ISAPP school 21

DARK MATTER FROM STANDARD MODEL AND BEYOND, A SELECTION OF PRODUCTION MECHANISMS.

LAURA LOPEZ HONOREZ (ULB)
LLOPEZHO@ULB.BE.

PART I

Most of the talks introducing a dark matter (DM) particle will begin with a list of expected properties:

1/ DM is a beyond the Standard Model (BSM) particle

↳ let us check first the candidates for DM in the Standard Model. (SM)

2/ DM is essentially neutral ($Q=0$)

Carefull, neutral under $U(1)_Q$ for Q = electromagnetic charge. DM with non zero $SU(2)_L$ or $U(1)_Y$ is possible! (minimal DM, Inert doublet, etc) Now some contribution of millicharged DM could be allowed.

3/ DM is massive and stable ($\tau_{DM} > \tau_{universe}$ ↑ 10^{18} s)

↳ Massive to allow for bottom up structure formation and stable to account for Ω_{DM} measurements. Now there is still some room for a (fraction of) decaying DM, see e.g. 1610.1051, 2012.05276

I. The Standard Model of particle physics.

About charges and symmetries, the SM gauge group is:

$$\underbrace{SU(3)_c}_{\text{Strong interactions}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{WEAK and electro magnetism (EM)}}$$

$\equiv$ Glashow-Weinberg Salam Model

$$\longrightarrow SU(3)_c \times U(1)_Q$$

↳ spontaneous symmetry breaking.

Let me emphasize again that DM is expected to be essentially neutral under $U(1)_Q$ <u>but</u> could very well have e.g. a non zero $SU(2)_L$ charge, see e.g. "Minimal DM": 0903.3381.

In the table on the next page, I summarize the charges of all SM particles. It is very clear that among all $Q=0$ charged particles, the neutrino is our best candidate for DM as it interacts the most weakly with all other SM particles.

Charges of the SM particles:

		$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_Q$
Spin 1/2	$L_L = \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$	1	2	-1	$\begin{cases} 0 \\ -1 \end{cases}$
	e_R	1	1	-2	-1
	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2	$1/3$	$\begin{cases} 2/3 \\ -1/3 \end{cases}$
	u_R	3	1	$4/3$	$2/3$
	d_R	3	1	$-2/3$	$-1/3$
Spin 0	$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	1	2	1	$\begin{cases} 1 \\ 0 \end{cases}$
Spin 1	B_μ	1	1	0	To Be redefined as $A_\mu, Z_\mu, W^\pm_\mu$
	$W^{1,2,3}_\mu$	1	3	0	
	gluons	8	1	0	0.

Convention: $Q = \underbrace{T_3}_{\to \text{isospin.}} + Y/2$

At the very least, the ν_L interacts with the SM through gauge interactions:

lepton doublet.

$$\mathcal{L}_{SM} \supset \sum_i \bar{L}_{Li}\, i \not{D}\, L_{Li} \,. \quad ⊛ \qquad i = \text{flavour index}$$

Lorentz index.

where • $\not{D} = D_\mu \gamma^\mu$.

• γ^μ satisfy the Clifford Algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}\mathbb{1}$$

We have L_L charged under $SU(2)_L$ & $U(1)_Y$:

$a = 1\ldots3$ — Pauli matrix

$$D_\mu L_L = \Big(\partial_\mu - i\underset{\substack{\uparrow\\ SU(2)\\ \text{gauge coupl}}}{g}\, W_\mu^a \frac{\sigma^a}{2} - i\underset{\substack{\uparrow\\ U(1)_Y\\ \text{gauge coupling.}}}{g'}\, B_\mu \frac{Y}{2}\Big) L_L$$

Using

$$\boxed{\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right). \\ \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} &= \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \end{aligned}}$$

and

with $c_W = \cos\theta_W$ and $s_W = \sin\theta_W$

you can rewrite the $SU(2)_L \times U(1)$ part of the covariant derivative as:

$$\boxed{\begin{aligned} & -ig W_\mu^a \sigma^a/2 - ig' B_\mu Y/2 \\ & = -ig(W_\mu^+ T^+ + W_\mu^- T^-) - \frac{ig}{c_W} Z_\mu (T^3 - s_W^2 Q) - ieA_\mu Q \end{aligned}}$$

with $T^\pm = \frac{1}{2}(\sigma^1 \pm i\sigma^2)$ $\begin{cases} T^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\ T^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \end{cases}$

and $\begin{cases} Q = T^3 + Y/2 \quad \text{with } T^3 = \sigma^3/2 \text{ weak isospin} \\ e = g s_W \text{ and } g'/g = \tan\theta_W \end{cases}$

