GGI LECTURES 2025
ON
DARK MATTER COSMOLOGY.

From a selection of DM production mechanisms to a selection of cosmology probes.

PART I.

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REFERENCES :

BOOKS :

- KOLB & TURNER : the early universe
- DODELSON : Modern Cosmology
- BAUMAN : Cosmology
- LESGOURGUES, MANGANO, MIELE & PASTOR :
  Neutrino Cosmology.

DM reviews :

1605.04909 BERTONE
1705.01987. PLEHN
1807.08749 CLINE
1812.02029 HOOPER.

1009.3690 GELMINI-GONDOLO.

2406.01705 CIRELLI-STRUMIA-ZUPAN.

Menu

I. Brief Introduction to Dark Matter (DM)

II Cosmology tools.

III Standard Model and Beyond.

IV Boltzmann equations.

V The Freeze-out mechanism.

VI Feebly interacting particles.

VII DM imprint on cosmology & Constraints.

I Brief introduction to Dark Matter (DM)

I.1.. Brief highlights of DM history.

see e.g. Bertone, Hooper 1605.04909
. Cirelli, Strumia & Zupan 2406.01705.

First DM evidences are dynamical evidences :

- 1900's : KELVIN apply the concepts of theory of gas to stars :

"If stars in the milky way (MW) can be described as a gas of particles acting under influence of gravity, one can relate the size of the system to its velocity dispersion" (1904)

- 1915 : Öpik and later J. Kapteyn (1922) & Oort (1932)

Publish discussions on the abundance of "DM"

(the word "matière obscure" ≡ DM was already used by Poincaré in 1906)

based on studying the relations between mass distribution, velocity dispersion and luminosity.

The DM was supposed to be faint stars with abundance smaller than the luminous one

- $\underline{1933}$: F. Zwicky uses the virial theorem (for a system that is in a stationary state):

$$2\langle E_c\rangle + \langle E_p\rangle = 0$$

* He studied the galaxies (8) of the Coma cluster extracting velocities from Doppler shifts and got a velocity dispersion

$$\sigma_v = 1020 \quad km/s$$

(more recent value $\sigma_v = 1082$ km/s 1996)

* He compares this value to the one obtained equating

mass of the Coma cluster

$M_c = M(R)$

$$M_c \sigma_v^2 = 2\langle E_c\rangle = -\langle E_p\rangle = +\frac{3}{5}^{*} \frac{G_N M_c^2}{R}$$

- assuming that the coma cluster includes 800 galaxies of mass of $10^9 M_\odot$

from obs.

$$\rightarrow M_c = 800 \times 10^9 \times 2.\ 10^{33}\ g$$
$$= 1.6\ 10^{45} g$$

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* $\frac{3}{5}$ arises assuming cluster ≡ sphere of cst density ρ and radius R and tot mass M

$$\langle E_p\rangle_t = -\int_0^R \frac{G\,\overbrace{(\rho 4\pi r^2 dr)}^{\text{element of mass in } r+dr}\ \underbrace{M(r)}_{\frac{4}{3}\pi r^3 \rho \,=\, \text{mass enclosed in } r}}{r}$$

from obs.

- and a cluster radius $R = 10^6$ lyr $\sim 10^{24}$ cm

$$\rightarrow \sigma_v = \left(\frac{3}{5} G_N \frac{M}{R}\right)^{1/2}$$

$$= \left(\frac{3}{5} \frac{6.67\,10^{-5}\,\mathrm{cm}^3/\mathrm{kg}/\mathrm{s}^2\ 1.6\,10^{42}\,\mathrm{kg}}{10^{24}\,\mathrm{cm}}\right)^{1/2}.$$

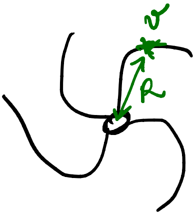
$$\simeq 80\,\mathrm{km/s}$$

→ the expected velocity dispersion from the luminous matter potential is one order of magnitude too small.

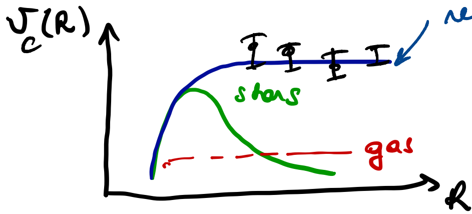
→ needs much more DM density than luminous matter. ($\times 100$.)

NB Zwicky also derives in 1937 the M/L ratio of the galaxies in the Coma Cluster and obtains $M/L = 500\ M_\odot/L_\odot$ while a $M/L = 3\ M_\odot/L_\odot$ was expected from the local Kapteyn System. With today's value of H_0 (and thus better distance eval) the would get $M/L \sim 160\ M_\odot/L_\odot$

- 1960: Penzias & Wilson discover the CMB background ($\equiv$ black body $T_{CMB} \simeq 2.7\,\mathrm{K}$ $= 0{,}2\,\mathrm{meV}$)
  → looked very smooth for ~ 30 years
  ↳ highly isotropic



1970 : • K. Ford & V. Rubin publish their spectroscopic observations of the M31 (Andromeda galaxy) allowing to track the stars motion to larger radius (up to ~22 kpc, further extended to ~50 kpc later)



$$F_g = \frac{G m M(R)}{R^2}$$

$$F_c = \frac{m \langle v_c^2 \rangle}{R}.$$

$$\langle v_c^2 \rangle = \frac{G M(R)}{R} \quad \text{and} \quad M(R) = 4\pi \int_0^R r^2 \rho(r)\, dr$$

- for centrally concentrated mass: $\langle v_c^2 \rangle \propto \frac{1}{R}$.
- for observations at large radius: $\langle v_c^2 \rangle \propto$ cst
  $\Rightarrow$ would need $\rho(r) \propto \frac{1}{r^2}$

e.g. isothermal DM profile: $\rho(r) = \dfrac{\rho_s}{1 + \left(\frac{r}{r_s}\right)^2}$

NB: • ∃ previous works on that topic since 1940's but Ford and Rubin provided the first precise measurement

• Note that rotation curves are quite diverse!

