The amplification of magnetic fields by the turbulent dynamo in the early Universe

Jacques M. Wagstaff

Phys. Rev. D**89**:103001 arXiv:1304.4723

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Service de Physique Théorique,
Université Libre de Bruxelles, May 23rd, 2014

Magnetic fields in the Universe

Observational evidence:

- Stars, solar system, Milky Way

  (Donati&Landstreet 2009; Weilebinski 2005)

- Galaxies - low and high redshift - $B = \mathcal{O}(10)\mu$G

  (Bernet et al. 2008; Beck 2012; Hammond et al. 2012)

- Clusters and superclusters of galaxies - $B = \mathcal{O}(1)\mu$G

  (Clarke et al. 2001; Ferretti et al. 2012; Xu et al. 2006)

- Voids of the Large Scale Structure - $\gtrsim 10^{-7}$nG

  (Neronov&Vovk 2010; Tavecchio et al. 2010, 2011)

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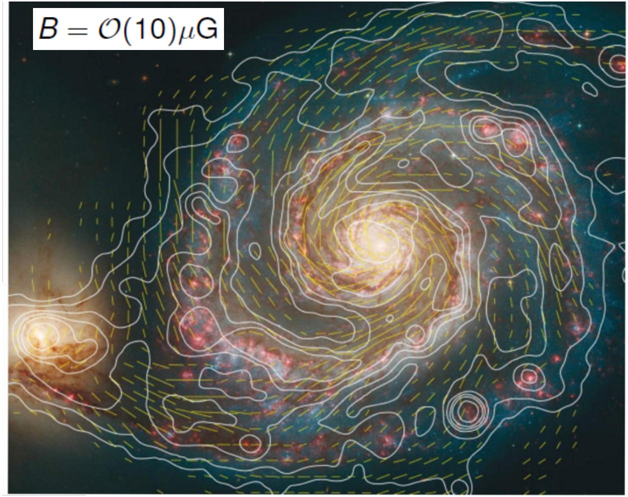
Synchrotron radiation + Faraday rotation of polarized component

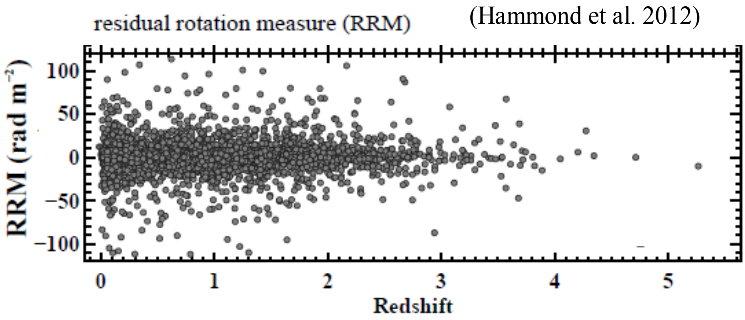
M51:
Radio emission
B-vectors
(Fletcher et al. 2011)



MPIfR Bonn and *Hubble Heritage Team*

Magnetic fields in low and high redshift Galaxies

Faraday Rotation Measures



- No significant evolution with redshift: $B = \mathcal{O}(10)\mu\text{G}$
- *B*-fields generated quickly at an early epoch

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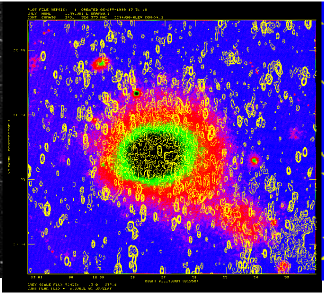
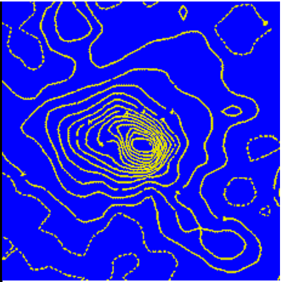
Magnetic fields in Clusters of Galaxies

The Coma galaxy cluster:





Optical

X-ray

Radio

$B \sim \mu\mathrm{G}$

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Extragalactic magnetic fields *(Neronov and Vovk 2010)*

- High energy γ-rays ($>$ TeV) cannot propagate over cosmological distances $\Longrightarrow$ Interact with extragalactic background light
  $\Longrightarrow$ create $e^{\pm}$ pairs
  $\Longrightarrow$ secondary cascade of lower energy γ-rays
  - No *B*-fields - flux of secondary cascade contributes to primary flux
  - With *B*-fields - extended emission and even non-observations of secondaries
- Deflection depends on field strength and coherence length.
- Fermi - non-observations of GeV γ-rays from distant blazars imply intergalactic $B \gtrsim 10^{-7}$ nG (λ_c dependent)