Some more details on spinors:

* one usefull representation of γ^μ matrices for high energy description is the WEYL ($\equiv$ chiral) representation

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad \text{with} \quad \overset{(-)}{\sigma^\mu} = (\mathbb{1}, (-)\sigma^i)$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1} & \\ & \mathbb{1} \end{pmatrix}$$

and the Pauli matrices are given by:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

* $S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$ provide a 4x4 representat° of the Lorentz group

with the Dirac spinor transforming

as: $\psi \to \exp(-i\, \underset{\text{antisymmetric tensor}}{\omega_{\mu\nu}}\, S^{\mu\nu})\, \psi$

You can check (exercice) that

$$\boxed{\begin{cases} \psi_L = L\psi = \frac{1-\gamma_5}{2}\psi \\ \psi_R = R\psi = \frac{1+\gamma_5}{2}\psi \end{cases}}$$ transform

differently under the Lorentz transform.

They correspond to 2 different irreducible representations of the Lorentz group.

II. Candidates for DM in SM and Beyond

DM from the SM?

As said above, within the SM, we have a potential candidate for DM, the SM neutrino ν_L with isospin.

$$Q_{\nu_L} = 0 \; ; \quad Y_{\nu_L} = -1/2 \; ; \quad T_{3\,\nu_L} = 1/2$$

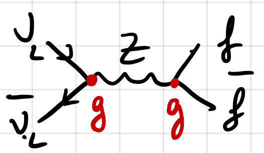
$$T_3 \, L_L = \frac{\sigma_3}{2} L_L = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \begin{pmatrix} 1/2 \; \nu_L \\ -1/2 \; \ell_L \end{pmatrix}$$

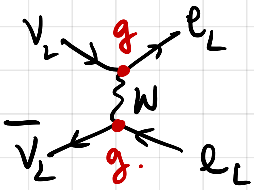
$\rightarrow$ the ν_L couples to Z and W bosons

We have thus:

$$\mathcal{L}_{SM} \supset \bar{L}_L (i \not{\partial}) L_L + \frac{g}{\sqrt{2}} \left(\left(\bar{\ell}_L \gamma^\mu \nu_L \right) W^-_\mu + h.c. \right) + \frac{g}{2 c_W} \bar{\nu}_L \gamma^\mu \nu_L Z_\mu$$

$\rightarrow$ Possible annihilation channels for ν_L





For the purpose of these lectures, let us look at ν_L abundance being agnostic on m_ν
(obviously, we have strong laboratory and cosmo constr.!)

As very rough estimates, when:

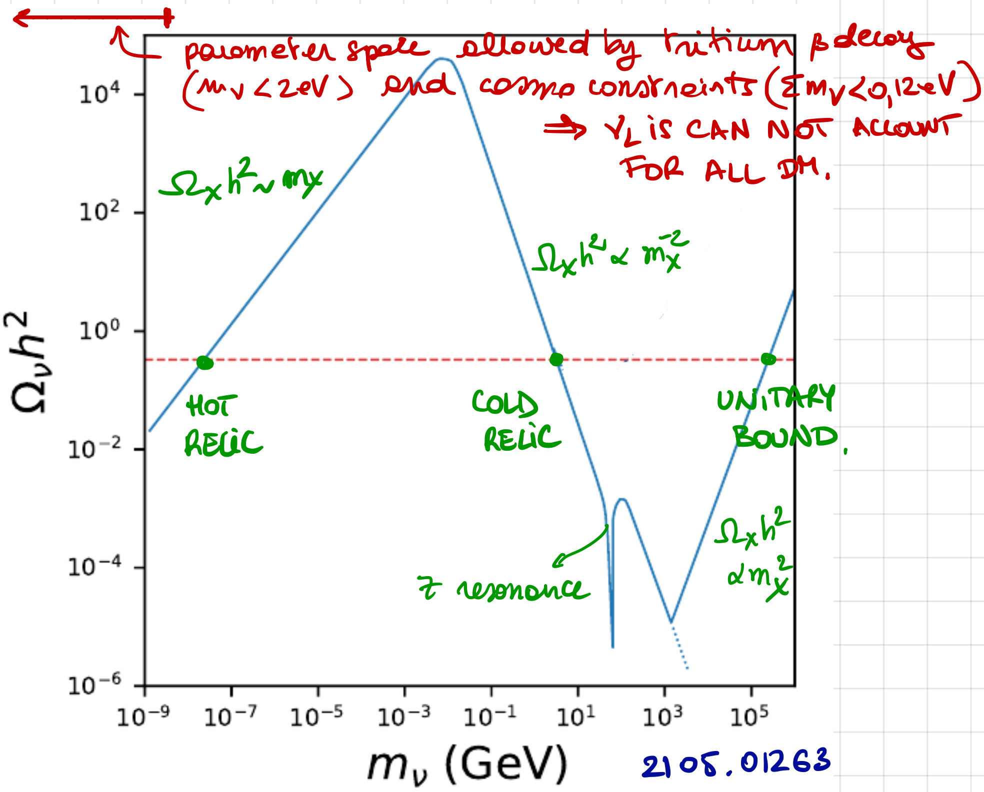
$\underline{T > m_\nu}$: $\sigma v_{\nu\nu \to ff} \sim G_F^2 T^5$