- 1982 : Peebles : absence of CMB fluctuations at the level 10^{-4} rule out baryons as being DM

The reason of the above statement is related to the growth of non relativistic matter perturbations, considering,

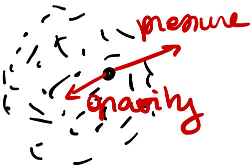
$$\rho_i(t,\bar{x}) = \bar{\rho}_i(t)\,(1+\underbrace{\delta_i(t,\bar{x})}))\,.$$

↳ fourier transf. $\delta_k(z)$

$\dot{x} = \partial_t x$

comoving wave nb. $\bar{r} = a\bar{x}$; $k \sim 1/x$

$$\hookrightarrow \ddot{\delta}_k + \overbrace{2H\dot{\delta}_k}^{\text{Hubble friction}} + \left(\underbrace{\frac{k^2}{a^2} v_s}_{\text{Pressure}} - \underbrace{\frac{3H^2}{2}}_{\text{Newtonian potential}}\right)\delta_k = 0 \,.\qquad \delta = \frac{\delta\rho}{\bar{\rho}}$$



When pressure term is small and working in matter dominated era :

$$\begin{cases} a \propto t^{2/3} \quad (\text{see later}) \\ \quad\ \ \uparrow \text{physical time.} \\ H = da/dt = \dfrac{8\pi G}{3}\bar{\rho} = \dfrac{2}{3t^2} \end{cases}$$

⇒ the growing mode solution scales as $\delta \propto a$ for negligible pressure term

NB one defines the "Jeans scale"

as $k_J = \frac{\sqrt{4\pi G \bar{\rho}(t)}}{v_s(t)}$

* $k/a^2 \gg k_J$ (small scales) pressure term wins.

  damped oscillator

* $k/a^2 \ll k_J$ (large scales) gravity wins.

  • if the friction term can be neglected.

    $\delta_\rho \propto \exp(\pm t/\tau_J)$ $\tau_J = \frac{1}{\sqrt{4\pi G \bar{\rho}}}$

    → exp growth of perturb $\equiv$ Jeans instability.

  • if the friction term can't be neglected

    and $\delta_k(t) = \delta_{+k} D_+(t) + \delta_{-k} D_-(k)$.

    → $\ddot{D} + \frac{4}{3t}\dot{D} - \frac{2}{3t^2} D = 0$.

    → $D_- \propto t^{-1} \sim a^{-3/2}$ & $D_+ \propto t^{2/3} \propto a$.

    → growing mode $\underline{\underline{\delta \propto a}}$

    NB $\delta\rho \propto a^{-2}$ while $\rho \propto a^{-3}$.
    ↳ still decreasing!

Considering matter perturbations made of baryons only, they can only grow since baryons–γ last scattering, ie $a_{CMB} \sim 10^{-3}$

$$\rightarrow \quad \delta_k(t_0) = \delta_k(t_{CMB}) \frac{a(t_0)}{a(t_{CMB})}$$

and $\delta_k(t_{CMB}) \sim 10^{-5} \rightarrow \delta_k(t_0) \sim 10^{-2} \ll 1$

→ baryon perturbations would not have grown non linear today!

→ we need other sources of sizeable gravitational potentials at CMB time.

<u>NB</u>: CMB fluctuations are expected to be related to the seeds of structure formation and are different than the observed $\Delta T/T \sim 10^{-3}$ due to dipole effect resulting from Doppler shift of the sun velocity wrt the CMB rest frame. [ApJ 1993 Kogut]

$$T(\theta) \cong T_0 (1 + \beta \cos\theta + \dots)$$

$\vec{\beta} = \frac{\vec{v}}{c}$ the $\odot$ velocity through the isotropic radiation field itensity of temperature T_0.

$\theta = \measuredangle$ between $\vec{\beta}$ and the direction of observation measured in the observer rest frame.

- **1984**: Numerical simulations to reproduce the cosmic web by Blumenthal, Faber, Primack & Rees

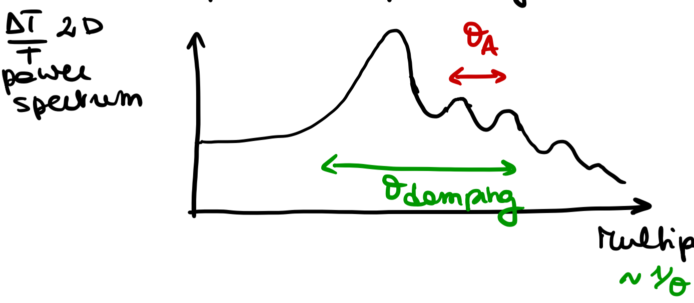
  $\Rightarrow \Omega_B / \Omega_{DM} \sim O(1/10)$ to account for observations

- **1985**: Davis, Esthathiou, Frenk, White obtain with a high resolution cold DM simulation a clear resemblance to CFA survey result

- **1992** Discovery of CMB anisotropies with COBE.

Since then WMAP, Planck, etc

$\rightarrow$ temperature, E-polarisation anisotropy spectrum precisely measured up to $\ell \sim 2500$)



Accoustic oscillations due to Pressure – gravity effect on the strongly coupled B-γ fluid.

$*\ \theta_A = \frac{r_s}{D_A}$ with $D_A \equiv$ distance to last scattering

sound speed

$$r_s = \int c_s \, d\eta$$

comoving time

$*$ ∃ also a Damping due to photon diffusion and their mean free path near decoupling.

$\lambda_{damping} \propto (n_e \sigma_T H.)^{-1/2}$ ie $\propto 1/\Omega_b^{1/2}$

If $\Omega_b \uparrow \Rightarrow \lambda_{damping} \downarrow$.

* It is also posible to show that $\Delta T/T$ follow a forced harmonic oscillator with external force $\leftrightarrow$ gravity ($\leftrightarrow \rho_{dm}$) and and its frequency $\leftrightarrow$ sound speed

$$c_s = \left(3(1+3\rho_b/\rho_r)\right)^{-1/2}$$

non baryonic matter $\Rightarrow$

$$\left\{ \boxed{\begin{array}{l} \Omega_{nbm} h^2 \simeq 0{,}12 \equiv \Omega_{DM} h^2 \\ \Omega_b h^2 \simeq 0{,}022 \end{array}} \right.$$

$\rightarrow$ measure energy density.

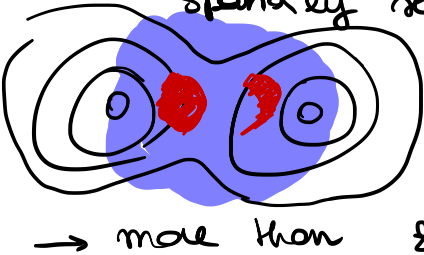
$\rightarrow H_0 \propto h$; $h \simeq 0{,}7$

NB Another probe sensitive to Ω_b is the Big Bang nucleosynthesis: $\Omega_b h^2 \sim 0{,}022$ from H, ^{3}He, D, ^{7}Li abundance measurements.

