COMPOSITENESS & CUSTODIAL SYMMETRY

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Outline

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What not to expect from the LHC

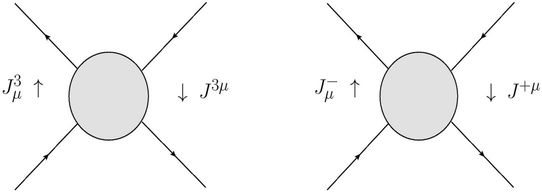
- What kind of new physics will be discovered at the LHC? Supersymmetry? Technicolor? Little Higgs? Extra dimension? Or maybe... the Standard Model (SM). Or even worse, nothing! No one knows, of course.
- A simpler question to answer is: what kind of new physics will NOT be discovered at the LHC?

No new sources of large weak isospin violation!

What not to expect from the LHC

ρ parameter

$$1 + \Delta\rho \equiv \lim_{q^2 \to 0} \frac{d\left(q^2 \mathcal{M}_{\mathrm{NC}}(q^2)\right)/dq^2}{d\left(q^2 \mathcal{M}_{\mathrm{CC}}(q^2)\right)/dq^2}$$



What not to expect from the LHC

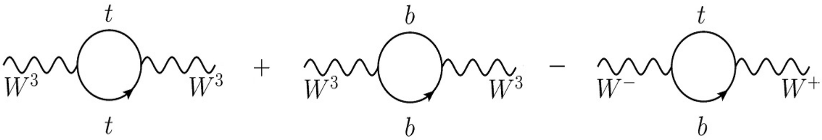
Experimental bounds from LEP1 and LEP2 (light Higgs)

$$\Delta\rho - (\Delta\rho)_{\mathrm{SM}} = (0.1 \pm 0.9)\, 10^{-3}$$

- The SM has sources of weak isospin violation in the hypercharge interactions and the Yukawa interactions. These are sufficient to account for nearly all the observed weak isospin violation at zero momentum.
- Therefore, whatever the new physics is, it should not carry large contributions to ρ.

Custodial symmetry

- The SM contribution to ρ comes from radiative corrections of the $\langle W^3 W^3 \rangle$ and $\langle W^- W^+ \rangle$ propagators.



- *How does the SM evade tree-level contributions to $\Delta\rho$?*

Custodial symmetry

Higgs Lagrangian

$$\mathcal{L}_{\text{Higgs}} = |D_\mu \phi|^2 - \frac{\lambda}{4}\left(|\phi|^2 - v^2\right)^2$$

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} i\sqrt{2}\,\pi^+ \\ v + h - i\,\pi^0 \end{pmatrix}$$

- This is of course invariant under the spontaneously broken electroweak symmetry, $SU(2)_L \times U(1)_Y \to U(1)_Q$.
- In the limit of zero hypercharge interactions this symmetry-breaking pattern is enhanced to $SU(2)_L \times SU(2)_R \to SU(2)_c$.

Custodial symmetry

Manifestly custodial Higgs Lagrangian

$$\mathcal{L}_{\text{Higgs}} = \frac{1}{2}\mathrm{Tr}|D_\mu \Sigma|^2 - \frac{\lambda}{4}\left(\mathrm{Tr}|\Sigma|^2 - v^2\right)^2$$

$$\Sigma \equiv \left(i\sigma^2\phi^*, \phi\right) = \frac{1}{\sqrt{2}}\left(v + h + 2iT^a\pi^a\right)$$

$$\Sigma \to u_L \Sigma u_R^\dagger \quad u_L \in SU(2)_L, u_R \in SU(2)_R$$

- The symmetry-breaking pattern is manifestly $SU(2)_L \times SU(2)_R \to SU(2)_c$.
- This *custodial* $SU(2)_c$ symmetry guarantees that $\Delta\rho = 0$ (Sikivie, Susskind, Voloshin and Zakharov, 1980).

Custodial symmetry

- There are no renormalizable terms we can write which violate this symmetry structure: $\mathrm{Tr}(\Sigma\Sigma^\dagger T^3) = 0$, $\mathrm{Tr}(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger T^3) = 0$
- Therefore, the SM is very succesful in predicting a small value for $\Delta\rho$.
- This suggests that viable extensions of the SM should be custodially symmetric (or possess some discrete symmetry which prevents large isospin violation effects from showing up at low momenta).

What not to expect from the LHC

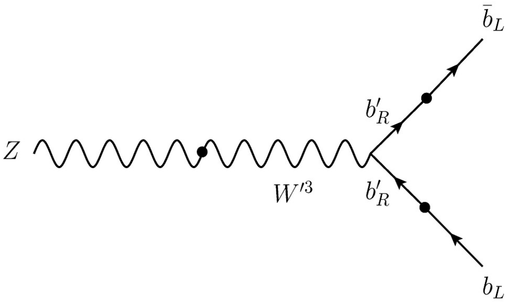
- New physics should also not dramatically affect the SM fermionic gauge interactions.