Fermi Constant: $G_F = \frac{\sqrt{2} g^2}{8 m_W^2}$

$\underline{m_\nu > T, m_W}$: $\sigma v_{\nu\nu \to ff} \sim \frac{\alpha_W}{m_\nu^2}$

$\alpha_W = \frac{g^2}{4\pi} = \frac{1}{S_W^2} \frac{e^2}{4\pi} = \frac{\alpha_{EM}}{4\pi S_W^2}$

In the next sections, we will see that $\Omega_\nu h^2 (m_\nu)$ takes the following form:



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DM candidate BSM.

As we will see, the ν_L can't account for all the DM of the universe.

As a result, we will have to go BSM. You might be aware that we have many possible candidates for DM.

As my purpose is both to introduce you to BSM DM and its mechanisms of production, I will consider a minimal extension of the SM that could give rise to DM candidates and allow me to explore a large range of DM couplings to the SM.

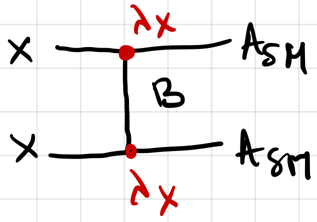
The discussion presented in these notes may be directly translated to any model where:

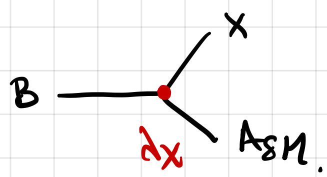
- DM is a singlet under the SM gauge group, say X
- Some symmetry prevents the DM to decay to SM. Here we assume a Z_2 symmetry under which DM $\equiv Z_2$ odd, SM $\equiv Z_2$ even.
- I will assume that DM is not the only new particle BSM. There will be

a "partner" odd under the Z_2 symmetry that will help to mediate DM – SM interactions. Let me refer to this "mediator" as B and assume that B is charged under the SM gauge group (B = bath particle)

NB: more minimal extensions of the SM, yet very rich, could be the scalar singlet interacting through H-portal with SM or e.g. "minimal DM" which is a $Q=0$ of a BSM $SU(2)_L$ multiplet

With the above considerations, assuming a cubic interaction: $\mathcal{L} \supset \lambda_\chi B \chi A_{SM}$ where A_{SM} = SM particle and λ_χ is some coupling I can have the following annih. decay channels:





there are multiple possibilities for such models (sometimes referred to as "t-channel" models for WIMP).
For definitiveness, here I will focus on fermionic Majorana DM.