↱ 1E0657-558

• 2006 Bullet Cluster: due to the recent collision of clusters of galaxies the mass, galactic stellar components and X ray emmiting gas are spatialy separated

DM $\sigma_{self} \leq 1\ cm^2 \left(\frac{m_{DM}}{g}\right)$



— mass from weak lensing.

X ray emmiting gas

stellar component

$\rightarrow$ more than 8σ offset between luminous matter and the gravitational potential (difficult to agree with MOND-like theories stating that $F=ma$ scales like $F = m\, a^2/a_0$ at low acceleration $a < a_0 = 1.2\,10^{-10}\ m/s^2$ see 0608407)

I.2. DM properties.

Let us roughly derive some generic constraints on the DM mass and interactions:

* if DM is a fermion, Pauli exclusion bound induces a lower bound on the DM mass.

Indeed $f_{FD}(E) = \frac{g}{e^{E/T}+1} \leq \frac{g}{2}$

$$\Rightarrow \quad n = \int \frac{d^3p}{(2\pi)^3} f_{FD}(E) \underset{g\sim 1}{\leq} \frac{1}{4\pi^2} \frac{p_{max}^3}{3}$$

in a galaxy in which DM is gravitationally bounded, i.e., $p_{max} = m_\chi v_{esc}$

↳ escape velocity
~ 100km/s
~ 10^{-3}

$$\Rightarrow \quad \boxed{m_\chi \gtrsim 30\,\text{eV} \left(\frac{\rho_\chi}{\text{GeV}/\text{cm}^3} \times \left(\frac{v_{esc}}{10^{-3}}\right)^{-3} \right)^{1/4}}$$

$\frac{\text{GeV}}{\text{cm}^3} = 7.7\, 10^{-6}\,\text{eV}^4$

fermionic DM due to PAULI exclusion principle in astro objects.

* If DM is a boson, it is allowed to go to much lower masses.
Now to describe $m \lesssim$ few eV, one should account for the wave-like nature of the particle* and we need its macroscopic de Broglie wave length to fit into a galaxy:

$$\lambda_{dB} = \frac{h}{p_X} \overset{\hbar=1}{=} \frac{2\pi}{m_X v_X} \lesssim \text{size of a galaxy}$$

$v_X \sim 10^{-3}$

$\sim \text{kpc} = 1{,}6\ 10^{26}\ \text{eV}^{-1} = 3.1\ 10^{21}\ \text{cm}.$

$$\Rightarrow \boxed{m_X \gtrsim 4.10^{-22}\,\text{eV}.}$$

bosonic DM.

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* when a particle is expected to follow a wave-like behavior:

- size : $\lambda_{dB} = \frac{2\pi}{m_X v_X} \sim 2\pi\ 10^3 \left(\frac{v_X}{10^{-3}}\right) \frac{1}{m_X}$

- distance between X : $n_X^{-1/3} \sim \left(\frac{\rho_X}{m_X}\right)^{-1/3} \sim \left(\frac{m_X}{\rho_X/(\text{GeV/cm}^3)}\right)^{1/3} \times \frac{1}{(7.7\,10^{-6}\,\text{eV})^{1/3}}$

$\rightsquigarrow \lambda_{dB} \gtrsim$ interdistance $\Leftrightarrow m \lesssim 37\,\text{eV}$
$\rightsquigarrow$ quasi-continuous field behavior.

\# of relativ. dof

* Cosmological limits on ΔN_{eff} arise from CMB ($z \sim 1000$, $T \sim eV$) and BBN ($z \sim 10^{10}$, $T \sim 10\,MeV$)

  Currently we have $\Delta N_{eff} \lesssim 0{,}3$.

  → allows to probe the content of the hidden sector, light DM decay products or the DM properties.

* In these lectures, we focus on DM particles with masses > few eV.

  For bosonic ALP, see e.g. E. Hardi GGi lectures.

I.3 Further inputs

Main evidences for DM from cosmology!
(well beyond rotation curves!)

Expected DM properties:

1/ DM is a beyond the Standard Model (BSM) particle

↳ let us check first the candidates for DM in the Standard Model. (SM)

2/ DM is essentially neutral ($Q=0$)

Carefull, neutral under $U(1)_Q$ for Q = electromagnetic charge. DM with non zero $SU(2)_L$ or $U(1)_Y$ is possible!
(minimal DM, Inert doublet, etc)
Now some contribution of millicharged DM could be allowed.

3/ DM is massive and stable ($\tau_{DM} > \tau_{universe}$) ↑ 10^{18} s

↳ Massive to allow for bottom up structure formation and stable to account for Ω_{DM} measurements. Now there is still some room for a (fraction of) decaying DM, see e.g. 1610.1051, 2012.05276

II Short intro to Cosmology Tools

Let me start with a short introduction to our expanding universe in cosmology.

II 1. The metric.

At large scales ($\gtrsim 100$ Mpc) our universe appear to be relatively "smooth", it looks quite isotropic (same in all directions) and homogeneous (same at every point in space). This is referred to as the cosmological principle.

Such an homogeneous and isotropic universe is relatively simple to describe using some basic concepts of special and general relativity (GR).
In particular infinitesimal distance squared can be described in terms of a metric, which in our case is the Friedmann-Lemaître-Robertson-Walker (FLRW)

metric.

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

physical time.

$$= -c^2 dt^2 + \underbrace{a^2(t)}_{\text{scale factor}}\, d\bar{\ell}^2$$

speed of light

$$\mu, \nu = 0, \ldots, 3$$

<u>NB</u> in these lectures, I will use the metric signature $(-+++)$ <u>and</u> use natural units in which $c = 1 = \hbar$

Among the possible external comoving line element squared CMB has demonstrated that our universe is relatively <u>flat</u>, ie

<u>ie</u> $$d\bar{\ell}^2 = \frac{dr^2}{1-kr^2} + r^2 d\Omega^2$$

with $k=0$ in comoving spherical coord or equivalently.

$$d\bar{\ell}^2 = \delta_{ij}\, \underbrace{dx^i dx^j}_{\text{comoving spatial coord.}}$$

$$i = 1, 2, 3$$

One can also define the conformal time η, related to the physical time though the scale factor :

$$a \, d\eta = dt$$

So that at the end of the day, we will mainly use:

$$\boxed{ds^2 = a^2(t) \left(d\eta + \delta_{ij} \, dx^i dx^j \right)}$$

In our expanding universe, the expansion rate is defined as

$$\boxed{H(t) = \frac{d \ln a}{dt} = \frac{1}{a^2} \frac{da}{d\eta}}$$

We denote $a(t_0) = a_0$ and $H(t_0) = H_0$. We conventionally set $a_0 = 1$ and the Hubble parameter H_0 is usually described in terms of the adimentional h parameter as :

$$H_0 = 100 \times h \quad km/s \; Mpc^{-1}$$

with $h = 0,674 \pm 0,005$ from CMB obs.

