universe.*

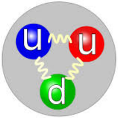
Right: The life-time of the EW VEV, with 2 different assumptions for future cosmology: universes dominated by the cosmological constant (ΛCDM) or by dark matter (CDM)

Main motivations for agravity

Motivation 1: EW symmetry breaking

Most of the mass of the matter we see has a dynamical origin

example: only few % of the proton mass is due to quark masses, which comes from an ad hoc mass parameter in the *h* mechanism



Is it possible to generate all the mass dynamically?
Is it possible to have a dynamical EW symmetry breaking?

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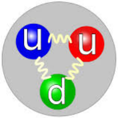

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Cosmological observations suggest inflation. However, it requires special models with flat potentials. *What is the reason for this flatness?*
The agravity scenario provides us with an explanation:
As we saw, the Einstein frame potential of a scalar S in agravity is

$$U(S) = \frac{\lambda_S |S|^4}{(2\xi_S |S|^2)^2} M_P^4 = \frac{\lambda_S}{4\xi_S^2} M_P^4$$

The potential is flat at tree-level, but at quantum level λ_S and ξ_S run

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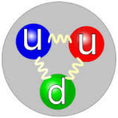

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what we need to have inflation!

Agravity scenario

The *most general* agravity action compatible with the assumed symmetries ... :

$$S = \int d^4x \sqrt{|\det g|} \left[\frac{R^2}{6f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} + \mathcal{L}_{\rm SM}^{\rm adim} + \mathcal{L}_{\rm BSM}^{\rm adim} \right]$$

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Non-gravitational interactions

- $\mathscr{L}_{\rm SM}^{\rm adim}$ is the no-scale part of the SM Lagrangian:

$$\mathscr{L}_{\rm SM}^{\rm adim} = -\frac{F_{\mu\nu}^2}{4} + \bar{\psi} i \not{D} \psi + |D_\mu H|^2 - (yH\psi\psi + \text{h.c.}) - \lambda_H |H|^4 + \xi_H |H|^2 R$$

- $\mathscr{L}_{\rm BSM}^{\rm adim}$ describes physics beyond the SM (BSM). it generates the EW scale

example: adding a scalar $S \ \rightarrow \ \mathscr{L}_{\rm BSM}^{\rm adim} = |D_\mu S|^2 - \lambda_S |S|^4 + \lambda_{HS} |S|^2 |H|^2 + \xi_S |S|^2 R$

extension: vectors interacting with S can be dark matter *[Hambye, Strumia (2013)]*

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Gravitational interactions

- M_P can be generated via a quantum $\langle S\rangle$: $\xi_S |S|^2 R \rightarrow M_P^2 = 2\xi_S |\langle S\rangle|^2$
- *Agravity is renormalizable* *[Stelle (1977)]*: there are all the terms allowed by the symmetries with coefficients having dimension of non-negative powers of energy
- *Linearizing around $\eta_{\mu\nu}$*: *(i)* massless graviton, *(ii)* scalar with mass $M_0^2 \sim \frac{1}{2} f_0^2 M_P^2$ *(iii)* massive graviton with mass $M_2^2 = \frac{1}{2} f_2^2 M_P^2$ and negative norm (a ghost), however with quantum energy bounded from below ...

The literature is controversial

Quantum agravity

Quantum effects are mostly encoded in the RGEs ...