Parameter	Average	Correlations						
		$g_{L\nu}$	$g_{L\ell}$	g_{Lb}	g_{Lc}	$g_{R\ell}$	g_{Rb}	g_{Rc}
$g_{L\nu}$	$+0.50075\pm0.00077$	1.00						
$g_{L\ell}$	-0.26939 ± 0.00022	−0.32	1.00					
g_{Lb}	-0.4182 ± 0.0015	0.05	−0.27	1.00				
g_{Lc}	$+0.3453\pm0.0036$	−0.02	0.04	−0.09	1.00			
$g_{R\ell}$	$+0.23186\pm0.00023$	0.25	0.34	−0.37	0.07	1.00		
g_{Rb}	$+0.0962\pm0.0063$	0.00	−0.33	0.88	−0.14	−0.35	1.00	
g_{Rc}	-0.1580 ± 0.0051	0.00	0.08	−0.17	0.30	0.08	−0.13	1.00

ALEPH, DELPHI, L3, OPAL, SLD Collaborations, LEP Electroweak Working Group, SLD Electroweak and Heavy Flavour Groups (2006)

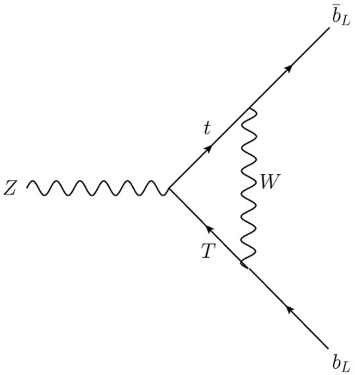
The $Zb_L\bar{b}_L$ coupling

- The most dangerous contribution is to the left-handed $Zb\bar{b}$ coupling.
- Tree-level contributions can show up through simultaneous mass mixings of the Z boson and the left-handed bottom with new heavy states:



The $Zb_L\bar{b}_L$ coupling

- One loop contributions can show up if there are heavy replicas of the top.



The $Zb_L\bar{b}_L$ coupling

- Important tree-level and loop corrections to the $Zb_L\bar{b}_L$ coupling occur in large classes of models: Little Higgs, Extra-dimension, composite fermions.
- In particular loop corrections are dengerous, and are present in any model with heavy replicas of the top quark.
- Surprisingly, **custodial symmetry can protect the $Zb_L\bar{b}_L$ coupling from large corrections!**

Enhanced custodial symmetry

- A conserved charge receives no non-universal corrections.
- Therefore, custodial symmetry $SU(2)_c$ implies that the vectorial T_V^3 charge receives no non-universal corrections.

$$\delta T_V^3 \equiv \delta(T_L^3 + T_R^3) = 0$$

- Add a parity symmetry P_{LR} which exchanges $L \leftrightarrow R$. If ψ is an eigenstate of this parity operator, then for ψ

$$\delta T_L^3 = \delta T_R^3$$

Enhanced custodial symmetry

- Therefore, if:
  1. there is a global symmetry $SU(2)_L \times SU(2)_R \times P_{LR} \sim SO(4) \times P_{LR} = O(4)$,
  2. this $O(4)$ symmetry is spontaneously broken to $SU(2)_c \times P_{LR} \sim O(3)$, and
  3. ψ is an eigenstate of P_{LR},

  then for ψ (Agashe, Contino, Da Rold and Pomarol, 2006)

Custodial protection of g_{Lb}

$$\delta T_L^3 = 0$$

- Then in order to protect the $Zb_L\bar{b}_L$ coupling from large corrections, enhance custodial symmetry to $SU(2)_c \times P_{LR} \sim O(3)$ and make b_L an eigenstate of P_{LR}

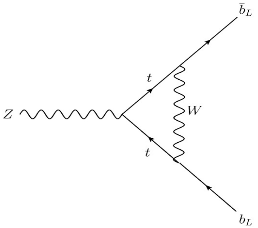
The $Zb_L\bar{b}_L$ coupling in the SM

- The SM violates $O(3)$ even in the limit of zero hypercharge and Yukawa interactions, since the left-handed top-bottom doublet would necessarily have to be a singlet of $SU(2)_R$, and this breaks P_{LR}.
- Hyperchearge and (especially) Yukawa interactions bring in an additional source of $O(3)$ breaking.
- The breaking of custodial symmetry in the SM is sufficient to account for the measured ρ (unless the Higgs is heavy).
- However, the $O(3)$ breaking in the SM is not enough to fully account for the observed $Zb_L\bar{b}_L$ coupling: the latter is 2σ above the SM prediction.

The $Zb_L\bar{b}_L$ coupling in the SM

The $Zb_L\bar{b}_L$ coupling

$$g_{Lb} \simeq \frac{e}{\sin\theta_W \cos\theta_W}\left[-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W + \delta g_{Lb}\right]$$



SM δg_{Lb}

$$\delta g_{Lb}^{SM} = \frac{m_t^2}{16\pi^2 v^2}$$

$g_{\rm Lb}$	-0.4182 ± 0.0015
$g_{\rm Lc}$	$+0.3453 \pm 0.0036$
$g_{\rm R\ell}$	$+0.23186 \pm 0.00023$

$$g_{Lb}^{SM} = -0.42114$$

The $Zb_L\bar{b}_L$ coupling in the SM

- Maybe we can extend the SM in a custodially symmetric fashion, and hope that the amount of custodial isospin violation necessary to introduce top-bottom mass splitting
  1. does not lead to large corrections to $\Delta\rho$, and
  2. gives the missing contribution to g_{Lb} to recover 1σ agreement with experiment.

Let's try!

Doublet-extended Standard Model

- Consider a model with a global symmetry $O(4) \times U(1)_X$. This is spontaneously broken to $O(3) \times U(1)_X$ by the vacuum expectation value of the SM Higgs. (Chivukula, Di Chiara, RF, Simmons, 2009)
- Hypercharge and electromagnetic charge are then given by

$$Y = T^3_R + Q_X \ , \qquad Q = T^3_L + Y = T^3_L + T^3_R + Q_X$$

- For the left-handed bottom we must impose:
  1. $T^3_L(b_L) = -1/2$, $Y(b_L) = 1/6$, $Q(b_L) = -1/3$, by SM charge assignments.
  2. $T^3_R(b_L) = T^3_L(b_L)$ since b_L must be an eigenstate of P_{LR}.
  3. It follows that $Q_X(b_L) = 2/3$.