Note that the wave length of light scales as:

$$\lambda_{obs} = \lambda_{emit} \frac{a_{obs}}{a_{emit}} .$$

Let us define the "cosmological red shift" as:

observed today

$$z = \frac{\lambda_0 - \lambda_e}{\lambda_e} = \frac{a_0 - a_e}{a_e}$$

emitted.

$$\rightarrow \boxed{1+z = \frac{a_0}{a(t)}}$$

II.2. Space time dynamics.

2.1 Conservation equation

Here we consider perfect fluids described in terms of their pressure P and energy ρ densities within the stress energy tensor of the form:

$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho(t) & & & 0 \\ & p(t) & & \\ & & p(t) & \\ 0 & & & p(t) \end{pmatrix}$$

for an observer comoving with the fluid.

$u^{\mu} = \frac{dx^{\mu}}{dt} = (1, \vec{0})$

which respects homogeneity and isotropy criterium

One usually introduces the equation of state (eos) to relate p to ρ:

$$\boxed{P = w \rho.}$$

with
$$\begin{cases} w_m = 0 & \text{for matter} \\ w_r = 1/3 & \text{for radiation} \\ w_\Lambda = -1 & \text{for a cosmological constant.} \end{cases}$$

- The covariant conservation eq. takes the form:

$$\boxed{D_\mu T^\mu{}_\nu = 0}$$

with $D_\mu \to \partial_\mu$ in Minkowski space while, in general, it takes the form of:

$$D_\mu T^\mu{}_\nu = \partial_\mu T^\mu{}_\nu + \Gamma^\mu_{\mu\alpha} T^\alpha{}_\nu - \Gamma^\alpha_{\mu\nu} T^\mu{}_\alpha$$

also denoted as $T^\mu{}_{\nu;\mu}$.

where the Christoffel coefficient are:

$$\Gamma^\mu_{\alpha\beta} = \frac{g^{\mu\nu}}{2}\left[\frac{\partial g_{\alpha\nu}}{\partial x^\beta} + \frac{\partial g_{\beta\nu}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\nu}\right]$$

$\Rightarrow$ For the FLRW metric, we get:

$$\boxed{\partial_t \rho = -3H(\rho + P).}$$

$$\Rightarrow \quad \rho \propto a^{-3(1+w)}.$$

ie: $\rho_R \propto a^{-4}$, $\rho_m \propto a^{-3}$; $\rho_\Lambda \propto cst$

NB: you can compute the Christoffel symbols for the FLRW metric:

$$ds^2 = -dt^2 + a^2 \delta_{ij}\, dx^i dx^j \,, \quad i = 1,2,3$$

$$\begin{cases} \Gamma^0_{ij} = \delta_{ij}\, a^2 H \\ \Gamma^i_{0j} = \Gamma^i_{j0} = \delta^i_j H \end{cases}$$

all others are zero.

or with conformal time

$$ds^2 = a^2 (d\eta^2 + \delta_{ij}\, dx^i dx^j)$$

$$\begin{cases} \Gamma^0_{00} = \mathcal{H} \quad \text{where } \mathcal{H} = \dfrac{1}{a}\dfrac{da}{d\eta} = \dfrac{\dot{a}}{a} \\ \Gamma^i_{j0} = \mathcal{H}\, \delta^i_j \quad \text{and} \quad \Gamma^0_{ij} = \mathcal{H}\, \delta_{ij} \end{cases}$$

all others are zero.

2.2. Friedman equations.

Einstein Equations, relating the space-time content to its geometry,

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}.$$

Einstein tensor ↕ double deriv of the metric $g_{\mu\nu}$

Newton cst.

Stress energy tensor, including DE.

$G = 6.67 \, 10^{-11} \, m^3 \, kg^{-1} \, s^{-2}$.
$= M_p^{-2}$ with M_p = Planck mass $= 1.22 \, 10^{19}$ GeV,

allow us to derive some $a(t)$ – universe content relations. known under the name of "Friedman-equations"

$$\boxed{\begin{aligned} H^2 &= H_0^2 \, \Sigma_i \, \Omega_{i,0} \, a^{-3(1+\omega_i)}. \\ \frac{\partial_t^2 a}{a} &= -\frac{4\pi G}{3} \, \Sigma_i \, \rho_i \, (1+3\omega_i). \end{aligned}}$$

$$\Omega_i = \frac{\rho_i}{\rho_c} \qquad \rho_c = \frac{3H^2}{8\pi G}$$

$\Omega_{i,0} = \Omega_i(t=t_0)$ & $\rho_i < \rho_c \rightarrow \Omega_i < 1$

$$\rho_{c,0} = 1.1 \; 10^{-5} \, h^2 \; \text{GeV cm}^{-3}$$

+ From CMB data (Planck'18, A&A 641, A6 2020)

$$\Omega_{m,0}h^2 = \Omega_{b,0}h^2 + \Omega_{dm,0}h^2.$$

$$\begin{cases} \Omega_{b,0}h^2 = 0{,}0224 \pm 0{,}0001 \\ \Omega_{c,0}h^2 = 0{,}120 \pm 0{,}001 \end{cases} \Rightarrow \frac{\Omega_{b,0}}{\Omega_{c,0}} = 0{,}16.$$

at 68% CL.

* Assuming ΛCDM, they get

$$\begin{cases} \Omega_{m,0} = 0{,}315 \pm 0{,}007. \\ H_0 = 67.4 \pm 0{,}5 \text{ km/s Mpc}^{-1} \\ \Omega_{\Lambda,0} = 0{,}684 \pm 0{,}007 \end{cases}$$

NB

$$\begin{cases} \Omega_{\gamma,0} = 5\ 10^{-4} \\ w_\Lambda = -1.03 \pm 0{,}03. \\ \Omega_{k,0} = 0{,}001 \pm 0{,}002 \\ \Omega_{r,0} = \Omega_{\gamma,0} + \Omega_{\nu,0}. \end{cases}$$

↳ to define in a few pages.

