Dark matter distribution around massive black holes and in phase-space

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Washington University in St. Louis

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Outline

Introduction: detecting dark matter

The DM distribution around a supermassive BH

DM annihilation in phase-space

Phase space distribution: Eddington's inversion

Conclusions

Laleh Sadeghian, FF & Clifford M. Will, PRD88, 063522 (2013) [arXiv:1305.2619]

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The case for dark matter

Most economical explanation of:

- The rate of expansion of the universe.
- The formation of large scale structure.
- The dynamics of galaxies, clusters, . . .

Expected in natural extensions of the SM.

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An example: WIMPs

Similar to a heavy neutrino, $m_\chi \approx 100$ GeV, weak-scale interactions produce observed abundance from thermal decoupling:

$$\Rightarrow < \sigma v > \approx 3 \times 10^{-26}\ \mathrm{cm}^3\ \mathrm{s}^{-1}$$

The same interactions make it potentially detectable:

- $\chi\chi \to \gamma\gamma,\ \pi^0,\ e^\pm, \ldots$
- $\chi N \to \chi N$

Other examples include axions, MeV particles, ...

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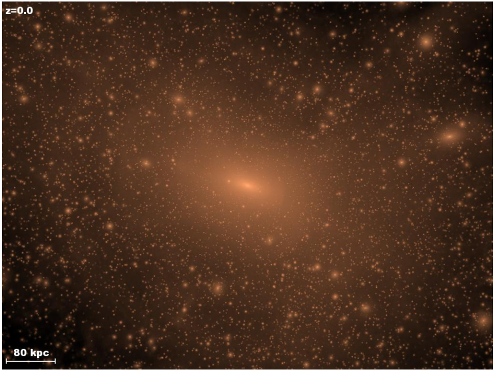
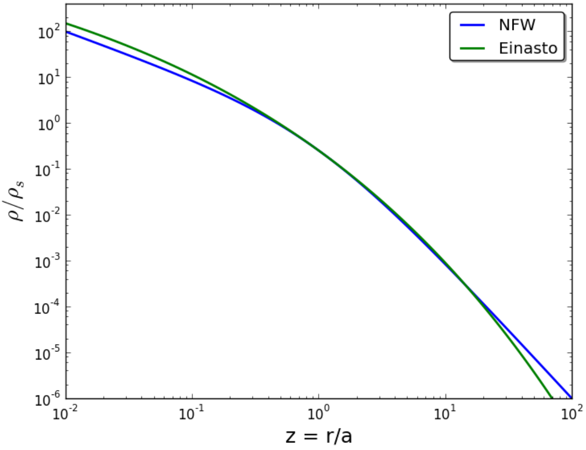
$$Flux = \underbrace{\frac{<\sigma v>}{4\pi m_{dm}^2}\frac{dN_\gamma}{dE_\gamma}}_{\text{Number of SM particles}} \times \underbrace{\int_0^\infty \rho^2(r)dl}_{\text{Amount of DM}^2}$$

- *Astrophysical factor* suggests looking at GC, dwarf spheroidals, ...
- Photons and neutrinos point back to the source, while charged particles diffuse.

The distribution of DM: simulations



1 billion 4,100 $M_\odot$ particles. 0.5 kpc in the host halo.


NFW
Einasto
ρ/ρ_s
z = r/a
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
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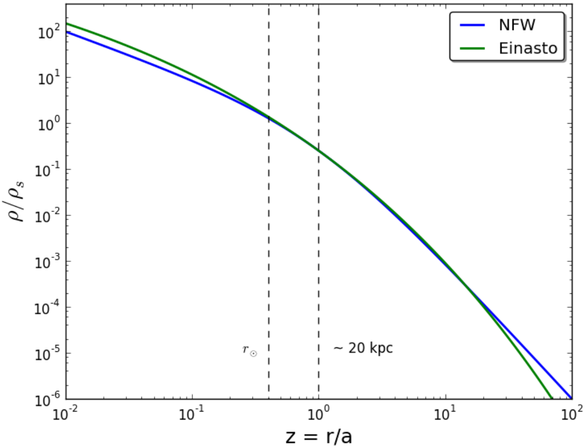

