Scalar perturbations in conformal rolling scenario with intermediate stage

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Properties of primordial perturbations

Primordial perturbations are nearly Gaussian: obey Wick theorem.

Nearly flat power spectrum

$$\langle \zeta^2(\mathbf{x}) \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_\zeta(k)$$

$\mathcal{P}_\zeta(k) \approx \text{const.}$

There must be some symmetry behind this property!

In case of inflation: de Sitter symmetry. A novel idea is to relate flatness of the spectrum to conformal symmetry.

Conformal rolling

V. Rubakov, 2009

Toy model possessing conformal symmetry.

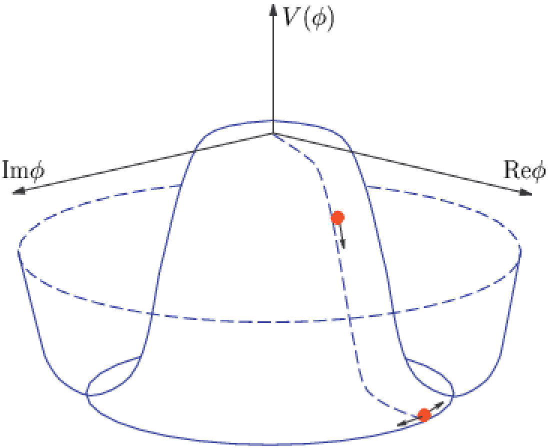
Field ϕ evolving at some stage preceding Hot Big Bang.

$$S_\phi = \int d^4x \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \phi^\star \partial_\nu \phi + \frac{R}{6} \phi^\star \phi + h^2 |\phi|^4 \right)$$

$$ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$$

Field ϕ is a spectator, does not affect the evolution of the Universe.

Conformal symmetry is broken at some $\phi \sim f$



$V(\phi)$
Imϕ
Reϕ

Background evolution

Action

$$\chi = a\phi, \qquad S[\chi] = \int d^3x d\eta [\eta^{\mu\nu}\partial^{\star}_{\mu}\chi\partial_{\nu}\chi + h^2|\chi|^4]$$

Equation of motion:

$$-\chi'' + 2h^2|\chi^2|\chi = 0, \quad \chi = \rho e^{i\theta}.$$

Conservation of current $\qquad \frac{d}{d\eta}(\rho^2\theta') = 0.$

Hence, as the value of ρ icreases, the phase θ freezes out,

$$\theta' \to 0, \quad \theta \to \theta_0.$$

Using U(1) symmetry, $\theta_0 = 0$.

$$\rho_0(\eta) = \frac{1}{h(\eta_\star - \eta)}$$

$$\chi = \mathrm{Re}\chi + i\mathrm{Im}\chi = \rho e^{i\theta}$$

$$\mathrm{Re}\chi = \rho_0 + \frac{\delta\chi_1}{\sqrt{2}} \qquad \mathrm{Im}\chi = \frac{\delta\chi_2}{\sqrt{2}}$$

$$\theta = \frac{\mathrm{Im}\chi}{\mathrm{Re}\chi}$$

In the leading order approximation

$$\theta = \frac{\delta\chi_2}{\sqrt{2}\rho_0(\eta)}$$

Phase perturbations are the most relevant for us!
Perturbations of the usual matter at the RD stage inherit the properties of the phase perturbations.

$$\mathcal{P}_\zeta \sim \mathcal{P}_\theta.$$

$$(\delta\chi_2)'' + k^2\delta\chi_2 - \frac{2}{(\eta_\star - \eta)^2}\delta\chi_2 = 0, \quad \chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}}e^{ik(\eta_\star - \eta)}$$

$$\chi_2 = \frac{1}{4\pi}\sqrt{\frac{\eta_\star - \eta}{2}}H^{(1)}_{3/2}[k(\eta_\star - \eta)].$$

$$\chi_2 = \frac{i}{2\pi^{3/2}}\frac{1}{k^{3/2}(\eta_\star - \eta)}, \quad k(\eta_\star - \eta) \ll 1$$

$$\theta = \frac{ih}{2\sqrt{2}\pi^{3/2}k^{3/2}}, \qquad \mathcal{P}_\theta = \frac{h^2}{8\pi^2}$$

(flat power spectrum!!!)