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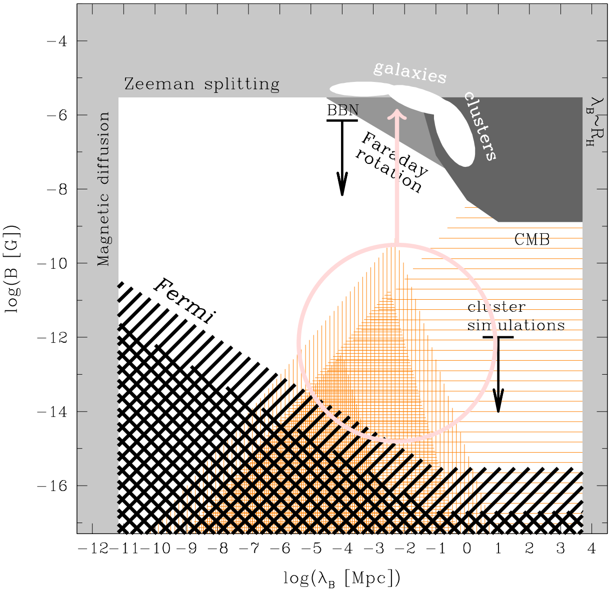

Theoretically:

Inflation
(Turner&Widrow 1988)
$\sim (10^{-25} - 10^{-1})$nG

Phase transitions
(Sigl et al. 1997)
$\sim 10^{-20}$nG (EW)
$\sim 10^{-11}$nG (QCD)

Battery Mechanisms
(Harrison 1970)
$\sim 10^{-20}$nG

Amplification required!

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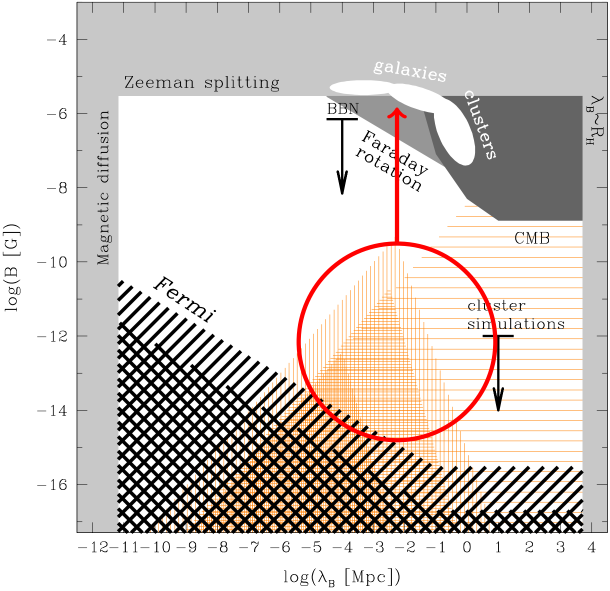

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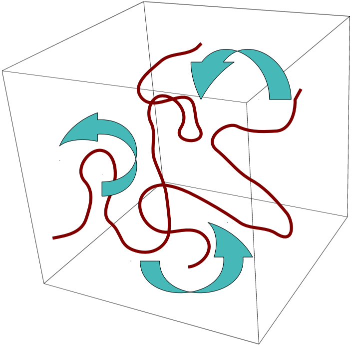
The dynamo mechanism

Converts turbulent kinetic energy into magnetic energy



Large-scale dynamo:

- Galaxy differential rotation

$$B_{\rm seed} \sim 10^{-30}\ \mathrm{G} \quad \Longrightarrow \quad B \sim \mu\mathrm{G}$$

(Davis et al. 1999)

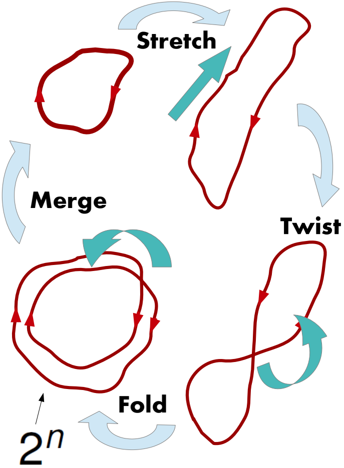
▷ difficult to explain strong fields in other types of galaxies, young galaxies, clusters, and in the voids

Small-scale dynamo:

- Structure formation:
  ▷ gravitational collapse, accretion, and supernovae explosions

The small-scale dynamo mechanism

Amplification of magnetic fields from turbulence



$\langle B^2 \rangle \propto \exp(2\Gamma t)$

Requires turbulence:

Reynolds number: $R_e = \frac{v(l)l}{\nu} \gg 1$

MHD:
$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} - (\mathbf{v}_A \cdot \nabla)\mathbf{v}_A = \nu \nabla^2 \mathbf{v}$

Spectrum: $v(l) \propto l^{\vartheta}$
$1/3 \leqslant \vartheta \leqslant 1/2$
Kolmogorov $\leqslant \vartheta \leqslant$ Burgers
(incompressible) (highly compressible)

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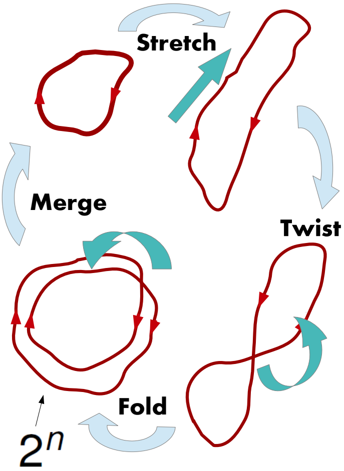
The small-scale dynamo mechanism

Amplification of magnetic fields from turbulence



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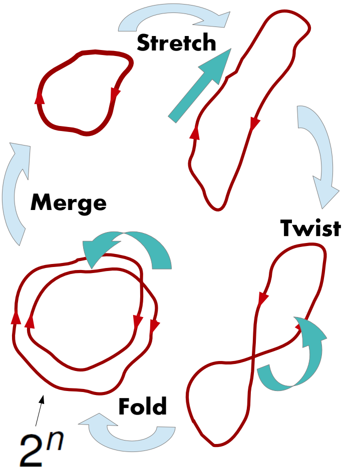

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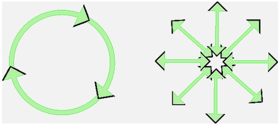

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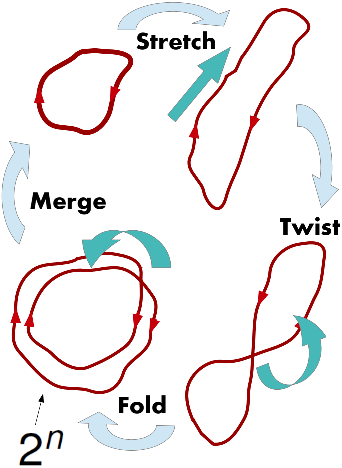
Amplification of magnetic fields from turbulence



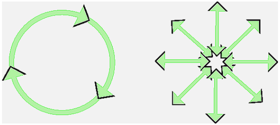
$\langle B^2 \rangle \propto \exp(2\Gamma t)$

Requires $R_m > R_m^{\mathrm{cr}}$:

Magnetic Reynolds no.: $R_m = \frac{v(l)l}{\eta}$

MHD:
$\frac{\partial \mathbf{v}_A}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v}_A - (\mathbf{v}_A \cdot \nabla)\mathbf{v} = \eta \nabla^2 \mathbf{v}_A$

Spectrum: $v(l) \propto l^{\vartheta}$
$60 \lesssim R_m^{\mathrm{cr}} \lesssim 2700$
Kolmogorov $\leqslant R_m^{\mathrm{cr}} \leqslant$ Burgers
(incompressible) (highly compressible)



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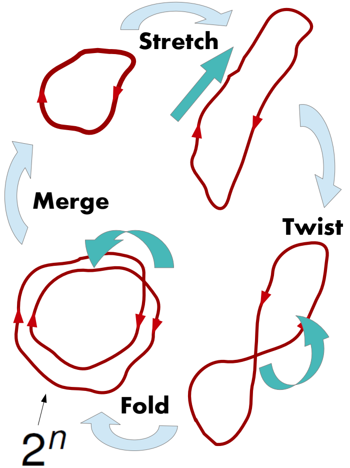

$\langle B^2 \rangle \propto \exp(2\Gamma t)$

Depends on environment:

Prandtl: $P_m \equiv R_m/R_e = \nu/\eta$

Grows on
eddy-turnover time scale
$\tau_{\rm eddy} = l/v$

Growth rate Γ depends on the
kinetic Reynolds number

$R_e = \tau_{\rm diss}/\tau_{\rm eddy} = vl/\nu$

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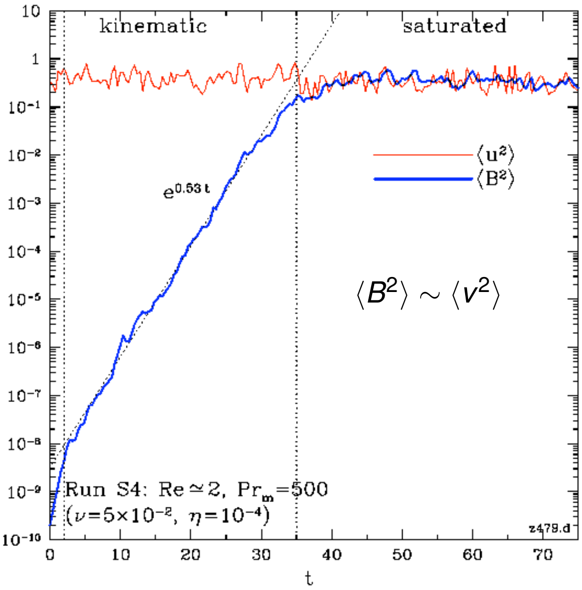
Simulations of the small-scale dynamo *(Schekochihin et al. 2004)*



$$\langle B^2 \rangle \sim \langle v^2 \rangle$$

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Decompose:

$$\boldsymbol{B} = \langle \boldsymbol{B} \rangle + \delta \boldsymbol{B} \qquad \boldsymbol{v} = \langle \boldsymbol{v} \rangle + \delta \boldsymbol{v}$$

Velocity spectrum:

$$\langle \delta v_i(\boldsymbol{r}_1, t_1) \delta v_j(\boldsymbol{r}_2, t_2) \rangle = T_{ij}(r) \delta(t_1 - t_2)$$

***Assumptions*:**

- Gaussian
- Homogeneous and Isotropic in space
- Instantaneously correlated in time

$$\boxed{\langle \delta B_i(\boldsymbol{r}_1, t) \delta B_j(\boldsymbol{r}_2, t) \rangle = M_{ij}(r, t)}$$

$\Rightarrow$ Induction equation $\Rightarrow$ Stochastic differential equation

The small-scale dynamo - Analytic model

The Kazantsev model *(Kazantsev 1967, Subramanian 1997, Brandenburg&Subramanian 2005)*

Kazantsev equation: Schrödinger equation-like equation:

$$\frac{\partial \Psi}{\partial t} = -\hat{H}\Psi \qquad \text{where} \qquad \begin{cases} T_{ij}(r) & \Longrightarrow U(r) \\ \Psi(t) & \Longrightarrow M_{ij}(t) \end{cases}$$

Find solution for fluctuating field:

$$\langle B^2 \rangle \propto \exp(2\Gamma t)$$

General types of turbulence: $v(l) \propto l^{\vartheta}$ and $P_m = \nu/\eta \gg 1$

$$\Gamma = \frac{(163-304\vartheta)}{60} R_e^{(1-\vartheta)/(1+\vartheta)} / \tau_{\rm eddy}$$ *(Schober et al. 2012)*

Velocity spectrum modelled so that: *(Schober et al. 2012)*

- if $\vartheta = 1/3$ Kolmogorov turbulence (incompressible)
- if $\vartheta = 1/2$ Burgers turbulence (highly compressible)

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The small-scale dynamo

Applications

The SSD mechanism requires turbulence

Turbulence driven by:
gravitational collapse, accretion and Supernovae explosions

- First stars *(e.g. Schleicher et al. 2010)*
- First galaxies *(e.g. Schober et al. 2012)*
- Clusters of Galaxies *(Subramanian et al. 2012)*

What about before structure formation?

- Turbulence from primordial density perturbations
- Turbulence injected during first-order phase transitions

Turbulence from primordial density perturbations

Cosmological perturbations *(Kodama&Sasaki 1984)*

The primordial density perturbations generate velocity fluctuations

- Fluid 3-velocity perturbations:

$$\boxed{\boldsymbol{v}(\boldsymbol{k},\eta) = -\frac{i\boldsymbol{k}}{2\mathcal{H}^2}\left[\Phi'(\boldsymbol{k},\eta) + \mathcal{H}\Phi(\boldsymbol{k},\eta)\right]}$$

$$\boldsymbol{v}(\boldsymbol{k},\eta) = -i\frac{3\sqrt{3}}{2}\hat{\boldsymbol{k}}\left[\sin(k\eta/\sqrt{3}) - 2j_1(k\eta/\sqrt{3})\right]\Phi_0(k).$$