Doublet-extended Standard Model

- Then also $Q_X(t_L) = 2/3$, and the full left-handed top-bottom doublet, $q_L \equiv (t'_L, b_L)$, must have $T^3_R = -1/2$.
- As a consequence we need introduce a new left-handed doublet, $\Psi_L \equiv (\psi_L, t''_L)$, with $T^3_R = +1/2$.
- q_L and Ψ_L form a bi-doublet under $SU(2)_L \times SU(2)_R$, with Q_X charge $2/3$:

$$\mathcal{Q}_L \equiv \begin{pmatrix} t'_L & \psi_L \\ b_L & t''_L \end{pmatrix} = \begin{pmatrix} q_L & \Psi_L \end{pmatrix}$$

- The action of P_{LR} on $\mathcal{Q}_L$ is

$$P_{LR}\mathcal{Q}_L = -\left[(i\sigma_2)\,\mathcal{Q}_L\,(i\sigma_2)\right]^T = \begin{pmatrix} t''_L & -\psi_L \\ -b_L & t'_L \end{pmatrix}$$

Doublet-extended Standard Model

- The SM right-handed fields have $T^3_L = 0$. This implies $T^3_R(t'_R) = 0$, $T^3_R(b_R) = -1$. Then t'_R can be a singlet of $SU(2)_R$, whereas b_R is at least in a triplet of $SU(2)_R$.
- A right-handed partner $\Psi_R \equiv (\psi_R, t''_R)$ of the new Ψ_L doublet is necessary to give mass to the new fermions, and to break the custodial symmetry necessary to introduce top-bottom mass splitting.
- Since $y_b \ll y_t$, we ignore the bottom mass and do not include b_R (and its multiplet partners) in this analysis. Notice, however, that the SM prediction for g_{Rb} is more than 2σ below its experimental value, and thus some mechanism is needed to recover 1σ agreement. This problem will not be discussed in this talk.

Doublet-extended Standard Model

DESM Yukawa

$$\mathcal{L}_{\text{Yukawa}} = -\lambda_t \mathrm{Tr}\left(\overline{\mathcal{Q}}_L \cdot \Sigma\right) t'_R \;+ \text{h.c.}\;.$$

DESM hard mass

$$\mathcal{L}_{\text{mass}} = -M\;\bar{\Psi}_L \cdot \Psi_R + h.c.$$

	t'_L	b_L	Ω_L	T'_L	t'_R	b_R	Ω_R	T'_R
T^3_L	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
T^3_R	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	-1	0	0
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{5}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{5}{3}$	$\frac{2}{3}$
Y	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{7}{6}$	$\frac{7}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{7}{6}$	$\frac{7}{6}$
Q_X	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{7}{6}$	$\frac{7}{6}$

Doublet-extended Standard Model

Mass Lagrangian

$$\mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{mass}} \supset$$

$$-\begin{pmatrix} t'_L & t''_L \end{pmatrix} \begin{pmatrix} m & 0 \\ m & M \end{pmatrix} \begin{pmatrix} t'_R \\ t''_R \end{pmatrix} - M\bar{\psi}_L\psi_R + \text{h.c}$$

$$m \equiv \frac{\lambda_t v}{\sqrt{2}}$$

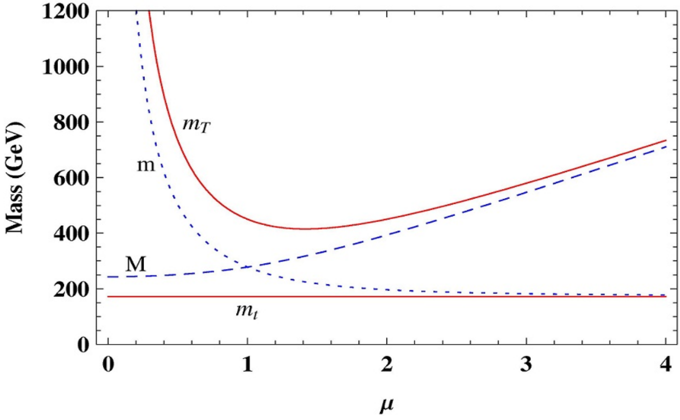
Diagonalization

$$m_t^2 = \frac{1}{2}\left[1 - \sqrt{1 + \frac{4m^4}{M^4}}\right] M^2 + m^2$$

$$m_T^2 = \frac{1}{2}\left[1 + \sqrt{1 + \frac{4m^4}{M^4}}\right] M^2 + m^2$$

Doublet-extended Standard Model

- Top-sector mas spectrum ($\mu \equiv M/m$)



$Zb_L\bar{b}_L$ coupling in DESM

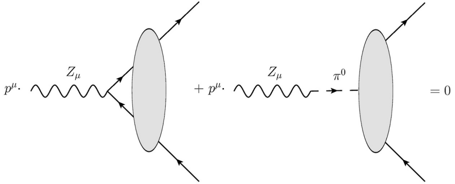
- There are no tree-level corrections.
- The dominant one-loop correction can be computed in *gaugeless limit* (Barbieri, Beccaria, Ciafaloni, Curci, and Viceré, 1992): the Z boson is treated as a classical field coupled to the neutral component of the isospin current. The latter gets a contribution from the eaten Goldstone boson:

Neutral weak current

$$J_\mu = \hat{J}_\mu + \frac{v}{2}\partial_\mu \pi^0$$

$Zb_L\bar{b}_L$ coupling in DESM

- Then the current conservation, $\partial^\mu J_\mu = 0$, translates into the equality



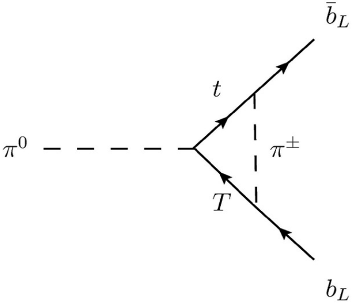
- This eventually leads to an expression of δg_{Lb} in terms of the amplitude for $\pi^0 \to b_L\bar{b}_L$, which is much easier to compute than $Z \to b_L\bar{b}_L$

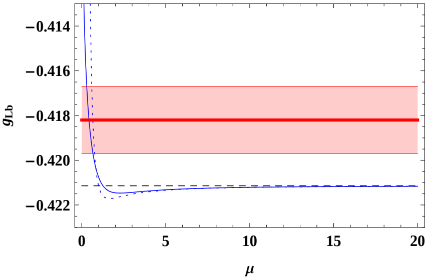
δg_{Lb} in gaugeless limit

$$\delta g_{Lb} = \frac{v}{2}\mathcal{M}(\pi^0 \to b_L\bar{b}_L)$$

$Zb_L\bar{b}_L$ coupling in DESM

- The dominant contribution comes from the loop with one SM top quark and one heavy top.