Considering B as a fermion or a scalar, the "minimal" cubic interactions I can write are:

$A_{\rm SM}$	Spin DM	Spin B	Interaction	Label
$\psi_{\rm SM}$	0	1/2	$\bar{\psi}_{\rm SM}\Psi_B\phi$	$\mathcal{F}_{\psi_{\rm SM}\phi}$
	1/2	0	$\bar{\psi}_{\rm SM}\chi\Phi_B$	$\mathcal{S}_{\psi_{\rm SM}\chi}$
$F^{\mu\nu}$	1/2	1/2	$\bar{\Psi}_B\sigma_{\mu\nu}\chi F^{\mu\nu}$	$\mathcal{F}_{F\chi}$
H	0	0	$H^\dagger\Phi_B\phi$	$\mathcal{S}_{H\phi}$
	1/2	1/2	$\bar{\Psi}_B\chi H$	$\mathcal{F}_{H\chi}$

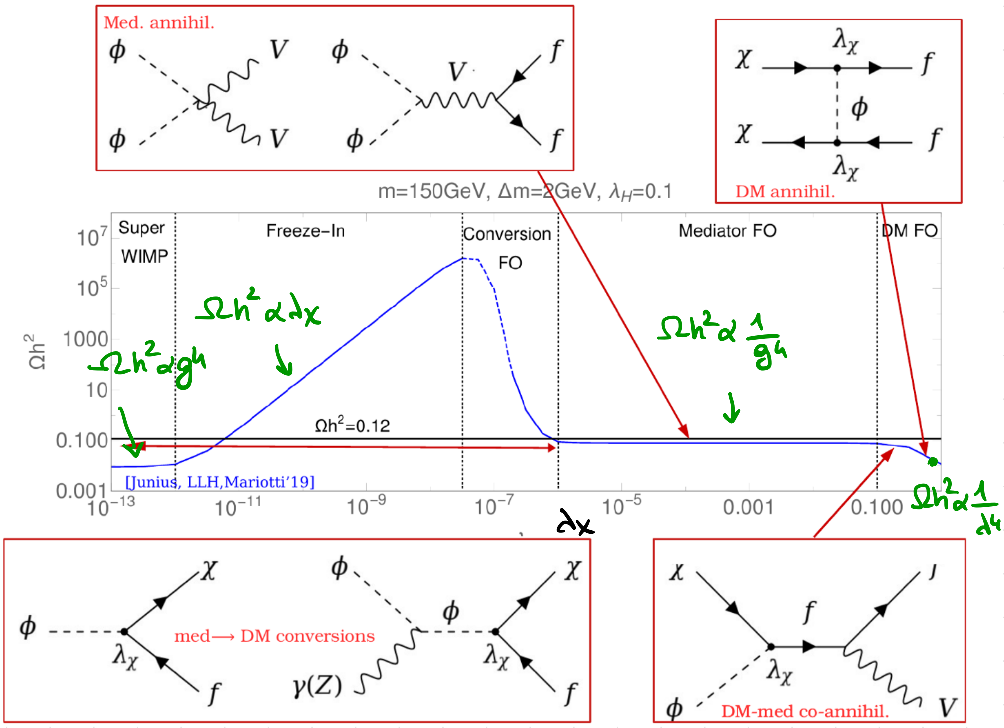
For more details see 2102.06221.

Here, for definitiveness, I will take the case of fermionic DM majorana fermion χ, coupling through "Yukawa" like interactions to a charged scalar ϕ and a right handed lepton

$$\mathcal{L} \supset \lambda_\chi \; \bar{\ell}_R \chi \phi$$

For "WIMP" behaviour see e.g. 1307.6480, 1503.01500
For "FIMP" see e.g. 2102.06221, 1904.07513

At the end of the day, our (enthousiastic) goal is to go through all the DM production mechanisms represented in the Fig below



see 1904.07513 for details.

III. Boltzmann equations.

The question that arises now is what would be the neutrino mass that would give rise to all the dark matter.

In order to answer this question, we first have to introduce Boltzmann equations describing the evolution of the DM phase space distribution function $f_X(p^\mu, x^\mu)$ that counts the number of particles in the phase space element $d^3x \; d^3p$ where x an p refer to position and momentum. Here in particular we follow the convention:

$$\begin{cases} n_X = \int \frac{d^3p}{(2\pi)^3} f_X \\ \rho_X = \int \frac{d^3p}{(2\pi)^3} E(p) f_X \end{cases}$$

where X refers to DM (scalar, fermion, ...). carefull, here we absorb the X number of dof into f_X.

The Boltzmann equation can be written as:

$$\boxed{L[f] = C[f]}$$

influenced by cosmology

influenced by particle phys.