NFW
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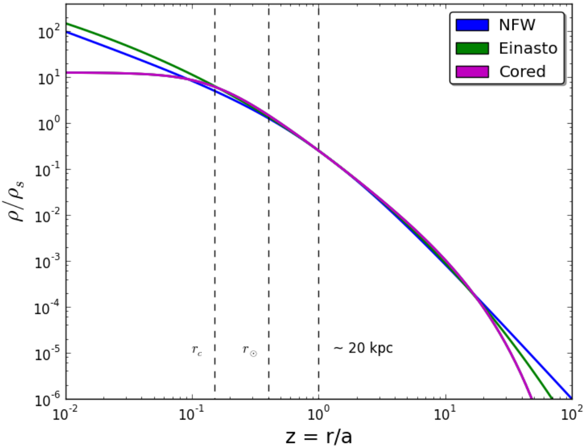
The distribution of DM: observations

Jeans' equation shows that $M/L \sim 1000$. Clean systems.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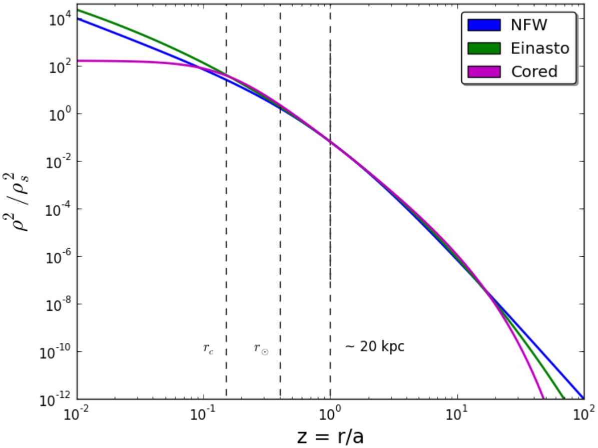

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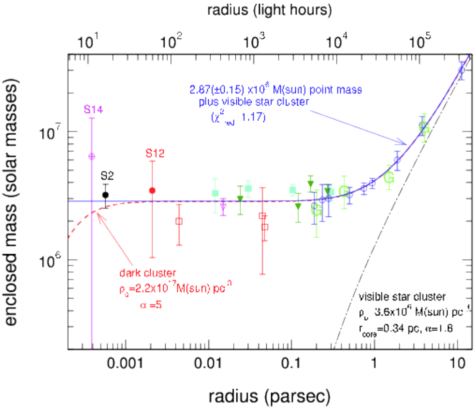
The central supermassive black hole

- Will focus on the super-massive BH at the center of the Galaxy.
- Similar effects will occur in the cores of AGNs, or in IMBHs.



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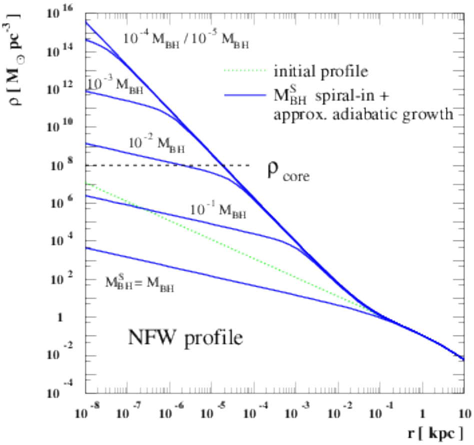
We will assume that a black hole of mass $4 \times 10^6 M_\odot$ grows adiabatically over $\sim 10^{10}$yr.



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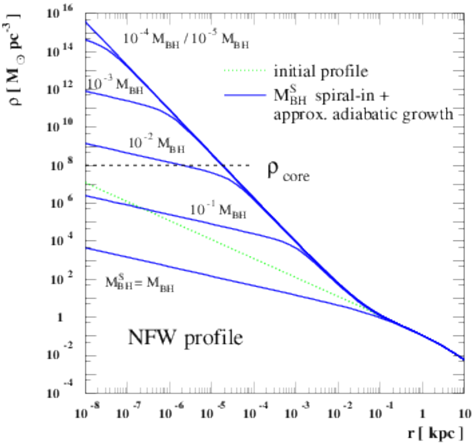

Is the growth adiabatic?