At linear order in Φ, turbulent velocity is purely irrotational $v_{\|}$

$$\boxed{\mathcal{P}_v(k) = \frac{27}{4}\left[\sin(k\eta/\sqrt{3}) - 2j_1(k\eta/\sqrt{3})\right]^2 \mathcal{P}_{\Phi_0}(k)}$$

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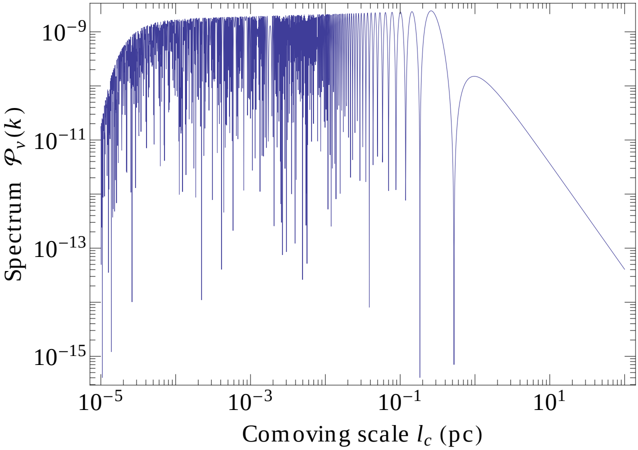
Velocity perturbations in the radiation era

Spectrum of velocity perturbations at $T \simeq 0.2$ GeV (neutrino era)



$$v_{\|} \sim \sqrt{\mathcal{P}_v} \simeq 5 \times 10^{-5} c$$

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Turbulence from first-order phase transitions

(Steinhardt 1982, Kamionkowski et al. 1994, Kosowsky et al. 2002)

- Bubbles of new phase collide and merge - injecting large kinetic energy into plasma.
- Phase boundary propagate via *detonations* which can be modelled analytically.

$$v_L^{\rm rms} \sim \sqrt{\alpha\kappa}$$

- $\alpha = \rho_{\rm vac}/\rho_{\rm thermal}$ - Strength of phase transition, $\alpha \sim (10^{-5} - 10^{-1})$
- $\kappa(\alpha) = \rho_{\rm kin}/\rho_{\rm vac}$ - Vacuum energy converted to kinetic energy

$$\boxed{v_L^{\rm rms} \sim (10^{-4} - 10^{-1})c}$$

Stirring scale L_c limited by Hubble horizon size at the time of PT:

- $1/aH|_{\rm QCD} \sim 0.1$ pc
- $1/aH|_{\rm EW} \sim 10^{-4}$ pc

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Injection of kinetic energy $\Longrightarrow$ Large enough $R_e \gg 1 \Longrightarrow$ Turbulence

$$R_e(L_c) \simeq 8 \frac{v(L_c) L_c}{l^{\nu,\gamma}_{\mathrm{mfp},c}} \gg 1 \,, \qquad \tau_{\mathrm{eddy}} = \frac{a L_c}{v(L_c)}$$

With the expansion R_e decreases and τ_{eddy} increases.

- Turbulence due to neutrinos ν, $R_e \gg 1$ - The neutrino era
- Viscous regime, $R_e < 1$
- Neutrinos decoupling at $T \simeq 2.6$ MeV
- Turbulence due to photons γ, $R_e \gg 1$ - The photon era
- $e^{\pm}$ annihilation from $m_e \gtrsim T \gtrsim 20$ keV
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Conditions:

$$L_c < l_H = 4H^{-1}/a \qquad \text{Coherence length}$$

$$\tau_{\text{eddy}} = aL_c/v < H^{-1} \qquad \text{Many turnover times}$$

$$R_e > R_e^{\text{cr}} \sim (10^2 - 10^3) \qquad \text{Highly turbulent}$$

hence

$$\boxed{R_e^{\text{cr}} l_c^{\gamma}/v \lesssim L_c \lesssim vH^{-1}/a}$$

Silk damping scale:

$$l_D^2 = \int \frac{l_c^{\gamma}}{a}\,\mathrm{d}t \qquad \Longrightarrow \qquad v \propto e^{-(l_D(t)/l)^2}.$$

Dissipation of small-scale perturbations caused by γ random walking out of overdense regions.