Radial perturbations

To the leading order in h, radial perturbations obey the following equation:

$$(\delta\chi_1)'' + p^2\delta\chi_1 - \frac{6}{(\eta_\star - \eta)^2}\delta\chi_1 = 0.$$

The solution is

$$\delta\chi_1 = \frac{1}{4\pi}\sqrt{\frac{\eta_\star - \eta}{2}} H^{(1)}_{5/2}[p(\eta_\star^{(0)} - \eta)]$$

The late time asymptotics,

$$\delta\chi_1 = \frac{3}{4\pi^{3/2}} \frac{1}{p^{5/2}(\eta_\star - \eta)^2} \quad (\textit{danger!})$$

Radial perturbations can be absorbed by the end time of roll,

$$\text{Re}\chi = \frac{1}{h[\eta_\star(\mathbf{x}) - \eta]},$$

where

$$\eta_\star \to \eta_\star + \delta\eta_\star(\mathbf{x}),$$

$\delta\eta_\star(\mathbf{x})$ is the random Gaussian field,

$$\mathcal{P}_{\delta\eta_\star} \sim \frac{1}{p^2}$$

Time shift does not affect the physical results.

$$v_i = -\partial_i \eta_\star(\mathbf{x})$$

has flat power spectrum.

The end of conformal rolling

The radius of the field ϕ settles down to the constant value f.

$$|\phi| = f$$

The phase θ is the massless scalar field minimally coupled to gravity.

Non trivial imprint of radial perturbations:

$$\theta = \theta(v_i, \partial_i \partial_j \eta_\star) \qquad \partial_\eta \theta = 0$$

In the leading order flat power spectrum

$$\mathcal{P}_\theta = \text{const.}$$

The end of conformal rolling

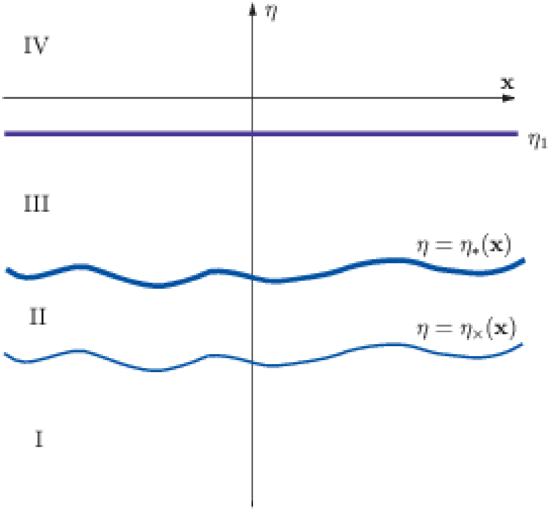
Two possible regimes:

1. Modes of interest are already superhorizon, $\frac{k}{a} < H$.

Considered in Libanov&Rubakov, Libanov&Mironov&Rubakov.

2. They are still subhorizon, $\frac{k}{a} > H$.

In the latter case phase perturbations proceed to evolve at the intermediate stage, which ends as phase perturbations freeze out.


η
IV
$\mathbf{x}$
η_1
III
$\eta = \eta_*(\mathbf{x})$
II
$\eta = \eta_\times(\mathbf{x})$
I

Assumptions

The duration of the intermediate stage is long enough,

$$k(\eta_1 - \eta_\star) \gg 1$$

Evolution of the phase perturbations at the intermediate stage must be nearly Minkowskian

$$(\delta\theta)'' - \partial_i^2 \delta\theta = 0$$

to do not spoil the flat power spectrum.

Background evolution:

1. Genesis.
2. Ekpyrosis with superstiff equation of state, $p \gg \rho$.

Cauchy problem

Minkowskian evolution,

$$(\delta\theta)'' - \partial_i \partial_i \delta\theta = 0.$$

Initial conditions defined at the curved hypersurface $\eta = \eta_\star(\mathbf{x})$,

$$\delta\theta(\eta, \mathbf{x})|_{\eta=\eta_\star(\mathbf{x})} = \delta\theta(\eta_\star(\mathbf{x}), \mathbf{x}) \qquad \partial_N \delta\theta = 0$$

Initial conditions carry the information about the dynamics at the conformal rolling stage and, in particular, about interaction of phase perturbations with radial ones.