III 1. The Liouville operator

$$L[f] = p^{\alpha}\frac{\partial f}{\partial x^{\alpha}} - \Gamma^{\alpha}_{\beta\gamma}\, p^{\beta} p^{\gamma} \frac{\partial f}{\partial p^{\alpha}}$$

connexion; Christoffel symbols $\sim \frac{\partial g_{\mu\rho}}{\partial x^{\nu}}$

this part of the Boltzmann equation is essentially influenced by the cosmology through $\Gamma^{\alpha}_{\beta\gamma}$.

Considering a Friedmann-Robertson-Walker space time ($ds^2 = dt^2 - a^2(t)\, d\vec{x}^2$) the homogeneity and isotropy implies at zeroth order in perturbations that*:

from now on $p = |\vec{p}|$

$$f(p^{\mu}, x^{\mu}) = f(|\vec{p}|, t) \equiv f(p,t)$$

and

$$L[f] = E\left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial E}\frac{dE}{dt}\right)$$

$$= E\left(\frac{\partial f}{\partial t} - H\frac{p^2}{E}\frac{\partial f}{\partial E}\right)$$

influence of the history of the Universe $H = \frac{da}{dt}\frac{1}{a}$

$$\Rightarrow \boxed{L[f] \equiv E\,\frac{df}{dt}.}$$

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* we leave the study of cosmological perturbations to the lectures of Cosmology. see also e.g. Dodelson, "Modern Cosmology"

- Notice that for most of the cases we are interested in the DM production will happen in a radiation dominated (RD) era:

$$\boxed{\begin{aligned} H &= \frac{T^2}{M_0(T)} \quad \text{where} \quad M_0(T) = M_p\sqrt{\frac{45}{4\pi^3 g_*(T)}} \\ &\simeq 1.66\,\frac{T^2}{g_*(T) M_p} \quad [\text{Radiation dominate (RD) era}]^* \end{aligned}}$$

where $g_*(T)$ denotes the number of relativistic dof contributing to radiation density at temperature T: $\rho_R = \frac{\pi^2}{30} g_* T^4$

- In addition the entropy density can be defined as: $\boxed{s = \frac{2\pi^2}{45} h_{eff} T^3}$

On general grounds g_* and h_{eff}. differ when ν decouple from the heat bath so that $T_\nu(T) < T$

$$\begin{cases} g_*(T) = \sum_i g_i^f \left(\frac{T_i}{T}\right)^4 . \\ h_{eff}(T) = \sum_i g_i^f \left(\frac{T_i}{T}\right)^3 \end{cases} ; \quad g_i^f = g_i \times \begin{cases} 1 & \text{bosons} \\ 7/8 & \text{fermions} \end{cases}$$

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* see e.g. 2102.06221 for e.g. FI in a matter dominated (M.D) era.

$g_*(T) = h_{eff}(T)$ at high T

for SM only $\left\{ \begin{array}{l} g_*(T) \simeq 100 \text{ for } T > T_{EW} \\ h_{eff}(T) \end{array} \right.$

while today. 2, 2, e, μ, τ.

$$\begin{cases} g_*(T_0) = g_\gamma^s + g_\nu^s \times 3 \left(\frac{T_\nu^0}{T_0}\right)^4 = 3.36, \\ h_{eff}(T_0) = g_\gamma^s + g_\nu^s \times 3 \left(\frac{T_\nu^0}{T^0}\right)^3 = 3.91 \end{cases}$$

$$\frac{T_\nu}{T}\Big|_{T > T_{CD}^\nu \sim MeV} = \left(\frac{h_{eff}(T_{CD}^\nu)}{h_{eff}(T^0)}\right)^{1/3} = \left(\frac{4}{11}\right)^{1/3}$$ by entropy conserv.

- Notice that the abundance of a given specie is usually expressed relatively to the critical density: $\left\{ \equiv \text{total} \right.$

NOT ESPECIALLY RD!

$$\begin{cases} \rho_c = H(T) \left(\frac{8\pi}{3 M_P^2}\right)^{-1} \\ \Omega_i(T) = \frac{\rho_i(T)}{\rho_c(T)} \end{cases}$$

<u>Today</u> $H_0 = 100\ h$ km/s/Mpc.

$h \simeq 0{,}7$ (Planck)

$\rho_c^0 \simeq 10^{-5}\ h^2$ GeV/cm^3

III 2. C[f] the collision term

C[f] is the collision term that captures the particle physics interactions, of your species χ.

On very general grounds, one should consider both elastic and inelastic processes:

$$C[f] = C_{el}[f] + C_{inel}[f]$$

Below, we provide order of magnitude estimates for a WIMP TOY MODEL with χ coupling to weak gauge bosons.