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The small-scale dynamo in the radiation era
The excited scales

Conditions:

$L_c < l_H = 4H^{-1}/a$	Coherence length
$\tau_{\rm eddy} = aL_c/v < H^{-1}$	Many turnover times
$R_e > R_e^{\rm cr} \sim (10^2 - 10^3)$	Highly turbulent

hence

$$\boxed{R_e^{\rm cr} l_c^{\gamma}/v \lesssim L_c \lesssim vH^{-1}/a}$$

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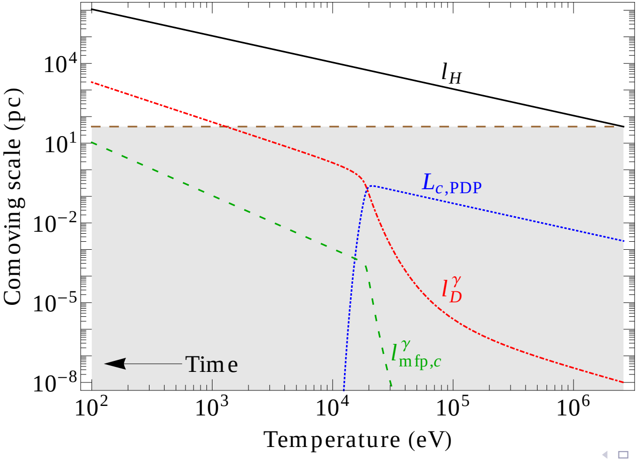
Dissipation of small-scale perturbations caused by γ random walking out of overdense regions.

The photon era: 100 eV $\lesssim T \lesssim$ 2.6 MeV



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$L_c < l_H = H^{-1}/a$

Many turnover times:

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Silk damping:

$l_D^2 = \int \frac{l_c^\gamma}{a}\,\mathrm{d}t$

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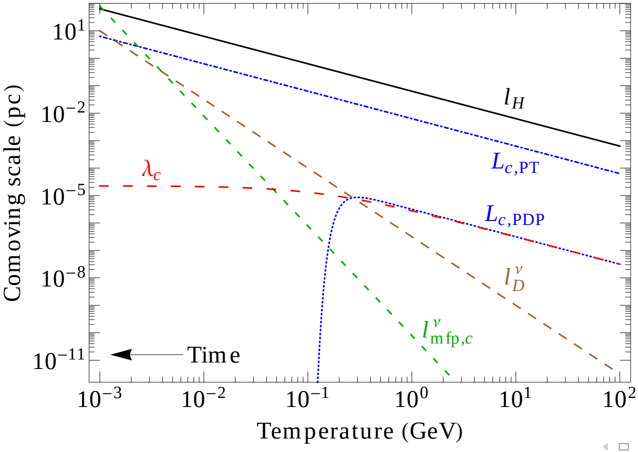
The neutrino era: 2.6 MeV $\lesssim T \lesssim$ 100 GeV



Coherence length:

$L_c < l_H = H^{-1}/a$

Many turnover times:

$\tau_{\mathrm{eddy}} = aL_c/v < H^{-1}$

Damping scale:

$l_D^2 = \int \frac{l_c^\nu}{a}\,\mathrm{d}t$

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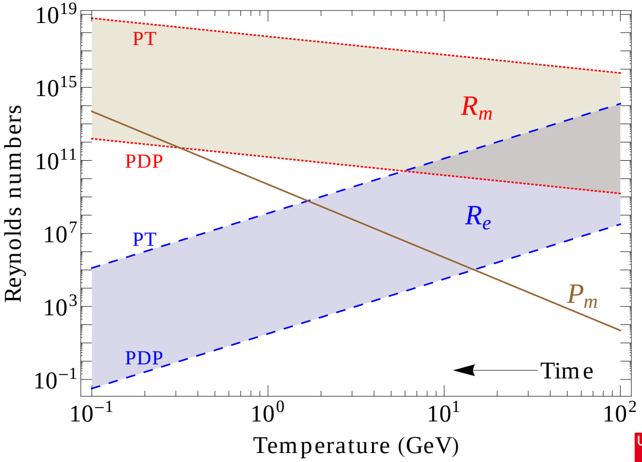
The small-scale dynamo in the radiation era

The Reynolds number

The neutrino era: $2.6\ \mathrm{MeV} \lesssim T \lesssim 100\ \mathrm{GeV}$



$P_m \gg 1$ we can neglect dissipative effects due to finite conductivity.