They are important to obtain n_s *and* r *and to dynamically generate* M_P *and* m

Quantum agravity

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The most general agravity can be parameterized by the following $\mathcal{L}$

$$\frac{R^2}{6f_0^2}+\frac{\frac{1}{3}R^2-R_{\mu\nu}^2}{f_2^2}-\frac{\left(F_{\mu\nu}^A\right)^2}{4}+\frac{(D_\mu\phi_a)^2}{2}-\frac{\xi_{ab}}{2}\phi_a\phi_b R-\frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d+\bar{\psi}_j i\not{D}\psi_j-Y_{ij}^a\psi_i\psi_j\phi_a+\text{h.c.}$$

We obtain the RGEs of this renormalizable quantum field theory:

$$\beta_p \equiv \frac{dp}{d\ln\mu} \qquad \text{(of all parameters } p\text{)}$$

Without gravity this was done before *[Machacek and Vaughn (1983,1984,1985)]*

We include gravity and use the one-loop approximation for $\mu > M_P$ (no-scale case)

Results for RGEs

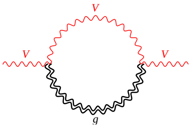
Gauge couplings

Possible new gravity contributions

Their contributions to the RGEs cancel!

This was previously noticed in *[Narain, Anishetty (2013)]*

Possible explanation:
the graviton is not charged



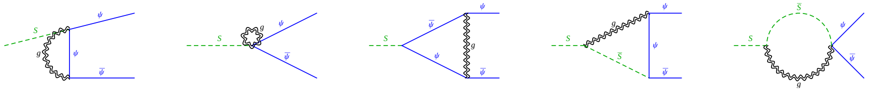
(Rainbow)

(Seagull)

Yukawa couplings

We find the one-loop RGE (where $C_{2F} \equiv t^A t^A$ and $t^A \equiv$ "fermion gauge generators"):

$$(4\pi)^2 \frac{dY^a}{d\ln\mu} = \frac{1}{2}(Y^{\dagger b}Y^b Y^a + Y^a Y^{\dagger b} Y^b) + 2Y^b Y^{\dagger a} Y^b + Y^b \mathrm{Tr}(Y^{\dagger b} Y^a) - 3\{C_{2F}, Y^a\} + \frac{15}{8} f_2^2 Y^a$$



All remaining RGEs

We also compute the RGEs for ▸ λ_{abcd} ▸ ξ_{ab} ▸ f_0 and f_2

Dynamical generation of the Planck scale

There must be a real scalar s (e.g. the modulus of the complex scalar S)

Agravity generates the Planck scale while keeping the vacuum energy small if

$$\begin{cases} \lambda_S(s) & \simeq & 0 & \leftrightarrow & \text{nearly vanishing cosmological constant (dark energy)} \\ \beta_{\lambda_S}(s) & = & 0 & \leftrightarrow & \text{minimum condition} \\ \xi_S(s)s^2 & = & M_P^2 & \leftrightarrow & \text{observed Planck mass} \end{cases}$$

Once M_P is generated:

One can use the RGEs to extract n_s and r

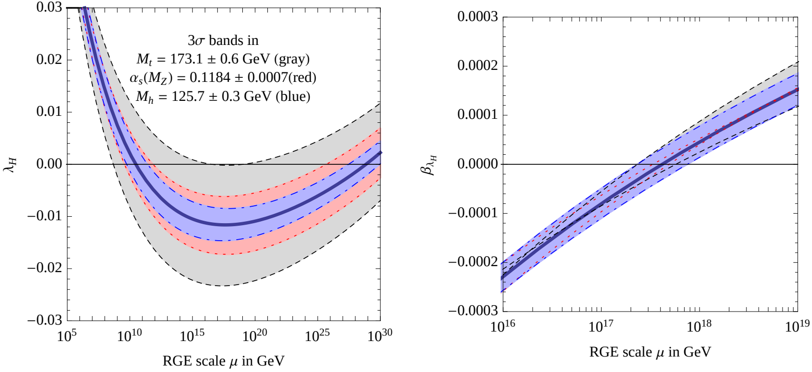
This is easy when the inflaton is s

Dynamical generation of the Planck scale: models

Are these conditions realized in the physics we know (the SM)?

example: λ_H in the SM for $M_h \simeq 125$ GeV and $M_t \simeq 171$ GeV

RGE running of the $\overline{\text{MS}}$ quartic Higgs coupling in the SM



... These conditions are possible! But in the pure gravity limit they cannot be satisfied