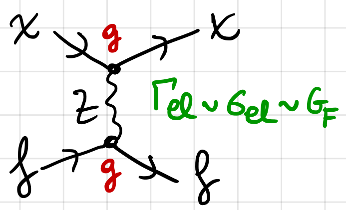
III 2.1. Elastic scatterings $\leftrightarrow C_{el}[f]$ and Thermal equilibrium

- for our "toy model":

$\chi f \to \chi f$

elastic processes

↳ drives momentum exchanges.



The χ particle is kept in LOCAL THERMAL EQUILIBRIUM (LTE) while the rate of elastic scatterings Γ_{el} gives rise to a relaxation time, τ_{LTE}, sufficiently small compared to the Hubble time, i.e.

for $\tau_{LTE} \propto \frac{1}{\Gamma_{el}} \ll H(T)^{-1}$

KINETIC DECOUPLING (KD) happens at a temperature T_{KD} at which : $\boxed{\tau_{LTE} = H(T_{KD})^{-1}}$

e.g $T_{KD} \sim \left(\frac{M_X}{M_P}\right)^{1/4} m_Z \, g^{-1} \sim$ few MeV for ~100 GeV DM

for the toy model under consideration.
with non relativistic DM (see also 0612238)

For weakly interacting particles (neutrinos, electrons, "vanilla WIMP", ...) KD usually happens well after chemical decoupling (see below) so that one usually directly make use of Fermi-Dirac, Bose-Einstein or Boltzmann (non relativistic) momentum dependent distribution functions.

- when particles are in thermal equilibrium it is enough to follow the first momentum of the distribution function

$$n(t) = \int \frac{d^3p}{(2\pi)^3} f(p,t)$$

or equivalently the yield

$$\boxed{Y = \frac{n(t)}{s(t)} \quad \text{where} \quad s(t) = \frac{2\pi^2}{45} h_{eff} T^3}$$

with Y dimensionless, s is the entropy density and h_{eff} is the number of relativistic dof contributing to the entropy.

- when particles are suspected to have a departure from thermal equilibrium with the heat bath, one should follow the "unintegrated" Boltzmann equations $\frac{d}{dt} f(p,t)$ which can be computationally expensive and tricky.
(integro partial differential equation, see e.g. 1706.07433)

- It has been shown though that when chemical and kinetic decoupling are intertwined it might be enough to follow

DS!

the first and second momenta only of distribution function introducing the dimensionless variable:

$$y_X = \frac{m_X}{3 s^{2/3}} \left\langle \frac{\bar{p}^2}{E} \right\rangle = \frac{m_X}{3 s^{2/3}} \frac{1}{n_X} \int \frac{d^3p}{(2\pi)^3} \frac{\bar{p}^2}{E} f_X(\bar{p},t)$$

Keeping in mind that for e.g. a non relativistic X particle : $y_X = \frac{m_X}{s^{2/3}} \underbrace{\frac{2}{3} \left\langle \frac{\bar{p}^2}{2 m_X} \right\rangle}_{= T_X \text{ for } X \text{ N.R.}}$

with T_X equals the heat bath temperature T when $T > T_{KD}$. For more details, see e.g. 0612238, 1706.07433, 2103.01944.

NB As a side note, notice that particles free-stream (FS) from the moment they are kinetically decoupled leaving an exponential cut on the power spectrum due to collision-less damping (= free streaming)

this assumes χ with $SU(2)_L$ interactions, non relativistic at the time of KD. $M_{FS} = \frac{4\pi}{3} \lambda_{FS}^3 \rho_X$

- For the DM toy model considered you get *
  $M_{FS} = 10^6 M_\odot$ for $\begin{cases} 100\,\text{GeV DM} \\ T_{KD} \sim 30\,\text{MeV} \end{cases}$.
- electrons get chemical decoupling from photons around $z \sim 10^3$ (last scattering surface) but get kinetically decoupled at $z \sim 10^2$ only (from which $T_{gaz} \propto 1/a^2$ instead of $T \propto 1/a$)

Let us emphasize that free-streaming is one possible source of damping. Another one is collisional damping (Silk Damping) due to e.g. interactions of massive species with lighter ones, as is the case of baryons or DM scattering with neutrinos or (dark) photons

see also eg: 0012504, 0410591, 0903.0189
1205.5809, 1603.04884

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* a small galaxy of ~ 100 kpc size ⟺ $M \sim 10^9 M_\odot$ which roughly corresponds to the smallest scales tested by Ly-α forest data.