Magnetic field amplification

Estimate the growth rate from the Kazantsev model:

$$\langle B^2 \rangle \propto \exp(2\Gamma t)$$

Turbulence with spectrum: $v(l) \propto l^{\vartheta}$ and $P_m \gg 1$
(Kolmogorov: $\vartheta = 1/3$), (Burgers: $\vartheta = 1/2$)

$$\Gamma = \frac{(163-304\vartheta)}{60} R_e^{(1-\vartheta)/(1+\vartheta)} / \tau_{\rm eddy}$$

(Schober et al. 2012)

Large amplification factor in very short time!

$$N \equiv \int \Gamma(t)\mathrm{d}t \qquad \longrightarrow \qquad \langle B^2 \rangle \propto e^{2N}$$

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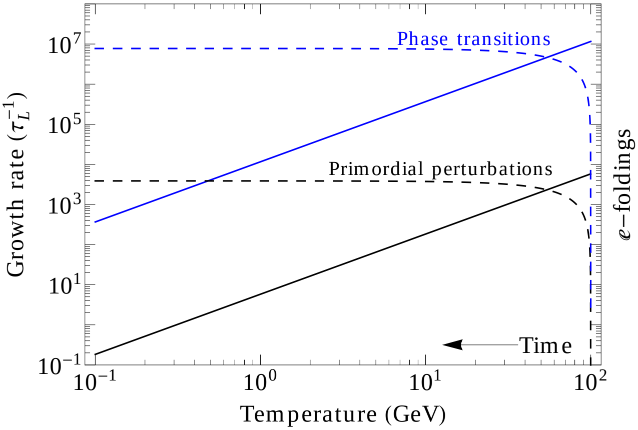


Growth rate:

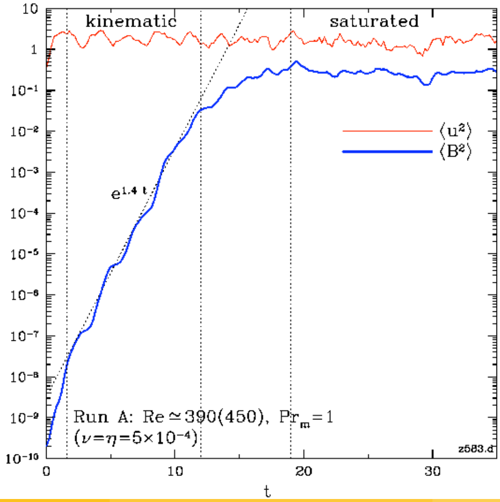
$\Gamma/\tau_{\mathrm{eddy}}^{-1}$

e-foldings:

$N = \int \Gamma(t)\,\mathrm{d}t$

Saturation given by equipartition of magnetic and kinetic energy

$$\langle B^2 \rangle \approx 4\pi\varepsilon(\rho + p)\langle v^2 \rangle$$



Magnetic field saturation *(Schekochihin et al. 2004, Federrath et al. 2011)*

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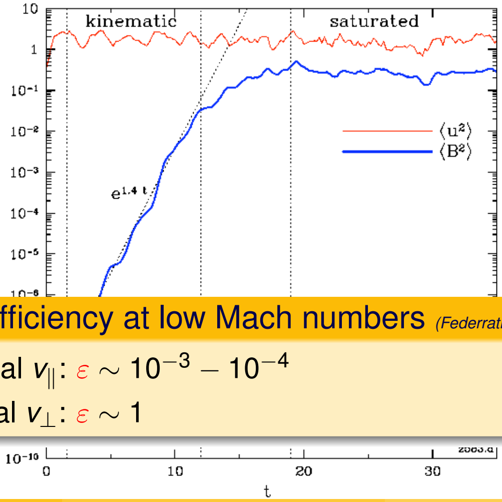

Saturation efficiency at low Mach numbers *(Federrath et al. 2011)*

- Irrotational $v_{\parallel}$: $\varepsilon \sim 10^{-3} - 10^{-4}$
- Rotational $v_{\perp}$: $\varepsilon \sim 1$

Results

The SSD mechanism can amplify to saturation tiny seed fields

$$B_0^{\rm seed} \simeq (10^{-30} - 10^{-20})\ {\rm nG}$$

Turbulence generated by primordial density perturbations:

$$a^2 B_{\rm rms} \sim 1\varepsilon^{\frac{1}{2}}\ {\rm nG} \quad \text{on scales up to} \quad \lambda_c \sim 10^{-5}\ {\rm pc}$$

Turbulence generated by first-order phase transitions:

$$a^2 B_{\rm rms} \sim (10^{-3} - 1)\varepsilon^{\frac{1}{2}}\ \mu{\rm G} \quad \text{on scales} \quad \lambda_c \sim (10^{-4} - 10^{-1})\ {\rm pc}$$