$\rightarrow$ the scalar S must have extra gauge and Yukawa interactions

$\rightarrow$ many models are possible

Natural dynamical generation of the weak scale

1) *Low energies* ($\mu < M_{0,2}$): agravity can be neglected and the SM RGE apply:

$$(4\pi)^2 \frac{dm^2}{d\ln\mu} = m^2 \beta_m^{\rm SM}, \qquad \beta_m^{\rm SM} = 12\lambda_H + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}$$

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2) Intermediate energies ($M_{0,2} < \mu < M_P$): agravity interactions cannot be neglected, but m and M_P appear in the effective Lagrangian. We find

$$(4\pi)^2 \frac{d}{d\ln\mu} \frac{m^2}{M_P^2} = -\xi_H[5f_2^4 + f_0^4(1+6\xi_H)] - \frac{1}{3}\left(\frac{m^2}{M_P^2}\right)^2 (1+6\xi_H) + + \frac{m^2}{M_P^2}\left[\beta_m^{\rm SM} + 5f_2^2 + \frac{5}{3}\frac{f_2^4}{f_0^2} + f_0^2\left(\frac{1}{3} + 6\xi_H + 6\xi_H^2\right)\right]$$

The first term is a non-multiplicative potentially dangerous correction to m

$$\text{naturalness} \;\rightarrow\; f_0, f_2 \simeq \sqrt{\frac{4\pi m}{M_{\rm Pl}}} \sim 10^{-8} \;\rightarrow\; M_2 = f_2 M_P/\sqrt{2} \sim 10^{10}\text{GeV}$$

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3) *Large energies* ($\mu > M_P$): the theory is no-scale and the previous RGEs apply

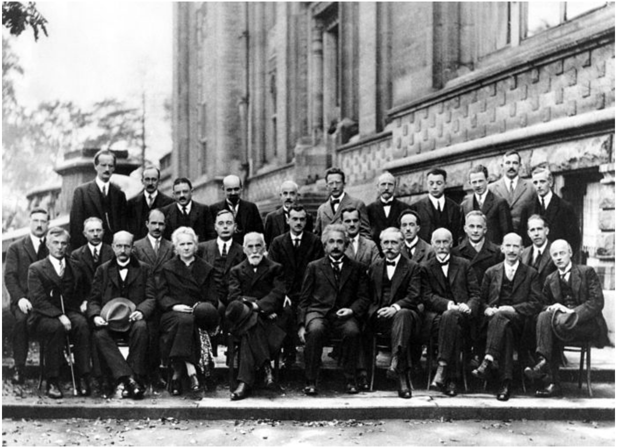
$$\lambda_{HS}|H|^2|S|^2 \quad\to\quad m^2 = \lambda_{HS}\langle s\rangle^2$$

Ignoring gravity, λ_{HS} can be naturally arbitrarily small, because it is the only interaction that couples the SM sector with the S sector. Within agravity

$$(4\pi)^2 \frac{d\lambda_{HS}}{d\ln\mu} = -\xi_H\xi_S[5f_2^4 + f_0^4(6\xi_S+1)(6\xi_H+1)] + \ldots \quad\to\quad \lambda_{HS} \sim f_{0,2}^4$$

Conclusions

- *We have presented the stability bound at full next-to-next-to-leading order*
- *Comparing the result obtained with the experimental values of the relevant parameters we have found some tension, which we have quantified (2.8σ)*
- *Data indicate that the EW VEV is metastable (the life-time is $>$ than the age of the universe) and h inflation is not possible (although the contrary is still allowed)*
- *A dynamical generation of m and a rationale for inflation can be achieved in theories of all interactions (including gravity) where fundamental scales are absent: agravity*



Thank you very much for your attention!