NB $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$; $R = R^{\nu}{}_{\nu} = g^{\nu\upsilon} R_{\upsilon\nu}$

$$R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\lambda} + \Gamma^\lambda_{\lambda\rho}\Gamma^\rho_{\mu\nu} - \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\rho}$$

NB: • FLRW in a single component universe i:

$$\partial_t \ln a = H_0 \sqrt{\Omega_i}\; a^{-\frac{3}{2}(1+w_i)}$$

$$\Rightarrow \boxed{a(t) \propto \begin{cases} t^{\frac{2}{3(1+w_i)}} & w_i \neq -1 \\ e^{H_0 \sqrt{\Omega_\Lambda}\, t} & w_i = -1 \end{cases}}$$

The bottom solution of a Λ dominated (ΛD) universe is just the de Sitter case. ($\Lambda > 0$)

The above solutions can e.g. describe a radiation dominated (RD) universe or a matter dominated (MD) universe:

$$a(t) \propto \begin{cases} t^{2/3} & \text{MD} \\ t^{1/2} & \text{RD.} \end{cases}$$

• in terms of conformal time
$a\, d\eta = dt$

$$\partial_\eta \ln a = H_0 \sqrt{\Omega_i}\; a^{-\frac{1}{2}(1+3w_i)}$$

$$\Rightarrow \boxed{a(\eta) \propto \eta^{2/(1+3w_i)}}$$

as a result $a(\eta) \propto \begin{cases} \eta^2 & \text{MD} \\ \eta & \text{RD} \\ \eta^{-1} & \Lambda\text{D} \end{cases}$

- the corresponding Hubble rates are given by

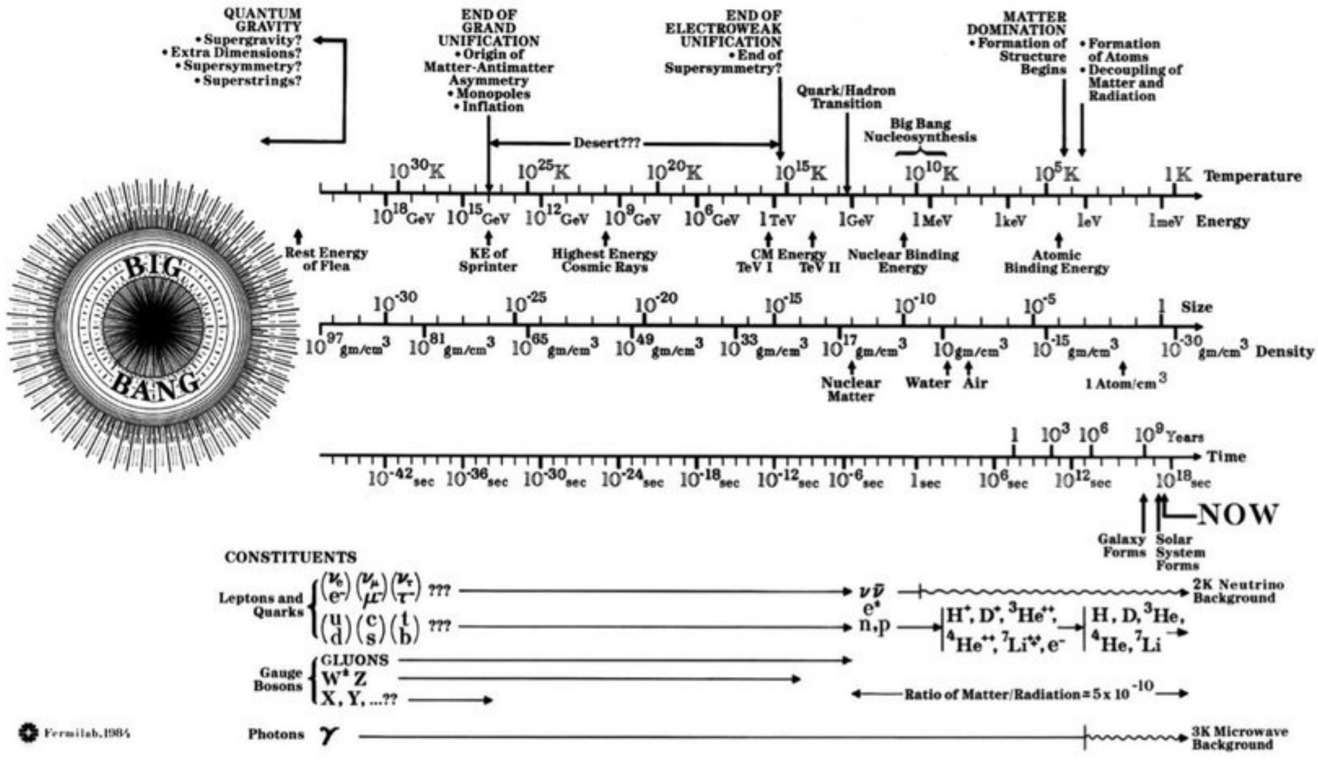
$$H = \partial_t \ln a = \begin{cases} 2/3 \ t^{-1} & \text{MD} \\ 1/2 \ t^{-1} & \text{RD.} \\ cst = H_0 \sqrt{\Omega_\Lambda} & \Lambda\text{D.} \end{cases}$$

$$\mathcal{H} = \partial_\eta \ln a = \frac{2}{1+3w_i} \eta^{-1}$$

NB in the next sections, we will often work in a R.D. era where $H \propto \sqrt{\frac{\rho_R}{M_p^2}} \propto g_*^{1/2} \, T^2/M_p$.

$$\Rightarrow \boxed{\frac{T}{1\,\text{MeV}} \simeq 1.5 \, g_*^{-1/4} \left(\frac{t}{1\text{s}}\right)^{-1/2} \quad [\text{RD}]}$$

II 12.