- Growth time
- Dynamical time

$$\frac{r_h}{\sigma} \leq \frac{m}{\dot{m}_{\mathrm{Edd}}}$$

$$r_h \approx \frac{Gm}{\sigma^2} \rightarrow t_{\mathrm{dyn}} \approx 10^4 \mathrm{yr} \leq t_{\mathrm{Salpeter}} \approx 5 \times 10^7 \mathrm{yr}$$

Caveats: Hierarchical mergers, initial BH seed off-center, kinetic heating of DM by stars, . . .

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Growing a BH: Newtonian analysis

We are interested in the DM density:

$$\begin{aligned}\rho &= \int f(E,L)d^3\boldsymbol{v} \\ &= 4\int dE \int LdL \int dL_z \frac{f(E,L)}{r^4|v_r||v^\theta|\sin\theta} \\ &= 4\pi \int dE \int LdL \frac{f(E,L)}{r^2|v_r|}\end{aligned}$$

The limits of integration are set by the requirements:

- $|v_r| = (2E - 2\Phi - L^2/r^2)^{1/2}$ real $\Rightarrow 0 \leq L \leq [2r^2(E-\Phi)]^{1/2}$.
- DM particle is bound to the halo $\Rightarrow \Phi(r) \leq E \leq 0$.

Take into account particles trapped inside the event horizon by modifying boundary conditions in an *ad hoc* manner: $L \geq 2cR_S$.

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Each particle in an initial DM distribution $f(E, L)$, will react to the change in Φ caused by the growth of the BH by altering its E, L and L_z. However, *the adiabatic invariants remain fixed*:

$$\begin{aligned} I_r(E,L) &\equiv \oint v_r dr = \oint dr \sqrt{2E - 2\Phi - L^2/r^2}\,, \\ I_\theta(L,L_z) &\equiv \oint v_\theta d\theta = \oint d\theta \sqrt{L^2 - L_z^2 \sin^{-2}\theta} = 2\pi(L - L_z)\,, \\ I_\phi(L_z) &\equiv \oint v_\phi d\phi = \oint L_z d\phi = 2\pi L_z\,. \end{aligned} \quad (1)$$

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Newtonian BH

For a Newtonian point mass,

$$I_r(E, L) = 2\pi \left(-L + \frac{Gm}{\sqrt{-2E}} \right)$$

And we can find the final DM density in the form:

$$\rho(r) = \frac{4\pi}{r^2} \int_{-Gm/r}^{0} dE \int_{0}^{L_{\max}} LdL \frac{f'(E'(E, L), L)}{\sqrt{2E + 2Gm/r - L^2/r^2}}$$

Young 80, Quinlan *et al.* 95, Gondolo & Silk 95

Growing a BH: Relativistic analysis

1. Generalize the definition of density:

$$J^\mu(x) \equiv \int f^{(4)}(p) \frac{p^\mu}{\mu} \sqrt{-g}\, d^4 p \,,$$

2. Use relativistic expressions to write it in terms of the invariants of motion (energy, angular momentum, ...).
3. Use relativistic expressions for the actions.

For Kerr:

$$\begin{aligned} \mathcal{E} &\equiv -u_0 = -g_{00} u^0 - g_{0\phi} u^\phi \,, & (2) \\ L_z &\equiv u_\phi = g_{0\phi} u^0 + g_{\phi\phi} u^\phi \,, & (3) \\ C &\equiv \Sigma^4 (u^\theta)^2 + \sin^{-2}\theta L_z^2 + a^2 \cos^2\theta (1 - \mathcal{E}^2) \,, & (4) \\ g_{\mu\nu} p^\mu p^\nu &= -\mu^2 \,. & (5) \end{aligned}$$

And need to calculate the jacobian

$$d^4 p = |J|^{-1} d\mathcal{E}\, dC dL_z d\mu$$

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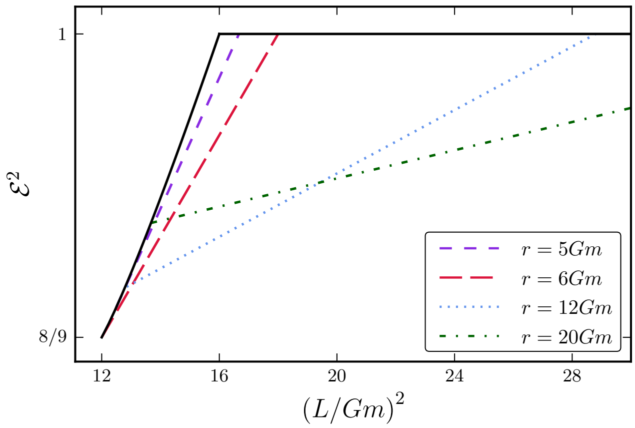
Example: Schwarzschild BH

The positivity of the radial action determines the boundary conditions, *including the effects of the horizon*.