II.2.2 Description of particles in thermal equil with a heat bath.

When following a thermal equilibrium evolution early on, one has:

$$\boxed{f_X(p,t) = \frac{g_X}{\exp\left(\frac{E-\mu}{T} \pm 1\right)}}$$

where $\pm$ stand for $\begin{cases} \text{BE} \\ \text{FD} \end{cases}$ species,

and μ is the chemical potential and g_X count the number of internal degrees of freedom of X.

number density.

Remember $\begin{cases} n_X = \int \frac{d^3p}{(2\pi)^2} f_X(p,t) \\ \rho_X = \int \frac{d^3p}{(2\pi)^3} E\, f_X(p,t) \end{cases}$

energy density

- In the $\begin{cases} \text{N.R.} \\ \text{relativistic} \end{cases}$ limit ($T \gg m_X, \mu$)

Riemon function $\zeta(3) \simeq 1.2$.

$$\begin{cases} n_X = g_X^n \dfrac{\zeta(3)}{\pi^2} T^3 \\ \rho_X = g_X^\rho \dfrac{\pi^2}{30} T^4 \end{cases}$$

__where__ $g_X^n = \begin{cases} 1 \\ 3/4 \end{cases} g_X$ and $g_X^\rho = \begin{cases} 1 \\ 7/8 \end{cases} g_X$ for $\begin{cases} \text{bosons} \\ \text{fermions} \end{cases}$

- In the non-relativistic limit ($m_\chi \gg T$)

$$f_\chi \simeq \exp\left(\frac{E-\mu}{T}\right) \equiv \text{Boltzmann distribution}$$

$$\begin{cases} n_\chi = g_\chi \left(\frac{m_\chi T}{2\pi}\right)^{3/2} \exp\left(-(m_\chi - \mu)/T\right) \\ \rho_\chi = m_\chi n_\chi \end{cases}$$

- It is convenient to introduce a dimensionless time variable $\sim m_{ref}/T$ where m_{ref} is some reference mass. Within the freeze-out, it is convenient to take $m_{ref} = m_\chi$ and we define

$$\boxed{x = \frac{m_\chi}{T}}$$

In the follow up, we refer to a equilibrium density n, n^{eq} the equ density with $\mu = 0$:

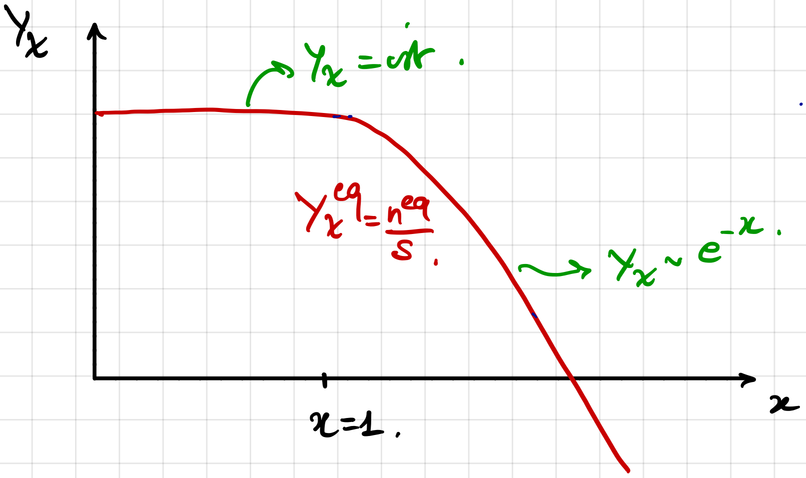
$$n_\chi^{eq}(x) = \frac{g_\chi}{2\pi^2} \frac{m_\chi^3}{x} \underbrace{K_2[x]}_{} = \begin{cases} g_\chi^{n} \frac{\zeta(3)}{\pi^2} \cdot \frac{m_\chi^3}{x^3} & \text{for } x \ll 1 \\ g_\chi \frac{m_\chi^3}{(2\pi x)^{3/2}} e^{-x} & \text{for } x \gg 1 . \end{cases}$$

Modified Bessel functⁿ of the 2d kind.