QUANTUM GRAVITY
• Supergravity?
• Extra Dimensions?
• Supersymmetry?
• Superstrings?
END OF GRAND UNIFICATION
• Origin of Matter-Antimatter Asymmetry
• Monopoles
• Inflation
Desert???
END OF ELECTROWEAK UNIFICATION
• End of Supersymmetry?
Quark/Hadron Transition
Big Bang Nucleosynthesis
MATTER DOMINATION
• Formation of Structure Begins
• Formation of Atoms
• Decoupling of Matter and Radiation
10^{30}K
10^{25}K
10^{20}K
10^{15}K
10^{10}K
10^{5}K
1K
Temperature
10^{18}GeV
10^{15}GeV
10^{12}GeV
10^{9}GeV
10^{6}GeV
1 TeV
1 GeV
1 MeV
1 keV
1 eV
1 meV
Energy
Rest Energy of Flea
KE of Sprinter
Highest Energy Cosmic Rays
CM Energy
TeV I
TeV II
Nuclear Binding Energy
Atomic Binding Energy
BIG BANG
10^{-30}
10^{-25}
10^{-20}
10^{-15}
10^{-10}
10^{-5}
1
Size
10^{97} gm/cm^3
10^{81} gm/cm^3
10^{65} gm/cm^3
10^{49} gm/cm^3
10^{33} gm/cm^3
10^{17} gm/cm^3
10 gm/cm^3
10^{-15} gm/cm^3
10^{-30} gm/cm^3
Density
Nuclear Matter
Water
Air
1 Atom/cm^3
1
10^3
10^6
10^9 Years
Time
10^{-42} sec
10^{-36} sec
10^{-30} sec
10^{-24} sec
10^{-18} sec
10^{-12} sec
10^{-6} sec
1 sec
10^{6} sec
10^{12} sec
10^{18} sec
NOW
Galaxy Forms
Solar System Forms
CONSTITUENTS
Leptons and Quarks
$\binom{\nu_e}{e^-}\binom{\nu_\mu}{\mu^-}\binom{\nu_\tau}{\tau^-}$???
$\binom{u}{d}\binom{c}{s}\binom{t}{b}$???
$\nu\bar{\nu}$
$e^{\pm}$
n,p
$H^+, D^+, {}^3He^{++}, {}^4He^{++}, {}^7Li^{+++}, e^-$
$H, D, {}^3He, {}^4He, {}^7Li$
2K Neutrino Background
Gauge Bosons
GLUONS
$W^{\pm}$ Z
X, Y, ...??
Ratio of Matter/Radiation = 5×10^{-10}
Photons γ
3K Microwave Background
Fermilab, 1984

III The Standard Model (SM) and beyond.

III. 1 Standard Model.

About charges and symmetries, the SM gauge group is:

$$\underbrace{SU(3)_c}_{\text{strong interactions}} \times \underbrace{SU(2)_L \times U(1)_Y}_{\text{WEAK and electromagnetism (EM)} \;=\; \text{Glashow-Weinberg-Salam Model}}$$

$$\longrightarrow SU(3)_c \times U(1)_Q$$

↳ spontaneous symmetry breaking.

Let me emphasize again that DM is expected to be essentially neutral under $U(1)_Q$ but could very well have e.g. a non zero $SU(2)_L$ charge, see eg. "Minimal DM": 0903.3381.

In the table on the next page, I summarize the charges of all SM particles. It is very clear that among all $Q=0$ charged particles, the neutrino is our best candidate for DM as it interacts the most weakly with all other SM particles.

Charges of the SM particles:

		$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_Q$	# dof.
Spin 1/2	$L_L = \begin{pmatrix} \nu_{e_L} \\ e_L \end{pmatrix}$	1	2	-1	$\begin{cases} 0 \\ -1 \end{cases}$	$g_\nu = 2$
	e_R	1	1	-2	-1	$g_e = 2\times 2$
	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	3	2	$1/3$	$\begin{cases} 2/3 \\ -1/3 \end{cases}$	
	u_R	3	1	$4/3$	$2/3$	$g_q = N_c \times 2 \times 2 = 12$
	d_R	3	1	$-2/3$	$-1/3$	
Spin 0	$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$, $H_0 = \frac{(h + iA_0)}{\sqrt{2}}$	1	2	1	$\begin{cases} 1 \\ 0 \end{cases}$	$g_h = 1$
Spin 1	B_μ	1	1	0	To Be redefined as $A_\mu, Z_\mu, W^\pm_\mu$ after SSB	$g_\gamma = 2$
	$W^{1,2,3}_\mu$	1	3	0		$g_{Z,W} = 3$
	gluons	8	1	0	0.	$g_g = 8 \times 2$

Convention: $Q = \underbrace{T_3}_{\text{isospin.}} + \overbrace{Y/2}^{\text{hypercharge.}}$

The last column give you the # of dof associated to each species relevant for later. Some extra comments.

- Leptons and quarks come in families: $\boxed{N_f = 3}$

(e, μ, τ) charged leptons
(u, c, t) u-type quarks
(d, s, b) d-type quarks.

they all come with spin 1/2 $\Rightarrow$ 2 dof and with particles and antiparticles $\Rightarrow$ for each species 2×2

Quarks come with colors $\rightarrow$ extra $N_c = 3$ factor.

In total you have thus.

$$\boxed{\begin{array}{ll} \text{charged leptons}: & N_f \times 2 \times 2 = 12 \\ \text{quarks}: & \underbrace{2}_{u\,\&\,d} N_f \times N_c \times 2 \times 2 = 72. \end{array}}$$

Let us emphasize that the top mass is much larger than the others

$$m_t = 175 \text{ GeV} \gg m_b \doteq 5 \text{ GeV}.$$
$$\text{or } m_c \cong 2 \text{ GeV}.$$

- In addition you have neutral leptons (neutrinos)

$$(\nu_e, \nu_\mu, \nu_\tau)$$

the fundamental nature of neutrinos and exact mass properties are still unknown. In particular, we do not know if they are of majorana nature $(\nu = \bar{\nu})$ or Dirac $(\nu \neq \bar{\nu})$. In the majorana case, we clearly have $g_\nu = 2$. In the Dirac case, we know that anyway only $\nu = \nu_L$

participate to SM interactions so that in practice, you also have $g_\nu = 2$.

$$\boxed{\text{Thus, in total for neutrinos you can count : } N_f \times g_\nu = 3 \times 2 = 6.}$$

- Massless spin 1 bosons (γ and gluons) come with 2 polarisations.
  In addition, because gluons mediate strong interactions driven by $SU(3)_c$ with 8 generators you have an extra factor 8 for g_g.

  Massive spin 1 bosons have 3 polarization states so that $g_{Z,W} = 3$.

  What is left is only the scalar component of H_0, denoted with $h \equiv$ Higgs boson, with one dof.