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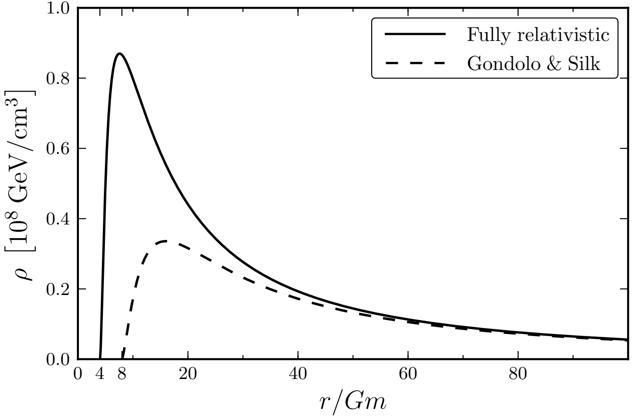
For a constant phase-space distribution:



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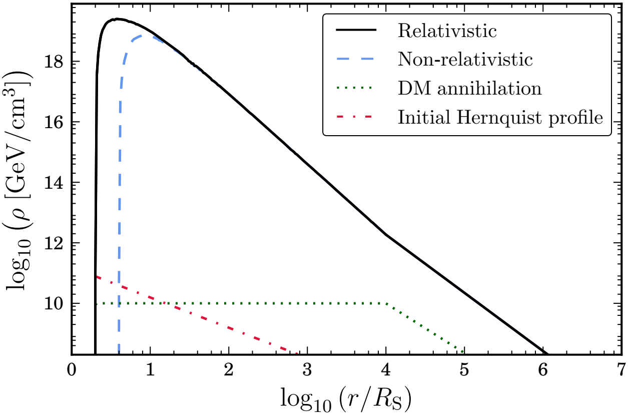
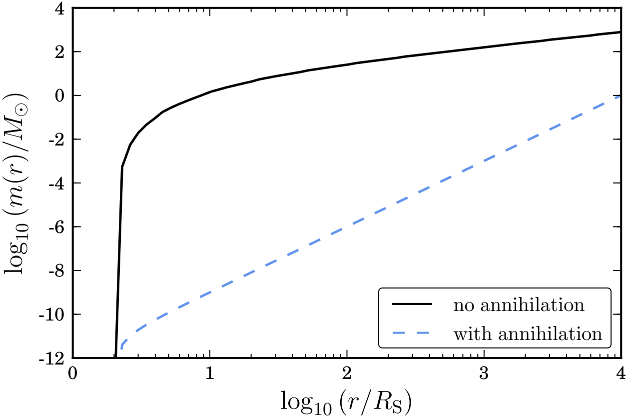
For a, more realistic, cuspy DM distribution:



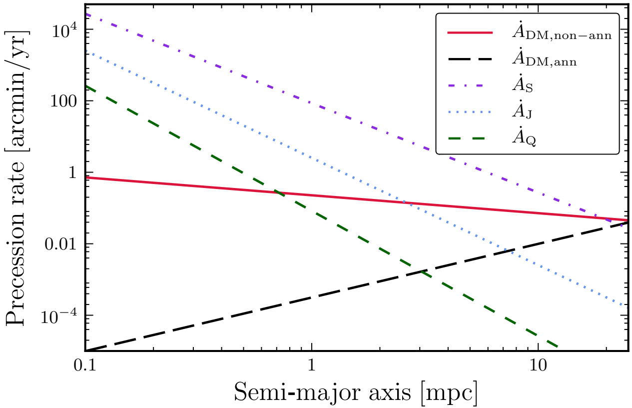
Consequences

The gravitational potential is still dominated by the BH:



Consequences

No big changes for DM annihilation, but precession rates could be observable.