Solution

The solution for a given $\mathbf{k}$: a linear combination of two waves coming from direction of $\mathbf{k}$ and from opposite direction and travelling distance $r = \eta_1 - \eta_\star$,

$$\delta\theta(\mathbf{k}) = F[\mathbf{v}(\pm \mathbf{n}_k r)] A_{\mathbf{k}} + \text{h.c.}$$

$$\mathbf{v} = -\partial \eta_\star$$

Imprint is via a random Gaussian field $\mathbf{v}$.

Statistical anisotropy

Calculate the two-point product $\delta\theta(\mathbf{k})\delta\theta(\mathbf{k}')$ over the realizations of $A_{\mathbf{k}}$ and $A^{\star}_{\mathbf{k}}$.
Treat $\mathbf{v}$ as a classical field.

$$\mathcal{P}(\mathbf{k}) = \mathcal{P}_0 \left(1 + n_{k_i} \left[v_i(\mathbf{n}_k r) - v_i(-\mathbf{n}_k r)\right]\right), \qquad \mathcal{P}_0 = \frac{h^2}{8\pi^2}$$

$$= \sum_{LM} q_{LM} Y_{LM}(\mathbf{n}_k)$$

Remember that $\mathbf{v}$ is an isotropic Gaussian field,

$$\langle q_{LM} q^{\star}_{L'M'} \rangle = Q_L \delta_{LL'} \delta_{MM'}, \quad Q_L = \frac{3h^2}{\pi} \frac{1}{(L-1)(L+2)}$$

Non-Gaussianity

Since $\delta\theta = -\delta\theta$, bispectrum vanishes.

Non-Gaussianity arises in trispectrum,

$$\langle \delta\theta(\mathbf{k})\delta\theta(\mathbf{k}')\delta\theta(\mathbf{q})\delta\theta(\mathbf{q}')\rangle = \frac{\mathcal{P}_0(k)}{4\pi k^3}\frac{\mathcal{P}_0(q)}{4\pi q^3}\delta(\mathbf{k}+\mathbf{k}')\delta(\mathbf{q}+\mathbf{q}')\left[1 + F_{NG}(\mathbf{n}_k, \mathbf{n}_q)\right]$$

$$F_{NG} = \frac{3h^2}{\pi^2} \ln \frac{\text{const}}{|\mathbf{n}_k - \mathbf{n}_q|}$$

Logarithmically amplified in the folded limit.

Tilt

Small negative tilt

$$n_s - 1 = -\frac{3h^2}{4\pi^2}$$

This is not a strong result.

Other sources of scalar negative tilt are possible.
What if the observed tilt was due to this contribution?

$$n_s - 1 \approx -0.04$$

Then, $h^2 \sim 1$. However, disfavored by observations of statistical anisotropy.

How to constrain parameter h^2 of our model?

As mentioned, tilt is flexible to slight modifications of the theory.

Non-Gaussianity in trispectrum is not testable now.

Thus, statistical anisotropy.

Imprint on CMB spectrum

Having fluctuations $\delta T(\mathbf{n})$ of the CMB sky, we define

$$a_{lm} = \int d\mathbf{n} \delta T(\mathbf{n}) Y^{\star}_{lm}(\mathbf{n}), \quad C_{lm;l'm'} = \langle a_{lm} a^{\star}_{l'm'} \rangle$$

In case of statistical anisotropy off-diagonal elements appear,

$$C_{lm;l'm'} = C_l \delta_{ll'} \delta_{mm'} + \sum_{LM} q_{LM} D^{LM}_{lm;l'm'}$$

$$\langle a_{lm} a^{\star}_{l+2,m} \rangle, \quad \langle a_{lm} a^{\star}_{l+4,m} \rangle$$

are non zero.

NB These correlations are non zero already due to noise contamination. We focus on the cosmological origin.

First step. Model independent analysis.