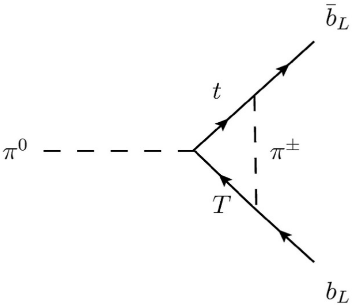


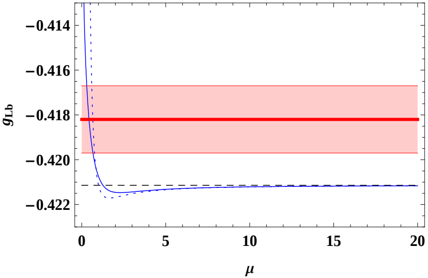
DESM δg_{LB}: large μ

$$\delta g_{Lb}(\mu \to \infty) = \frac{m_t^2}{16\pi^2 v^2}\left[1 + \frac{\log(1/\mu^2)}{2\mu^2} + \mathcal{O}(1/\mu^4)\right] \;,\; \mu \equiv M/m$$

$Zb_L\bar{b}_L$ coupling in DESM

- The dominant contribution comes from the loop with one SM top quark and one heavy top.



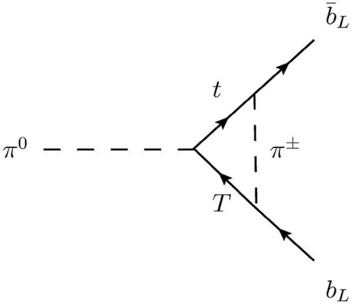


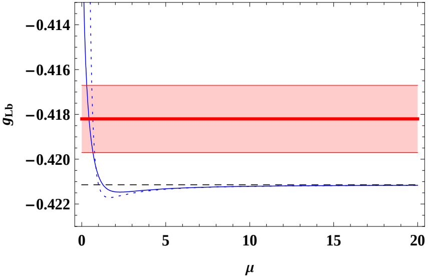
DESM δg_{LB}: small μ

$$\delta g_{Lb}(\mu \to 0) = \frac{m_t^2}{16\pi^2 v^2}\left[\frac{3\log(2/\mu) - 1}{2} + \mathcal{O}(\mu^2)\right]$$

$Zb_L\bar{b}_L$ coupling in DESM

- The dominant contribution comes from the loop with one SM top quark and one heavy top.

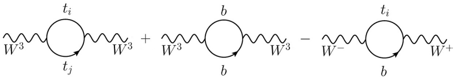


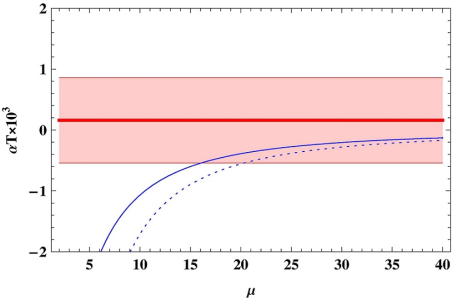


- Positive contribution for small μ! Maybe this works, but we need checking $\Delta\rho$!

$\Delta\rho$ in DESM

- The heavy top contributes to $\Delta\rho$.

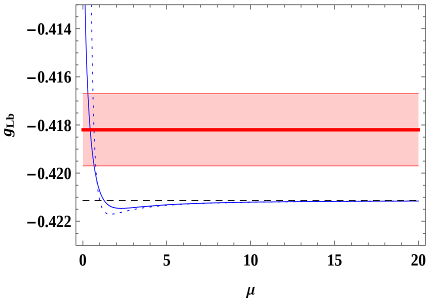


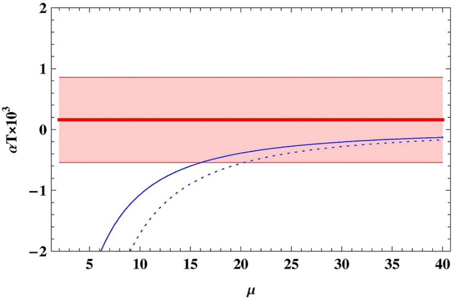


DESM $\Delta\rho$: large μ

$$\Delta\rho(\mu \to \infty) = -\frac{3m_t^2}{4\pi^2 v^2}\frac{\log(\mu^2)}{\mu^2} + \mathcal{O}(1/\mu^4)\ ,\ \ \mu \equiv M/m$$

Isospin violation in DESM





- The isospin violation is fine at low values of μ for g_{Lb}, but goes in the wrong direction for $\Delta\rho$!
- It is a generic feature that bidoublets of $SU(2)_L \times SU(2)_R$ give negative $\Delta\rho$.