Indirect detection

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The annihilation cross-section

Annihilations in the halo are non-relativistic, $v \approx 10^{-3}$.

The amplitude is analytical for $k \to 0$

$$\mathcal{M} \propto \int e^{ikx} V_{Born}(x)$$

Including factors of $k^l Y_l^m$ in a partial wave expansion, $\sigma \propto k^{2l-1}$

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More complicated velocity dependence

If there are new light particles mediating long-range forces between the dark matter, an enhancement occurs at low velocities:

$$\sigma \to \sigma \times \frac{\pi\alpha}{v}$$

If annihilation proceeds near a resonant state,

$$v\sigma \propto \frac{1}{(v^2/4 + \Delta)^2 + \Gamma_A^2(1-\Delta)/4m_\chi^2}$$

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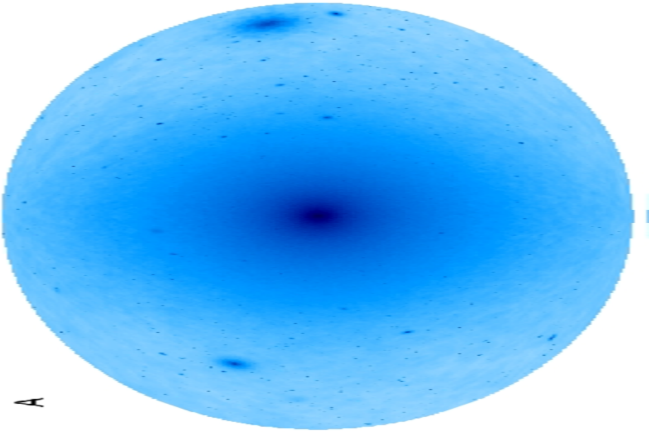

Lattanzi & Silk

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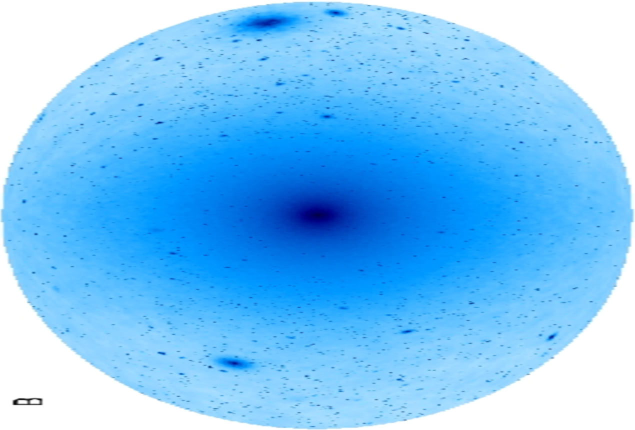

Lattanzi & Silk

What went in calculating the flux?

The averaged cross-section

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But, the flux is

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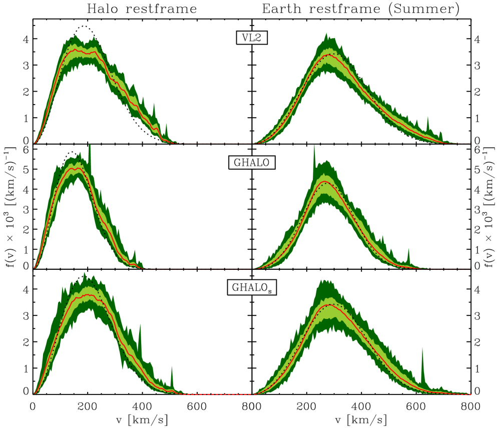

Kuhlen et al.

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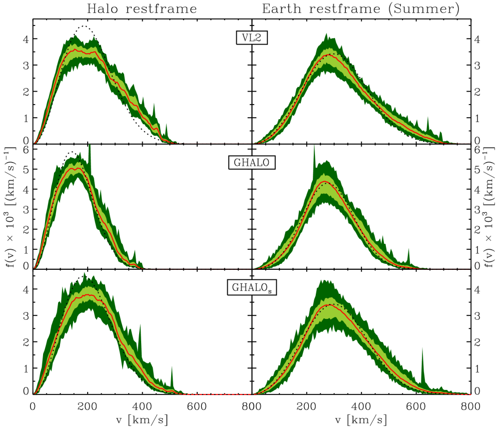

Kuhlen et al.