Extra slides

Outlook

- *Three loop QCD contribution to the threshold corrections*
- *Analyze the stability bound in BSM models and find one where the bound is fulfilled without tension and there is a natural inflaton (in progress)*
- *Can ghosts have a sensible physics?* (The literature is controversial)
- *Full analysis of inflation in agravity (for generic values of the parameters) (in progress)*
- *Inclusion of axions and right-handed neutrinos (generically see-saw) in agravity*
- *Inclusion of unified theories in agravity: e.g. Pati-Salam or trinification*

Step 1: effective potential

RG-improved tree level potential (V): classical potential with couplings replaced by the running ones

One loop (V_1): $V_{\rm eff}$ depends mainly on the top, W, Z, h and Goldstone squared masses in the classical background ϕ: in the Landau gauge ... they are

$$t \equiv \frac{y_t^2 \phi^2}{2}, \quad w \equiv \frac{g_2^2 \phi^2}{4}, \quad z \equiv \frac{(g_2^2 + 3g_1^2/5)\phi^2}{4}, \quad m_h^2 \equiv 3\lambda\phi^2 - m^2, \quad g \equiv \lambda\phi^2 - m^2$$

$\rightarrow (4\pi)^2 V_1$ is (in the $\overline{\rm MS}$ scheme)

$$\frac{3w^2}{2}\left(\ln\frac{w}{\mu^2} - \frac{5}{6}\right) + \frac{3z^2}{4}\left(\ln\frac{z}{\mu^2} - \frac{5}{6}\right) - 3t^2\left(\ln\frac{t}{\mu^2} - \frac{3}{2}\right) + \frac{m_h^4}{4}\left(\ln\frac{m_h^2}{\mu^2} - \frac{3}{2}\right) + \frac{3g^2}{4}\left(\ln\frac{g}{\mu^2} - \frac{3}{2}\right)$$

In order to keep the logarithms in the effective potential small we choose

$$\mu = \phi$$

Indeed, t, w, z, m_h^2 and g are $\propto \phi^2$ for $\phi \gg m$

Two loop (V_2): is very complicated, but always depend on t, w, z, m_h^2, g plus g_i

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Step 2: running couplings

For a generic parameter p we write the RGE as

$$\frac{dp}{d\ln\mu^2} = \frac{\beta_p^{(1)}}{(4\pi)^2} + \frac{\beta_p^{(2)}}{(4\pi)^4} + \ldots$$

They were computed before in the literature up to three loops

(very long and not very illuminating expressions at three loops)

One loop RGEs for λ, y_t^2, g_i^2 and m^2

$$\begin{aligned}
\beta_\lambda^{(1)} &= \lambda\left(12\lambda + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}\right) - 3y_t^4 + \frac{9g_2^4}{16} + \frac{27g_1^4}{400} + \frac{9g_2^2 g_1^2}{40}, \\
\beta_{y_t^2}^{(1)} &= y_t^2\left(\frac{9y_t^2}{2} - 8g_3^2 - \frac{9g_2^2}{4} - \frac{17g_1^2}{20}\right), \\
\beta_{g_1^2}^{(1)} &= \frac{41}{10}g_1^4, \quad \beta_{g_2^2}^{(1)} = -\frac{19}{6}g_2^4, \quad \beta_{g_3^2}^{(1)} = -7g_3^4, \\
\beta_{m^2}^{(1)} &= m^2\left(6\lambda + 3y_t^2 - \frac{9g_2^2}{4} - \frac{9g_1^2}{20}\right)
\end{aligned}$$