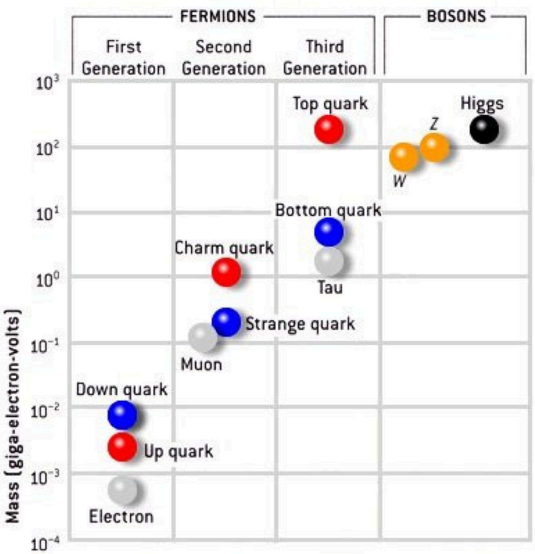
→ PDG'22 values.

- About masses, the most massive SM particles are the h, W, Z bosons and the top with

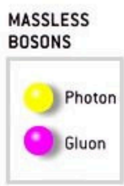
$$m_t = 172.69 \pm 0.30 \text{ GeV}$$
$$m_h = 125.25 \pm 0.17 \text{ GeV}$$
$$m_Z = 91.1876 \pm 0.0021 \text{ GeV}$$
$$m_W = 80.377 \pm 0.012 \text{ GeV}$$

we have
$$\begin{cases} m_\tau = 1776{,}86 \pm 0{,}12 \text{ MeV}, \\ m_\mu = 105.6583755 \pm 0.0000023 \text{ MeV} \\ m_e = 0.511 \pm 1.5\ 10^{-8} \text{ MeV}. \end{cases}$$

and*
$$m_b = 4.18 \,{}^{+0.03}_{-0.02} \text{ GeV}$$
$$m_c = 1.27 \pm 0{,}02 \text{ GeV}$$
$$m_u = 2.16 \pm^{0.49}_{0.26} \text{ MeV}.$$
$$m_d = 4.67 \pm^{0{,}48}_{0.17} \text{ MeV}.$$
$$m_s = 3.45 \pm^{0.35}_{0.15} \text{ MeV}.$$



+ 3 neutrinos
$\Sigma m_\nu > 0{,}06$ eV [oscill.]
$< 0{,}12$ eV [CMB]



* unlike leptons, quarks are confined inside hadrons and are not observed as free particles. their masses are determined indirectly through their influence on hadronic properties.

Some more details on ν_L.

At the very least, the ν_L interacts with the SM through gauge interactions:

lepton doublet.

$$\mathcal{L}_{SM} \supset \sum_i \bar{L}_{Li}\, i \not{D}\, L_{Li} \,. \qquad \circledast$$

i = flavour index

Lorentz index.

where • $\not{D} = D_\mu \gamma^\mu$.

• γ^μ satisfy the Clifford Algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\, \eta^{\mu\nu}\, \mathbb{1}$$

We have L_L charged under $SU(2)_L$ & $U(1)_Y$:

$a = 1 \ldots 3$ — Pauli matrix

$$D_\mu L_L = \left(\partial_\mu - i g\, W_\mu^a \frac{\sigma^a}{2} - i g'\, B_\mu \frac{Y}{2} \right) L_L$$

$SU(2)$ gauge couple — $U(1)_Y$ gauge coupling.

in flat space!

Using
$$\boxed{\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right). \\ \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} &= \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}. \end{aligned}}$$
and

with $c_W = \cos\theta_W$ and $s_W = \sin\theta_W$

you can rewrite the $SU(2)_L \times U(1)$ part of the covariant derivative as:

$$\boxed{\begin{aligned} & -ig W_\mu^a \frac{\sigma^a}{2} - ig' B_\mu \frac{Y}{2} \\ & = -ig\left(W_\mu^+ T^+ + W_\mu^- T^-\right) - \frac{ig}{c_W} Z_\mu \left(T^3 - s_W^2 Q\right) - ieA_\mu Q \end{aligned}}$$

with $T^\pm = \frac{1}{2}\left(\sigma^1 \pm i\sigma^2\right)$ $\quad T^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $\quad T^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

and $\begin{cases} Q = T^3 + Y/2 & \text{with } T^3 = \sigma^3/2 \text{ weak isospin} \\ e = g\, s_W \text{ and } g'/g = \tan\theta_W \end{cases}$

Some more details on spinors:

* one usefull representation of γ^μ matrices for high energy description is the WEYL (= chiral) representation

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad \text{with} \quad \overset{(-)}{\sigma}{}^\mu = (\mathbb{1}, (-)\sigma^i)$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1} & \\ & \mathbb{1} \end{pmatrix}$$

and the Pauli matrices are given by:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

+ $S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$ provide a 4x4 representat° of the Lorentz group

with the Dirac spinor transforming

antisymmetric tensor

as : $\Psi \to \exp(-i\, \omega_{\mu\nu} S^{\mu\nu})\, \Psi$

You can check (exercices) that

$$\boxed{\begin{cases} \psi_L = L\psi = \frac{1-\gamma_5}{2}\psi \\ \psi_R = R\psi = \frac{1+\gamma_5}{2}\psi \end{cases}}$$

transform differently under the Lorentz transform.

They correspond to 2 different irreducible representations of the Lorentz group.

III.2. Candidates for DM in SM and Beyond.

DM from the SM?

As said above, within the SM, we have a potential candidate for DM, the SM neutrino ν_L with

isospin.

$$Q_{\nu_L} = 0 \;; \quad Y_{\nu_L} = -1/2 \;; \quad T_{3\,\nu_L} = 1/2$$

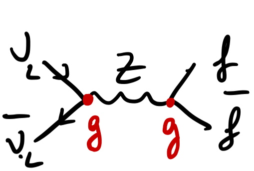
$$T_3 L_L = \frac{\sigma_3}{2} L_L = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} = \begin{pmatrix} 1/2 \; \nu_L \\ -1/2 \; \ell_L \end{pmatrix}$$

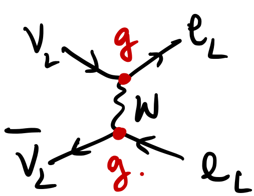
$\rightarrow$ the ν_L couples to Z and W bosons

We have thus:

$$\mathcal{L}_{SM} \supset \bar{L}_L (i \not{\partial}) L_L + \frac{g}{\sqrt{2}} \left(\left(\bar{\ell}_L \gamma^\mu \nu_L \right) W^-_\mu + h.c. \right) + \frac{g}{2 c_W} \bar{\nu}_L \gamma^\mu \nu_L Z_\mu$$

$\Rightarrow$ Possible annihilation channels for ν_L





For the purpose of these lectures, let us look at ν_L abundance being agnostic on m_ν (obviously, we have strong laboratory and cosmo constr!)