Narurality

- The DESM is anyway an unnatural theory: the Higgs is an *elementary* field, and thus suffers from radiative instability.
- In order to stabilize the Higgs we can either
  1. introduce a new symmetry which protects the mass from large radiative corrections,
  2. introduce a new strong force and make the Higgs a *composite* state of this new interaction.
- SUSY belongs to the first class, whereas Technicolor (TC) may belong to the second class. Little Higgs models belong to both classes.
- If, in addition to the Higgs, also the heavy quarks have a sizable amount of compositeness, then we might have solved two problems: the theory becomes natural, and the compositeness of the top-bottom doublet might explain why the $Zb_L\bar{b}_L$ coupling is not in full agreement with the SM prediction.

Light composite Higgs

- An light composite scalar isosinglet is possible in near-conformal TC theories (Sannino and Tuominen, 2005; Doff, Natale, da Silva, 2009).
- Traditional TC theories assume a heavy scalar isosinglet, and have a dual description in Higgsless models (Csaki, Grojean, Murajama, Pilo, Terning, 2005; RF, Gopalakrishna, Schmidt, 2004; Chivukula, Coleppa, Di Chiara, He, Kurachi, Simmons Tanabashi, 2006).
- The Randall-Sundrum model also provides a mechanism for a naturally light scalar isosinglet (Randall and Sundrum, 1999).
- We are only interested in a low-energy effective theory with the lowest-lying resonances: whether heavier resonances are described by a holographic theory or by a chiral resonance model is of no concern for us.

Compositeness

- In the model we want to build the Higgs has no elementary component, since otherwise this would suffer from radiative instability.
- All other SM particles are mixtures of elementary and composite states:

$$|\boldsymbol{\psi}\rangle = \mathbf{cos}\,\boldsymbol{\alpha}\ |\mathbf{elementary}\rangle\ +\ \mathbf{sin}\,\boldsymbol{\alpha}\ |\mathbf{composite}\rangle$$

$$|\boldsymbol{A_\mu}\rangle = \mathbf{cos}\,\boldsymbol{\theta}\ |\mathbf{elementary}\rangle\ +\ \mathbf{sin}\,\boldsymbol{\theta}\ |\mathbf{composite}\rangle$$

$$|\boldsymbol{h}\rangle = |\mathbf{composite}\rangle$$

Compositeness

- This might seem odd, but it is realized in Nature! The W boson, in the SM, is not fully elementary: it has a tiny component of ρ meson. The size of mixing is given by the m_ρ/m_W mass ratio:

$$\boldsymbol{|W_\mu\rangle = \cos\theta \; |W'_\mu\rangle \; + \; \sin\theta \; |\rho'_\mu\rangle}$$

$$\boldsymbol{\theta \sim \frac{m_\rho}{m_W} \sim 0.009}$$

Compositeness

- If GUT theories exist, and a four-fermion operator with three quarks and one lepton doublet mediates proton decay, then at low energy this will give rise to an electron-antiproton mass mixing. The size of the mixing is the m_e/m_{GUT} mass ratio:

$$|e\rangle = \cos\alpha\ |e'\rangle\ +\ \sin\alpha\ |\bar{p}'\rangle$$

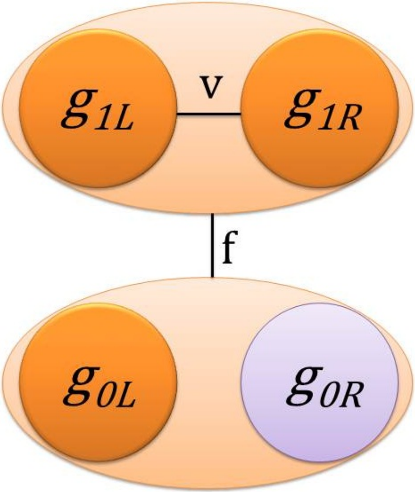
$$\alpha \sim \frac{m_e}{m_{\mathrm{GUT}}} \sim 10^{-19}$$

The model

- We would like to build a model of compositeness with custodial symmetry. In order to do this:
  1. The model must possess a $\boldsymbol{G_0 \equiv SO(4)_0 \times U(1)_{X0}}$ chiral symmetry, the $SU(2)_L \times U(1)_Y$ subgroup of which is gauged. Then we have $Y = T^3_{R0} + Q_{X0}$.
  2. There should be a mirror $\boldsymbol{G_1 \equiv SO(4)_1 \times U(1)_{X1}}$ gauge group, to describe the vector meson resonances.
  3. $\boldsymbol{G_0} \times \boldsymbol{G_1}$ is broken to a diagonal $\boldsymbol{G \equiv SO(4) \times U(1)_X}$ by the vacuum expectation value of a nonlinear sigma field Φ.
  4. The SM fermions are only charged under $\boldsymbol{G_0}$, whereas the composite fermions are only charged under $\boldsymbol{G_1}$.
  5. The Higgs bidoublet Σ, being fully composite, is only charged under $\boldsymbol{G_1}$.
- For the sake of simplicity, we un-gauge the $\boldsymbol{U(1)_{X1}}$ symmetry, and work in the broken phase of $\boldsymbol{U(1)_{X0} \times U(1)_{X1} \to U(1)_X}$. All fermions, elementary or composite, will be charged under this diagonal $\boldsymbol{U(1)_X}$.

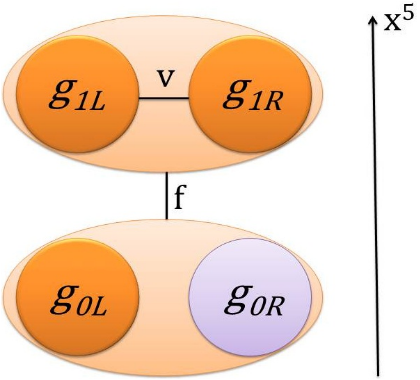
The model

- The model is conveniently depicted by a “moose” diagram:



The model

- This looks like a highly deconstructed extra-dimension:



- Extra-dimensional models of this type have been analyzed (Carena, Ponton, Santiago and Wagner, 2006).