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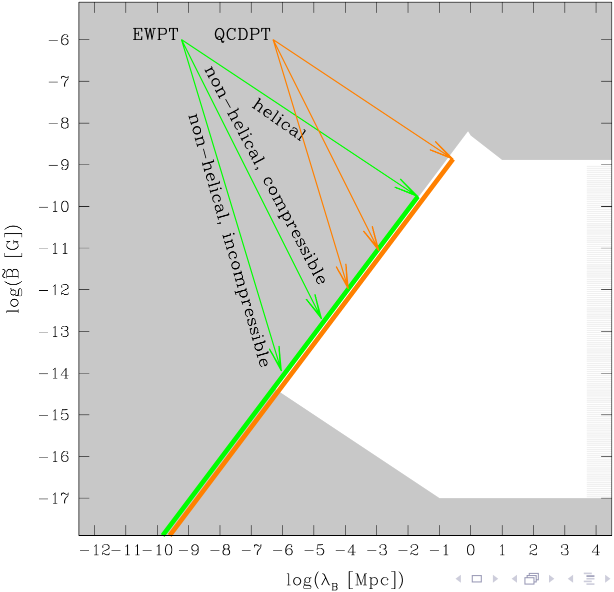
Subsequent evolution of magnetic fields *(Durrer&Neronov 2013)*

MHD turbulence decay



Turbulence generated by primordial density perturbations:

$$B_0^{\mathrm{rms}} \sim 10^{-6}\varepsilon^{\frac{1}{2}}\ \mathrm{nG} \quad \text{on scales up to} \quad \lambda_c \sim 10^{-1}\ \mathrm{pc}$$

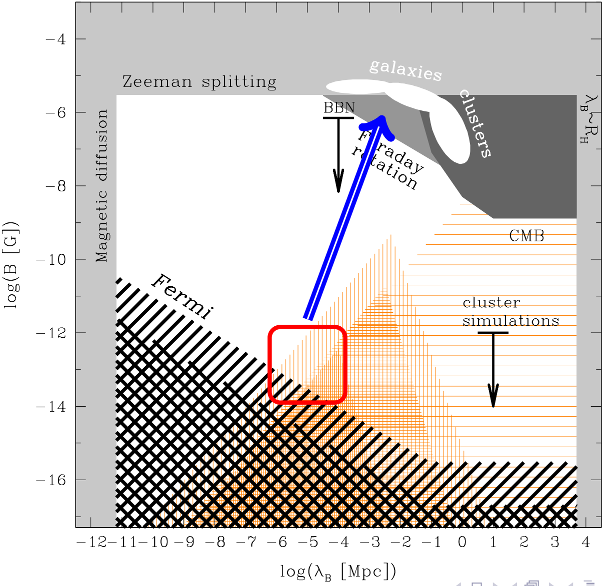
Too weak on too short scales to explain observed intergalactic magnetic fields

Turbulence generated by first-order phase transitions:

$$B_0^{\mathrm{rms}} \sim (10^{-6} - 10^{-3})\varepsilon^{\frac{1}{2}}\ \mathrm{nG} \quad \text{on scales} \quad \lambda_c \sim (10^{-1} - 10^{2})\ \mathrm{pc}$$

Strong enough to explain observed intergalactic magnetic fields

Intergalactic magnetic fields *(Neronov and Vovk 2010)*



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Summary

The small-scale dynamo in the radiation era

- Before structure formation the SSD can be effective at amplifying magnetic seed fields.
- Kinetic energy generated by primordial density perturbations and first-order phase transitions
- Fully developed turbulence is expected on scales $l_D \lesssim L_c \lesssim l_e$ where $R_e \gg 1$, many eddy-interactions and no damping
- Identified epoch in RD in which conditions are good for SSD
- Kazantsev model gives large growth rate of seed field: $\Gamma \propto R_e^{1/2}$
- For seed fields $10^{-30} \lesssim B_0^{\mathrm{seed}}/\mathrm{nG} \lesssim 10^{-20}$ magnetic energy saturates rapidly to:

$$B_0 \approx 10^{-3} \varepsilon^{1/2}\ \mathrm{nG} \quad \text{on scales up to} \quad \lambda_c \sim 100\ \mathrm{pc}$$

- These strong extragalactic fields can explain Fermi observations
- Provide initial magnetic fields for structure formation
- Other observable signatures?