Obtaining the phase-space distribution

Assume that dark matter satisfies the colisionless Boltzmann equation,

$$\frac{\mathrm{df}}{\mathrm{dt}} = 0$$

Very hard to solve! Only a few exact solutions known, found finding integrals of motion (*singular isothermal sphere*, Hernquist, Jaffe, . . .).

Taking velocity moments we obtain the Jeans' equation:

$$v_c^2 = \frac{GM(r)}{r} = -\bar{v_r^2}\left(\frac{d\log\nu}{d\log r} + \frac{d\log \bar{v_r^2}}{d\log r} + 2\beta\right).$$

Necessary condition, useful to obtain density profiles from observational data.

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Eddington's formula

Gives the phase space distribution, if we know the density profile:

$$f(\mathcal{E}) = \frac{1}{\sqrt{8}\pi^2} \int_0^{\mathcal{E}} \frac{\mathrm{d}\Psi}{\sqrt{\mathcal{E} - \Psi}} \frac{\mathrm{d}^2\rho}{\mathrm{d}\Psi^2}. \tag{6}$$

Given a profile (NFW, cored, ...) we obtain the phase space distribution, which provides a full description of the dark matter distribution.

Check that $\rho(r) \equiv \int \mathrm{d}^3\mathbf{v}\, f(\mathcal{E})$.

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$$f(\mathcal{E}) = \frac{1}{\sqrt{8}\pi^2} \int_0^{\mathcal{E}} \frac{\mathrm{d}\Psi}{\sqrt{\mathcal{E} - \Psi}} \frac{\mathrm{d}^2\rho}{\mathrm{d}\Psi^2}. \tag{6}$$

Given a profile (NFW, cored, ...) we obtain the phase space distribution, which provides a full description of the dark matter distribution.

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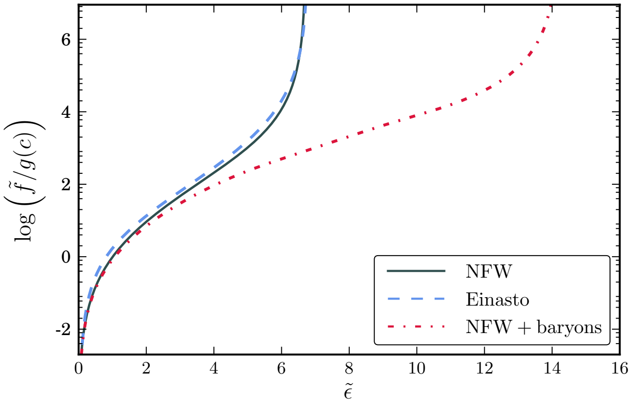
The phase-space distribution



Deriving the velocity distribution

Now that we have the full distribution function, we can find the velocity distribution at each point:

$$P(v) = \frac{f\left(\psi - v^2/2\right)}{\rho(\psi)}$$

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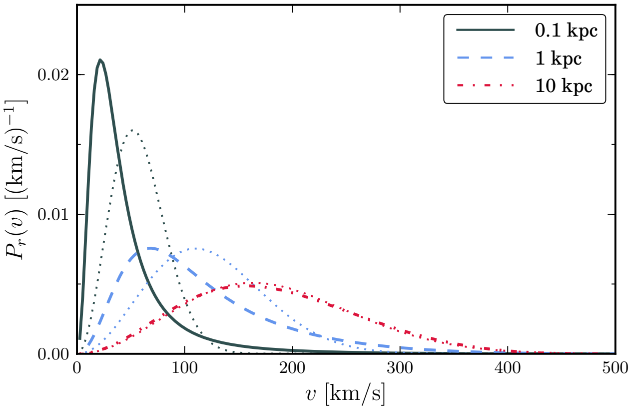