Assume general statistical anisotropy

$$\mathcal{P}_\zeta = \mathcal{P}_0 \left(1 + \sum q_{LM} Y_{LM}(\mathbf{n}_k)\right).$$

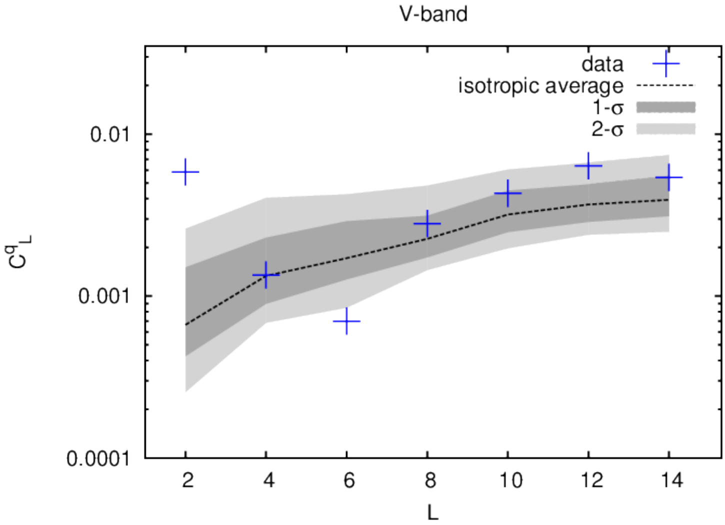
No assumptions about the quantities q_{LM}, except

- They do not depend on the magnitude of **k**
- They are small, $q_{LM} \ll 1$

Using likelihood methodology, extract q_{LM} from WMAP 7 data.

$$C_L^q = \frac{1}{2L+1} \sum_M |q_{LM}|^2$$

Hirata&Seljak (2002), Hanson&Lewis (2009)


V-band
data
isotropic average
1-σ
2-σ
0.01
0.001
0.0001
C^q_L
2
4
6
8
10
12
14
L

Unknown Systematics?

Strong indications that the quadrupole mode is largely contaminated by unknown systematics.

- The quadrupole preferred direction is aligned with the poles of the ecliptic plane
- The result is frequency dependent, i.e. the results in V- W-bands are different.

Estimator

Still we can put the upper constraint on the parameter h^2

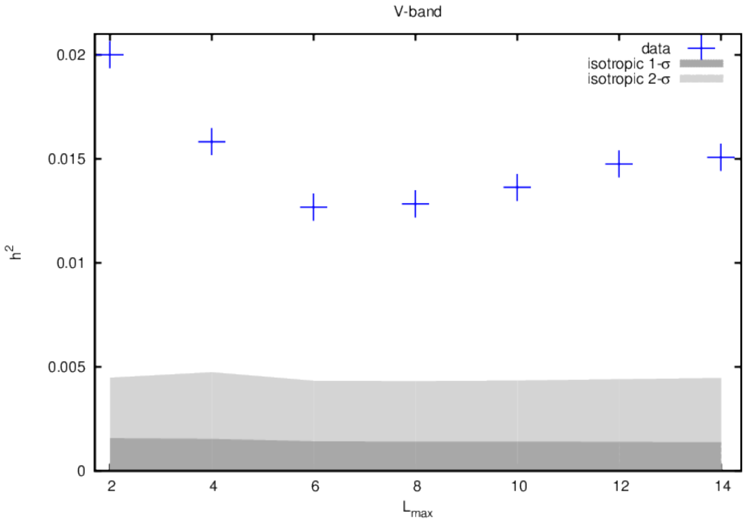
Using the likelihood technique, we construct the estimator of h^2, which best fits the observed CMB.

Remind: coefficients q_{LM} are random Gaussian quantities with dispersions

$$\langle q_{LM}^2 \rangle = \frac{3}{\pi} \frac{h^2}{(L-1)(L+2)}.$$

Of special interest: parameter h^2, which defines the strength of the statistical anisotropy.

$$h^2 = F(C_2^q, C_4^q, C_6^q, ...)$$


V-band
data
isotropic 1-σ
isotropic 2-σ
0.02
0.015
0.01
0.005
0
h2
2
4
6
8
10
12
14
Lmax

Constraint

- For a given h^2 one generates the sets $\{q_{LM}\}$.
- For a given set $\{q_{LM}\}$ one generates anisotropic maps.
- Apply our estimator.

For $h^2 = 0.045$ only 5% of all maps give $h^2 < 0.015$.

$$h^2 < 0.045$$

at the 95% CL.

Conclusions

- Statistical anisotropy of all even multipoles.

- Non-Gaussianity in trispectrum.

- Upper constraint on the paramater $h^2 < 0.045$ at the 95 % CL.