Let us now describe briefly the number density evolution in terms of the yield.

$$\boxed{Y_\chi = \frac{n_\chi}{s}}$$

which is again a dimensionless quantity.



Also, in the next section, we will want to account for $\Omega^0_\chi h^2 \simeq 0,12$ (Planck)

with
$$\boxed{\begin{aligned} &\Omega_\chi^0 h^2 = \frac{Y_\chi^\infty}{\rho_c^0} \times s_0 \times m_\chi \\ &s_0 = 0,74\, h_{eff}^0\, 10^3\, cm^{-3} \end{aligned}}$$
with

and we have assumed DM N.R. today because of large scale structures observations

$C_{inel}[f]$

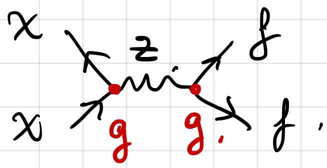
III 2.3. Inelastic Scatterings and Chemical decoupling

In addition to potentially fast momentum exchanges with the heat bath through elastic scatterings (see previous section), the DM might have inelastic interactions with particles of the heat bath such as, e.g. annihilations.

For our "toy" model, we would have e.g.:

$\chi\bar{\chi} \to f\bar{f}$

inelastic processes.



$\Gamma_{ann} \propto \sigma_{ann} \propto G_F$

Chemical decoupling (CD) happens when these inelastic processes, or particle number density changing processes, become slower than the expansion rate of the universe: ie

$$\boxed{\Gamma_{inel} \sim H(T_{CD})}$$

Notice that for CD of N.R DM of our toy model, you expect:

$$T_{CD} \sim \frac{m_\chi}{25} > T_{KD} \sim \text{few MeV} \quad \text{for } 100\,\text{GeV particle.}$$

Let us refer to $in \leftrightarrow fin + X$ as all the possible processes giving rise to a variation of your dark matter number density:

$$\boxed{\frac{1}{E_X} C_{inel}[f] = \frac{1}{2E_X}\int \prod_\alpha \left(\frac{d^3p_\alpha}{(2\pi)^3 2E_\alpha}\right)(2\pi)^4\delta^4(P_{fin}+p_X-P_{in}) \times\left[|M|^2_{in\to fin+X}\, f_{in}\,(1\pm f_X)(1\pm f_{fin}) - |M|^2_{fin+X\to in}\, f_{fin}\, f_X\,(1\pm f_{in})\right]}$$

where the above notations correspond to:

- the α index is not a Lorentz index here but runs over all species in the in and fin state
- $P_{fin} = \sum_{\alpha \ni fin} p_\alpha$; $P_{in} = \sum_{\alpha \ni in} p_\alpha$.

  ie P_{fin} is the sum of the 4-momenta of particles in the final state.

- $(1\pm f_X)$ is a $\begin{cases}\text{Pauli-Blocking (PB,−)}\\ \text{Bose-Einstein enhancement (BE,+)}\end{cases}$ factors for $\begin{cases}\text{fermionic DM}\\ \text{bosonic}\end{cases}$

while $(1\pm f_{in})$, $(1\pm f_{out})$ correspond to a product of PB, BE factors for particles in the in or fin states.

- f_{in}, f_{fin}: are the product of phase space distributions for particles in the $\genfrac{}{}{0pt}{}{in}{fin}$ state.
- $|M|^2$ are the transition matrix element squared summed over in an fin states (no averaging)

Assuming 1/ CP invariance so that

$$|M|^2 = |M|^2_{in \to out+X} = |M|^2_{out+X \to in}$$

and that 2/ we consider cases without Bose condensation or fermi degeneracy, i.e.

$$(1 \pm f_{\genfrac{}{}{0pt}{}{in}{fin}}) \simeq 1 .$$

The collision term, which we will refer to as $C[f]$ from now on, reduces to a simpler:

$$\boxed{\frac{1}{E_X} C[f_X] = \frac{1}{2E_X} \int \prod_\alpha \left(\frac{d^3 p_\alpha}{(2\pi)^3 2E_\alpha} \right) (2\pi)^4 \delta^4 (P_{fin} + p_X - P_{in}) \times |M|^2 \left[f_{in} - f_{fin} f_X \right]}$$

For the rest of the lecture, I will assume that 1/ and 2/ hold and I will use the above version of the Boltzmann equ.