The model

- The composite fermions are chosen to be vector-like.
- Here we only consider the third-generation quarks, and neglect the bottom Yukawa: the right-handed bottom is decoupled in this limit, and we need not consider it.
- The charge assignments for the localized fields are:

	Q_{0L}	t_{0R}	Q_1	t_1	Σ
$SU(2)_{L0} \times SU(2)_{R0}$	**(2,1)**	**1**	**1**	**1**	**1**
$SU(2)_{L1} \times SU(2)_{R1}$	**1**	**1**	**(2,2)**	**1**	**(2,2)**
$U(1)_X$	**2/3**	**2/3**	**2/3**	**2/3**	**0**

Two-component vs. four-component notation

- We can write the fields as 2×2 objects:

Composite bi-doublets: 2×2

$$Q_{0L} = (q_{0L}\ 0) = \begin{pmatrix} t^q_{0L} & 0 \\ b_{0L} & 0 \end{pmatrix},\ Q_1 = (q_1\ \chi_1) = \begin{pmatrix} t^q_1 & \psi_1 \\ b_1 & t^\chi_1 \end{pmatrix}$$

Composite Higgs: 2×2

$$\Sigma = (i\sigma^2\phi^*, \phi) = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h + \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & v + h - \pi^0 \end{pmatrix}$$

- There are four top-quark fields: two in the doublets, and two singlets.
- ψ_1 is a $5/3$ charge field.
- The b_1 field is an eigenstates of P_{LR}: the contribution to the $Zb_L\bar{b}_L$ coupling from the bidoublet Q_1 is expected to have comparable positive and negative contributions.

Two-component vs. four-component notation

- We can as well write the fields as 4-component objects:

Composite bi-doublets: 4

$$Q_{0L} = \begin{pmatrix} q_{0L} \\ 0 \end{pmatrix} = \begin{pmatrix} t^q_{0L} \\ b_{0L} \\ 0 \\ 0 \end{pmatrix} , \; Q_1 = \begin{pmatrix} q_1 \\ \chi_1 \end{pmatrix} = \begin{pmatrix} t^q_1 \\ b_1 \\ \psi_1 \\ t^\chi_1 \end{pmatrix}$$

Composite Higgs: 4

$$\Sigma = \begin{pmatrix} i\sigma^2\phi^* \\ \phi \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h + \pi^0 \\ \sqrt{2}\pi^- \\ \sqrt{2}\pi^+ \\ v + h - \pi^0 \end{pmatrix}$$

Lagrangian

- With the 4-component notation it is straightforward to write the mass and Yukawa Lagrangians:

Hard mass

$$\mathcal{L}_{\text{mass}} = -M_Q\ \bar{Q}_1\ Q_1 - M_t\ \bar{t}_1\ t_1 - \mu_t\left(\bar{t}_{0R}\ t_{1L}\ +\ \text{h.c.}\right)$$

Yukawa

$$\mathcal{L}_{\text{Yukawa}} = -y_t\ \bar{Q}_1\ \Sigma\ t_1\ -\ y_Q\ \bar{Q}_{0L}\ \Phi\ Q_{1R}\ +\ \text{h.c.}$$

Nonlinear sigma field

$$\Phi = \exp\left[i\left(\Pi_L^a T_L^a + \Pi_R^a T_R^a\right)\right]$$

Lagrangian

- With the 4-component notation it is straightforward to write the mass and Yukawa Lagrangians:

Hard mass

$$\mathcal{L}_{\text{mass}} = -M_Q\, \bar{Q}_1\, Q_1 - M_t\, \bar{t}_1\, t_1 - \mu_t\, (\bar{t}_{0R}\, t_{1L} \;+\; \text{h.c.})$$

Yukawa

$$\mathcal{L}_{\text{Yukawa}} = -y_t\, \bar{Q}_1\, \Sigma\, t_1 \;-\; y_Q\, \bar{Q}_{0L}\, \Phi\, Q_{1R} \;+\; \text{h.c.}$$

- There is one additional dimension-four term: $\bar{Q}_{0L}\, \Phi\, \Sigma\, t_{0R}$. We do not include this on the basis of a *moose locality principle.*
- Alternatively we could model this compositeness Lagrangian with a linear sigma field Φ, in which case we have included all renormalizable terms.

Relevant parameters

- The important parameters in the fermion sector are:
  1. $\sin\alpha \equiv \mu_Q \Big/ \sqrt{M_Q^2 + \mu_Q^2}$ = amount of compositeness of the left-handed top-bottom doublet.
  2. $\sin\beta \equiv \mu_t \Big/ \sqrt{M_t^2 + \mu_t^2}$ = amount of compositeness of the right-handed top.
  3. $\sqrt{M_Q^2 + \mu_Q^2}$ = mass scale of the composite bi-doublet.
  4. $\sqrt{M_t^2 + \mu_t^2}$ = mass scale of the composite singlet.
- In the gauge sector, the relevant parameters are
  1. $\sin\theta_L \equiv g_{0L} \Big/ \sqrt{g_{1L}^2 + g_{0L}^2}$ = amount of compositeness of the weak gauge bosons.
  2. $\sin\theta_R \equiv g_{0R} \Big/ \sqrt{g_{1R}^2 + g_{0R}^2}$ = amount of compositeness of the hypercharge gauge boson.