Obtaining the relative velocity distribution

We move to the CM, to obtain $P_{rel}(v_{rel})$:

$$f_{\rm sp}(\vec{v}_1) f_{\rm sp}(\vec{v}_2) \mathrm{d}\vec{v}_1 \mathrm{d}\vec{v}_2 = f_{\rm pair}(\vec{v}_{\rm cm}, \vec{v}_{\rm rel}) \mathrm{d}\vec{v}_{\rm cm} \mathrm{d}\vec{v}_{\rm rel} . \tag{7}$$

$$\begin{aligned} f_{\rm rv}(v_{\rm rel}) = {} & 4\pi v_{\rm rel}^2 2\pi \int_0^\infty \mathrm{d}v_{\rm cm} v_{\rm cm}^2 \int_0^\pi \mathrm{d}\theta \sin(\theta) \\ & \cdot f_{\rm sp}\left(\sqrt{v_{\rm rel}^2/4 + v_{\rm cm}^2 + v_{\rm rel} v_{\rm cm} \cos(\theta)}\right) \\ & \cdot f_{\rm sp}\left(\sqrt{v_{\rm rel}^2/4 + v_{\rm cm}^2 - v_{\rm rel} v_{\rm cm} \cos(\theta)}\right) . \end{aligned} \tag{8}$$

Recover the standard results for a Maxwell-Boltzmann distribution.

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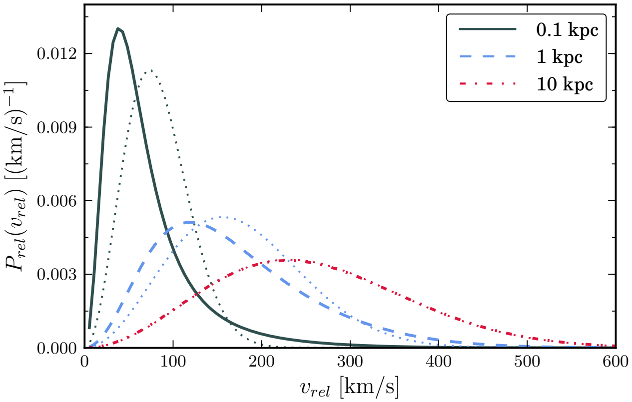

Fluxes

We have all the ingredients to perform the los integration:

$$Flux \propto \int dv_{rel} dl_{los} f_{pair}(r, v_{rel}) \times \sigma v_{rel}$$

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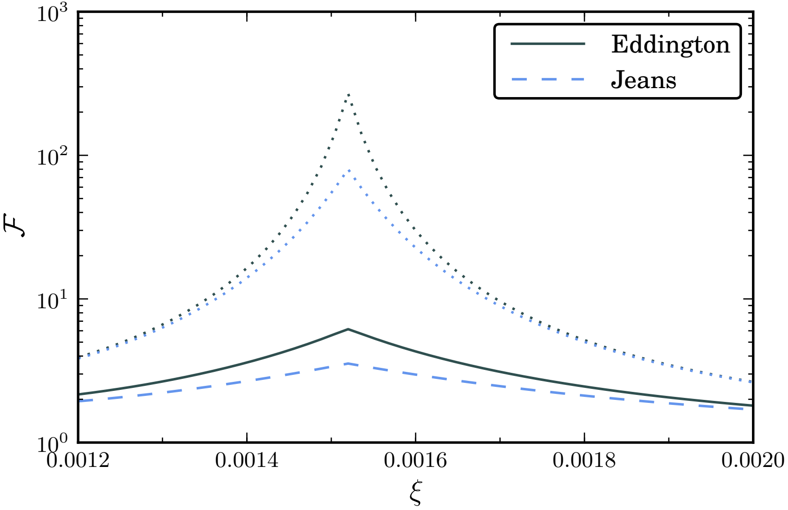
Boost factor



Enhancements up to 1000!

Conclusions

- A full general relativistic treatment shows significant deviations of the DM distribution around a black hole. They could affect tests of no-hair theorems.
- Gamma-ray and neutrino fluxes might depend on the velocity distribution, which generically deviates from the naive Maxwell-Boltzmann approximation.
- Using the full phase space distribution from the Eddington inversion suggests that fluxes from the center of the halo are up to 10^3 times larger.
- Constraints on Sommerfeld enhanced models from IC, synchroton or diffuse backgrounds might have to be re-evaluated.
- The velocity distribution also affects direct detection rates.