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Step 3: threshold corrections

$$
\begin{aligned}
\lambda(M_t) &= 0.12604 + 0.00206\left(\frac{M_h}{\mathrm{GeV}} - 125.15\right) - 0.00004\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) \pm 0.00030_{\mathrm{th}}\\
\frac{m(M_t)}{\mathrm{GeV}} &= 131.55 + 0.94\left(\frac{M_h}{\mathrm{GeV}} - 125.15\right) + 0.17\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) \pm 0.15_{\mathrm{th}}\\
y_t(M_t) &= 0.93690 + 0.00556\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) - 0.00042\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.00050_{\mathrm{th}}\\
g_2(M_t) &= 0.64779 + 0.00004\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) + 0.00011\,\frac{M_W - 80.384\,\mathrm{GeV}}{0.014\,\mathrm{GeV}}\\
g_Y(M_t) &= 0.35830 + 0.00011\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) - 0.00020\,\frac{M_W - 80.384\,\mathrm{GeV}}{0.014\,\mathrm{GeV}}\\
g_3(M_t) &= 1.1666 + 0.00314\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} - 0.00046\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right)
\end{aligned}
$$

The theoretical uncertainties on the quantities are much lower than those used in previous determinations of the stability bound

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Ghosts

Negative literature *[Ostrogradski (1850), Smilga (2009), ...]*

- *Classically* the energy is not bounded from below (Ostrogradski instability)
- *At quantum level* creation of negative energy $\sim$ destruction of positive energy: the Hamiltonian becomes positive, but some states ("ghosts") have negative norm

Positive literature

- *[Lee, Wick (1969)]* the introduction of negative norms can lead to a unitary S-matrix, provided that all stable particle states have positive norm
- *[Hawking, Hertog (2001)]* at least in a simple scalar field ϕ theory, the problem comes from regarding ϕ and $\Box\phi$ as independent and can be overcome by using the path integral, where they are dependent.

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RGEs for the quartic couplings

Tens of Feynman diagrams contribute to these RGEs ... we obtain

$$
\begin{aligned}
(4\pi)^2 \frac{d\lambda_{abcd}}{d\ln\mu} \quad = \quad & \sum_{\mathrm{perms}} \Bigg[\frac{1}{8}\lambda_{abef}\lambda_{efcd} + \frac{3}{8}\{\theta^A,\theta^B\}_{ab}\{\theta^A,\theta^B\}_{cd} - \mathrm{Tr}\, Y^a Y^{\dagger b} Y^c Y^{\dagger d} + \\
& + \frac{5}{8} f_2^4 \xi_{ab}\xi_{cd} + \frac{f_0^4}{8}\xi_{ae}\xi_{cf}(\delta_{eb}+6\xi_{eb})(\delta_{fd}+6\xi_{fd}) \\
& + \frac{f_0^2}{4!}(\delta_{ae}+6\xi_{ae})(\delta_{bf}+6\xi_{bf})\lambda_{efcd}\Bigg] + \lambda_{abcd}\Bigg[\sum_k (Y_2^k - 3C_{2S}^k) + 5f_2^2\Bigg],
\end{aligned}
$$

where the first sum runs over the 4! permutations of *abcd* and the second sum over $k = \{a,b,c,d\}$, with Y_2^k and C_2^k defined by

$$
\mathrm{Tr}(Y^{\dagger a} Y^b) = Y_2^a \delta^{ab}, \quad \theta^A_{ac}\theta^A_{cb} = C^a_{2S}\delta_{ab}
$$

(θ^A are the scalar gauge generators)

▸ back to main slides

RGEs for the quartic couplings: SM case

For the SM H plus the complex scalar singlet S the RGEs become:

$$
\begin{aligned}
(4\pi)^2 \frac{d\lambda_S}{d\ln\mu} &= 20\lambda_S^2 + 2\lambda_{HS}^2 + \frac{\xi_S^2}{2}\left[5f_2^4 + f_0^4(1+6\xi_S)^2\right] + \lambda_S\left[5f_2^2 + f_0^2(1+6\xi_S)^2\right] \\
(4\pi)^2 \frac{d\lambda_{HS}}{d\ln\mu} &= -\xi_H\xi_S\left[5f_2^4 + f_0^4(6\xi_S+1)(6\xi_H+1)\right] - 4\lambda_{HS}^2 + \lambda_{HS}\left\{8\lambda_S + 12\lambda_H + 6y_t^2\right. \\
&\quad \left. +5f_2^2 + \frac{f_0^2}{6}\left[(6\xi_S+1)^2 + (6\xi_H+1)^2 + 4(6\xi_S+1)(6\xi_H+1)\right]\right\} \\
(4\pi)^2 \frac{d\lambda_H}{d\ln\mu} &= \frac{9}{8}g_2^4 + \frac{9}{20}g_1^2g_2^2 + \frac{27}{200}g_1^4 - 6y_t^4 + 24\lambda_H^2 + \lambda_{HS}^2 + \frac{\xi_H^2}{2}\left[5f_2^4 + f_0^4(1+6\xi_H)^2\right] \\
&\quad +\lambda_H\left(5f_2^2 + f_0^2(1+6\xi_H)^2 + 12y_t^2 - 9g_2^2 - \frac{9}{5}g_1^2\right).
\end{aligned}
$$

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RGEs for the scalar/graviton couplings

Complicated calculation (but computer algebra helps!)

$$(4\pi)^2 \frac{d\xi_{ab}}{d\ln\mu} = \frac{1}{6}\lambda_{abcd}\left(6\xi_{cd}+\delta_{cd}\right) + \left(6\xi_{ab}+\delta_{ab}\right)\sum_k \left[\frac{Y_2^k}{3} - \frac{C_{2S}^k}{2}\right] + \\ -\frac{5f_2^4}{3f_0^2}\xi_{ab} + f_0^2\xi_{ac}\left(\xi_{cd}+\frac{2}{3}\delta_{cd}\right)\left(6\xi_{db}+\delta_{db}\right)$$

For the SM H plus the complex scalar singlet S the RGEs become:

$$(4\pi)^2 \frac{d\xi_S}{d\ln\mu} = (1+6\xi_S)\frac{4}{3}\lambda_S - \frac{2\lambda_{HS}}{3}(1+6\xi_H) + \frac{f_0^2}{3}\xi_S(1+6\xi_S)(2+3\xi_S) - \frac{5}{3}\frac{f_2^4}{f_0^2}\xi_S$$

$$(4\pi)^2 \frac{d\xi_H}{d\ln\mu} = (1+6\xi_H)(2y_t^2 - \frac{3}{4}g_2^2 - \frac{3}{20}g_1^2 + 2\lambda_H) - \frac{\lambda_{HS}}{3}(1+6\xi_S) + \\ +\frac{f_0^2}{3}\xi_H(1+6\xi_H)(2+3\xi_H) - \frac{5}{3}\frac{f_2^4}{f_0^2}\xi_H$$

back to main slides

RGE for the gravitational couplings

Huge calculation ... (computer algebra practically needed!!)

$$(4\pi)^2 \frac{df_2^2}{d\ln\mu} = -f_2^4\left(\frac{133}{10} + \frac{N_V}{5} + \frac{N_f}{20} + \frac{N_s}{60}\right)$$

$$(4\pi)^2 \frac{df_0^2}{d\ln\mu} = \frac{5}{3}f_2^4 + 5f_2^2 f_0^2 + \frac{5}{6}f_0^4 + \frac{f_0^4}{12}(\delta_{ab} + 6\xi_{ab})(\delta_{ab} + 6\xi_{ab})$$

Here N_V, N_f, N_s are the number of vectors, Weyl fermions and real scalars.

In the SM $N_V = 12$, $N_f = 45$, $N_s = 4$.