Top mass constraints

- For large values of the composite fermion masses, the top quark mass is

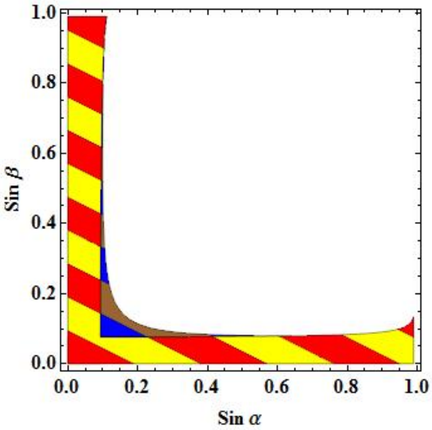
Top mass

$$m_t \simeq \frac{y_t \; v}{\sqrt{2}} \sin\alpha \;\; \sin\beta$$

- y_t is expected to be of order $1 \div 4\pi$, being the coupling of an interaction among composite states: in what follow I assume that y_t, g_{1L} and g_{1R} are sufficiently small to be perturbative.
- As a consequence $\sin\alpha$ and $\sin\beta$ cannot be too small.

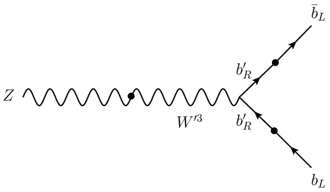
Top mass constraints

- For: $\sqrt{M_Q^2 + \mu_Q^2} = 4$ TeV, $\sqrt{M_t^2 + \mu_t^2} = 3$ TeV:



Tree-level correction to the $Zb_L\bar{b}_L$ coupling

- Custodial $SU(2)_c \times P_{LR} \sim O(3)$ symmetry is broken at site-0, and at site-1 by $g_{1L} \neq g_{1R}$: we expect tree-level contributions to the $Zb_L\bar{b}_L$ coupling, vanishing in the custodial limit $g' \to g$, $g_{1R} \to g_{1L}$.
- Direct computation gives



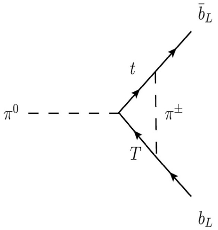
Tree level δg_{Lb}

$$\delta g_{Lb} = \frac{v^2}{2f^2} \sin^2\alpha \left(\sin^2\theta_R - \sin^2\theta_L\right)$$

- This is anyway negligible, for $f \gtrsim 1$ TeV: to recover 1σ agreement we need $\delta g_{Lb} \simeq 0.4$.

One-loop correction to the $Zb_L\bar{b}_L$ coupling

- The dominant contribution comes, as usual, from triangle diagrams with one SM top and one heavy top:



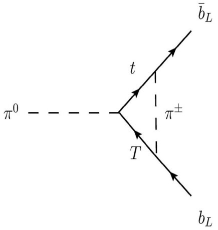
- The bi-doublet gives canceling contributions, due to the $SU(2)_c \times P_{LR} \sim O(3)$ custodial symmetry. But the overall result is slightly negative

Bi-doublet contribution to δg_{Lb}

$$\delta g_{Lb} \simeq \frac{m_t^2}{16\pi^2 v^2}\left[-\frac{1}{2}\left(\log\frac{M_Q^2+\mu_Q^2}{m_t^2} - \log\frac{M_Q^2}{m_t^2}\right)\frac{v^2}{f^2} - \log\frac{M_Q^2+\mu_Q^2}{m_t^2}\,\frac{m_t^2}{M_Q^2+\mu_Q^2}\right]$$

One-loop correction to the $Zb_L\bar{b}_L$ coupling

- The dominant contribution comes, as usual, from triangle diagrams with one SM top and one heavy top:

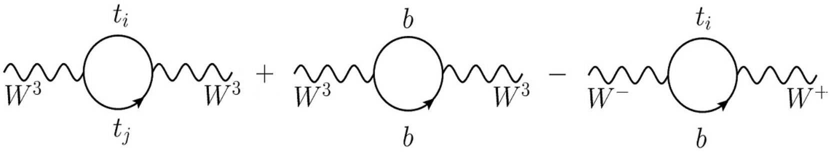


- The singlet gives a positive contribution, which becomes rather large for small values of β:

Singlet contribution to δg_{Lb}

$$\delta g_{Lb} \simeq \frac{m_t^2}{16\pi^2 v^2}\left(\cot^2\beta - 2 + \log\frac{M_t^2+\mu_t^2}{m_t^2}\right)\cot^2\beta\,\frac{m_t^2}{M_t^2+\mu_t^2}$$

One-loop correction to $\Delta\rho$

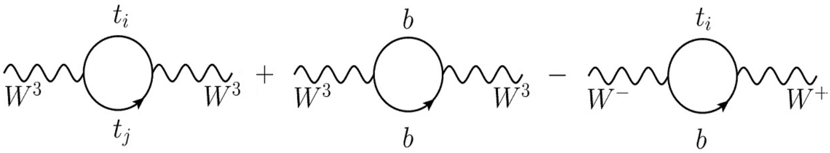


- The bi-doublet contribution is negative, as in DESM:

Bi-doublet contribution to $\Delta\rho$

$$\Delta\rho \simeq -\frac{3m_t^2}{16\pi^2 v^2} 8 \log \frac{M_Q^2 + \mu_Q^2}{m_t^2}$$

One-loop correction to $\Delta\rho$



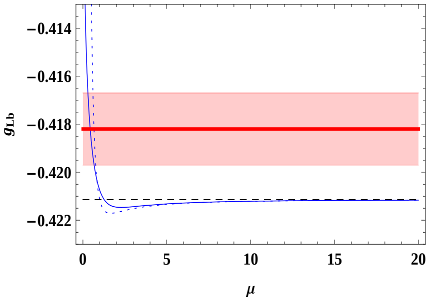
- The singlet contribution to $\Delta\rho$ is positive, and identical to the singlet contribution to δg_{Lb}:

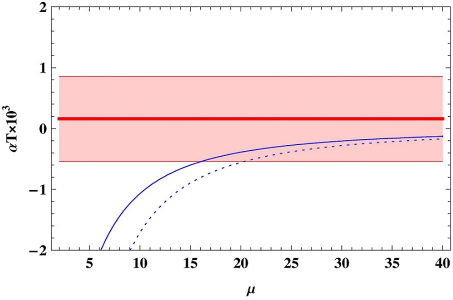
Singlet contribution to $\Delta\rho$

$$\Delta\rho \simeq \frac{3m_t^2}{16\pi^2 v^2}\left(\cot^2\beta - 2 + \log\frac{M_t^2+\mu_t^2}{m_t^2}\right)\cot^2\beta\,\frac{m_t^2}{M_t^2+\mu_t^2}$$

Bi-doublet effects

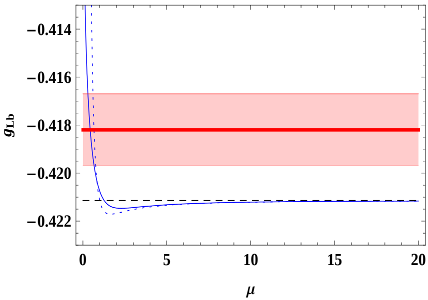
- Notice: the bidoublet gives very small (and negative) δg_{Lb}, and negative $\Delta\rho$.
- This is exactly like in DESM, where we only had the bi-doublet:

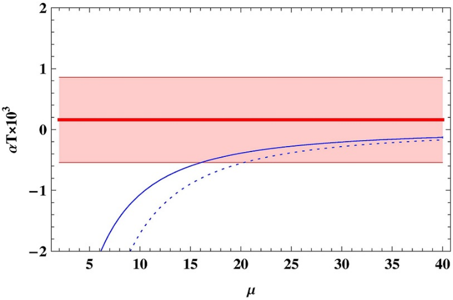




Bi-doublet effects

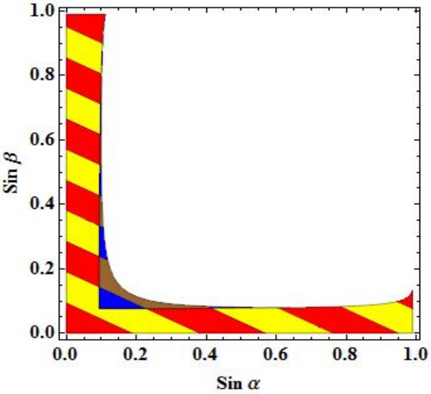
- Notice: the bidoublet gives very small (and negative) δg_{Lb}, and negative $\Delta\rho$.
- Now we have the singlet to give positive contributions to both δg_{Lb} and $\Delta\rho$: exactly what we need!





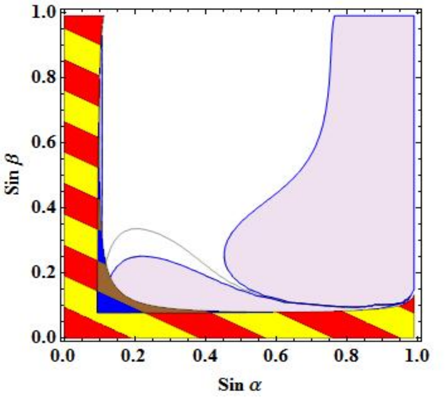
Constraints from top-mass and Yukawa coupling

- $\sqrt{M_Q^2 + \mu_Q^2} = 4$ TeV, $\sqrt{M_t^2 + \mu_t^2} = 3$ TeV.



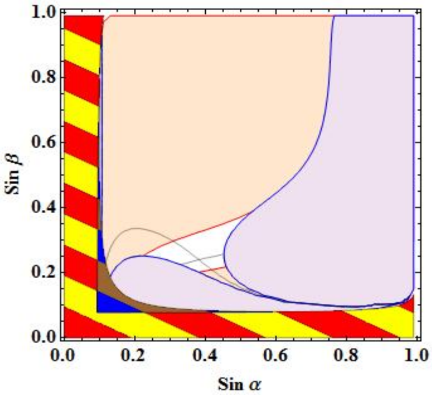
Constraints from $\Delta\rho$

- 1σ constraint from $\Delta\rho$, $\sqrt{M_Q^2+\mu_Q^2} = 4$ TeV, $\sqrt{M_t^2+\mu_t^2} =$ 3 TeV. Light Higgs ($m_H = 115$ GeV).



Constraints from g_{Lb}

- 1σ constraint from g_{Lb}, $\sqrt{M_Q^2+\mu_Q^2} = 4$ TeV, $\sqrt{M_t^2+\mu_t^2} = 3$ TeV, $f = 1$ TeV, $g_{1L} = g_{1R} = 4$ (weak dependence on f, g_{1L} and g_{1R}).



Conclusions

- In models with extended custodial symmetry, $SU(2)_c \times P_{LR} \sim O(3)$, the contributions to $\Delta\rho$ and the $Zb_L\bar{b}_L$ coupling are under control.
- Breaking of $O(3)$ in the SM accounts for essentially all the observed $\Delta\rho$, but not for all the observed δg_{Lb}.
- Extending the SM with a vector-like doublet χ, and embedding χ_L in an $SU(2)_L \times SU(2)_R$ bi-doublet with (t_L, b_L), yields a Yukawa interaction invariant under $O(3)$. Custodial symmetry is then broken by a hard mass term for the new doublet. This leads to 1σ agreement with the measured δg_{Lb}, but unfortunaltely in a region of the parameter space where $\Delta\rho$ is large and negative.

Conclusions

- $O(3)$-symmetric models of composite fermions feature new vector-like composite bi-doublets and singlets. These mix with the SM fermions, giving them mass.
- The contribution of the bi-doublet to the observables is as in DESM, but the singlet gives important positive contributions to both $\Delta\rho$ and δg_{Lb}. 1σ agreement with experiment is then easily attained for both observables.