We confirmed the calculations of *[Avramidi (1995)]*
rather than those of *[Fradkin and Tseytlin (1981,1982)]*

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Agravity inflation

All scalar fields in agravity are inflaton candidates

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example (the minimal model): h, the Higgs of gravity s, the scalar σ in $g_{\mu\nu}$

To see σ

$$\frac{R^2}{6f_0^2} \to \frac{R^2}{6f_0^2} - \underbrace{\frac{(R + 3f_0^2\sigma/2)^2}{6f_0^2}}_{\text{zero on-shell}}$$

By redefining $g^E_{\mu\nu} = g_{\mu\nu} \times f/M_P^2$ with $f = \xi_S s^2 + \xi_H h^2 + \sigma$ one obtains ...

$$\sqrt{|\det g_E|}\left\{\frac{M_P^2}{2}R_E + M_P^2\left[\frac{(\partial_\mu s)^2 + (\partial_\mu h)^2}{2f} + \frac{3(\partial_\mu f)^2}{4f^2}\right] - U\right\} + \cdots$$

as well as their effective potential:

$$U = \frac{M_P^4}{f^2}\left(V + \frac{3f_0^2}{8}\sigma^2\right)$$

Agravity inflation

We identify inflaton = s by taking the other scalar fields heavy ...

Then we can easily convert s into a scalar s_E with canonical kinetic term and find

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{1}{U}\frac{\partial U}{\partial s_E}\right)^2 = \frac{1}{2}\frac{\xi_S}{1+6\xi_S}\left(\frac{\beta_{\lambda_S}}{\lambda_S} - 2\frac{\beta_{\xi_S}}{\xi_S}\right)^2$$

$$\eta \equiv M_P^2\frac{1}{U}\frac{\partial^2 U}{\partial s_E^2} = \frac{\xi_S}{1+6\xi_S}\left(\frac{\beta(\beta_{\lambda_S})}{\lambda_S} - 2\frac{\beta(\beta_{\xi_S})}{\xi_S} + \frac{5+36\xi_S}{1+6\xi_S}\frac{\beta_{\xi_S}^2}{\xi_S^2} - \frac{7+48\xi_S}{1+6\xi_S}\frac{\beta_{\lambda_S}\beta_{\xi_S}}{2\lambda_S\xi_S}\right)$$

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We can insert them in the formulae for the observable parameters A_s, n_s and $r = \frac{A_t}{A_s}$:

$$n_s = 1 - 6\epsilon + 2\eta, \qquad A_s = \frac{U/\epsilon}{24\pi^2 M_P^4}, \qquad r = 16\epsilon$$

where everything is evaluated at about $N \approx 60$ e-foldings when the inflaton $s_E(N)$ was

$$N = \frac{1}{M_P^2}\int_0^{s_E(N)} \frac{U(s_E)}{U'(s_E)}\, ds_E$$

Agravity inflation: analytic approximation

$$\left\{\begin{array}{rcl}\lambda_S(s) & \simeq & 0\\ \beta_{\lambda_S}(s) & = & 0\\ \xi_S(s)s^2 & = & M_P^2\end{array}\right. \qquad \rightarrow \qquad \lambda_S(\mu\approx s)\approx \frac{b}{2}\ln^2\frac{s}{\langle s\rangle}\,, \qquad \underbrace{\xi_S(\mu)\approx\xi_S}_{\text{for simplicity}}$$

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The Einstein-frame potential is nearly quadratic around its minimum:

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Inserting s_E at $N \approx 60$ e-foldings, $s_E(N) \approx 2\sqrt{N} M_P$, ... **we obtain the predictions**

$$n_s \approx 1 - \frac{2}{N} \approx 0.967, \qquad r \approx \frac{8}{N} \approx 0.13, \qquad A_s \approx \frac{g^4 N^2}{24\pi^2 \xi_S(1 + 6\xi_S)}$$

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VEVs above M_P, $s_E\approx 2\sqrt{N}M_P$, are needed for a quadratic potential

Agravity predicts physics above M_P, and a quadratic potential is a good approximation, even at $s_E>M_P$, because coefficients of higher order terms are suppressed by extra powers of the loop expansion parameters, which are small at weak coupling

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