As very rough estimates, when:

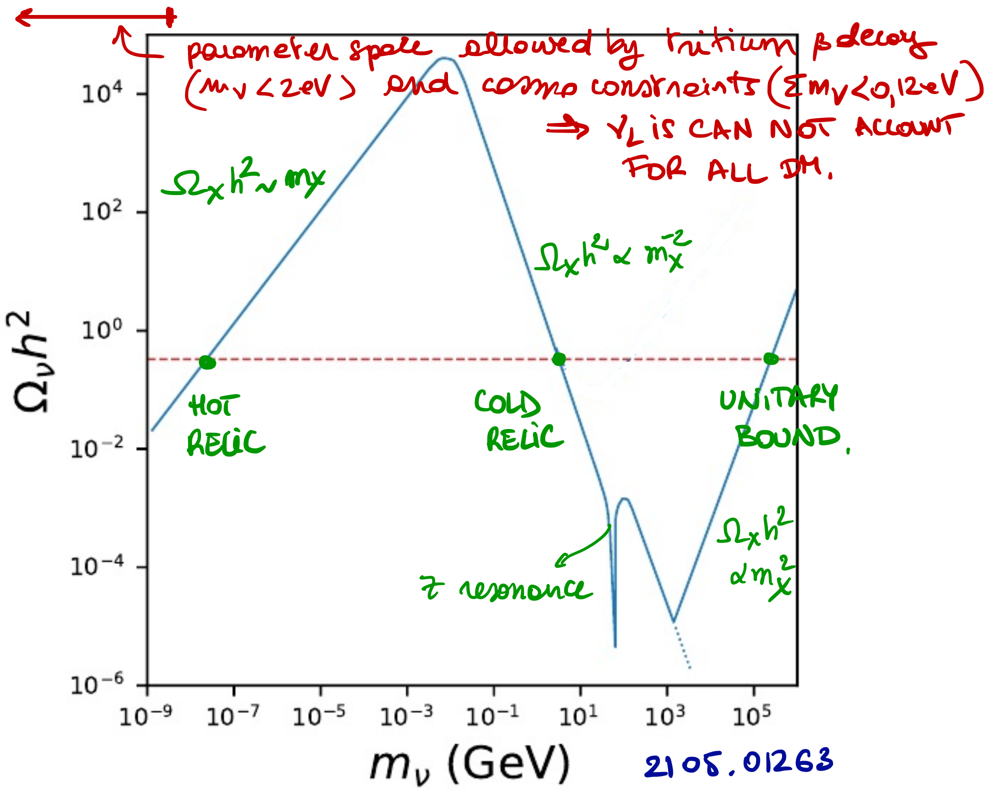
$\underline{T > m_\nu}$: $\sigma v_{\nu\nu \to ff} \sim G_F^2 T^2$

Fermi Constant: $G_F = \frac{\sqrt{2} g^2}{8 m_W^2}$

$\underline{m_\nu > T, m_W}$: $\sigma v_{\nu\nu \to ff} \sim \frac{\alpha_W}{m_\nu^2}$

$$\alpha_W = \frac{g^2}{4\pi} = \frac{1}{S_W^2} \frac{e^2}{4\pi} = \frac{\alpha_{EM}}{4\pi S_W^2}$$

In the next sections, we will see that $\Omega_\nu h^2 (m_\nu)$ takes the following form:



2105.01263
0206163

DM candidate BSM.

As we will see, the ν_L can't account for all the DM of the universe.

As a result, we will have to go BSM. You might be aware that we have many possible candidates for DM.

As my purpose is both to introduce you to BSM DM and its mechanisms of production, I will consider a minimal extension of the SM that could give rise to DM candidates and allow me to explore a large range of DM couplings to the SM.

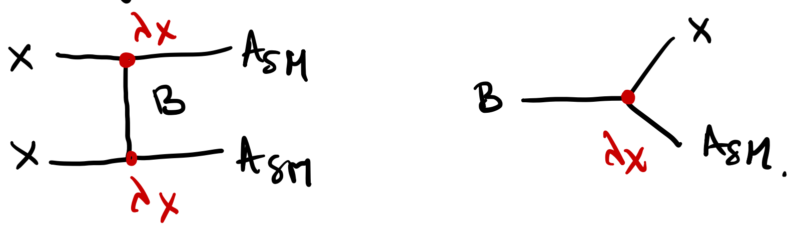
The discussion presented in these notes may be directly translated to any model where:

- DM is a singlet under the SM gauge group, say X
- Some symmetry prevents the DM to decay to SM. Here we assume a Z_2 symmetry under which DM $\equiv Z_2$ odd, SM $\equiv Z_2$ even.
- I will assume that DM is not the only new particle BSM. There will be

a "partner" odd under the Z_2 symmetry that will help to mediate DM – SM interactions. Let me refer to this "mediator" as B and assume that B is charged under the SM gauge group (B = bath particle)

<u>NB</u>: more minimal extensions of the SM, yet very rich, could be the scalar singlet interacting through H-portal with SM
<u>or</u> e.g. "minimal DM" which is a $Q=0$ of a BSM $SU(2)_L$ multiplet

With the above considerations, assuming a cubic interaction: $\mathcal{L} \supset \lambda_\chi B \chi A_{SM}$ where A_{SM} = SM particle and λ_χ is some coupling I can have the following annih. decay channels:



There are multiple possibilities for such models (sometimes referred to as "t-channel" models for WIMP).
For definitiveness, here I will focus on fermionic Majorana DM.

Considering B as a fermion or a scalar, the "minimal" cubic interactions I can write are:

$A_{\rm SM}$	Spin DM	Spin B	Interaction	Label
$\psi_{\rm SM}$	0	1/2	$\bar{\psi}_{\rm SM}\Psi_B\phi$	$\mathcal{F}_{\psi_{\rm SM}\phi}$
	1/2	0	$\bar{\psi}_{\rm SM}\chi\Phi_B$	$\mathcal{S}_{\psi_{\rm SM}\chi}$
$F^{\mu\nu}$	1/2	1/2	$\bar{\Psi}_B\sigma_{\mu\nu}\chi F^{\mu\nu}$	$\mathcal{F}_{F\chi}$
H	0	0	$H^\dagger\Phi_B\phi$	$\mathcal{S}_{H\phi}$
	1/2	1/2	$\bar{\Psi}_B\chi H$	$\mathcal{F}_{H\chi}$

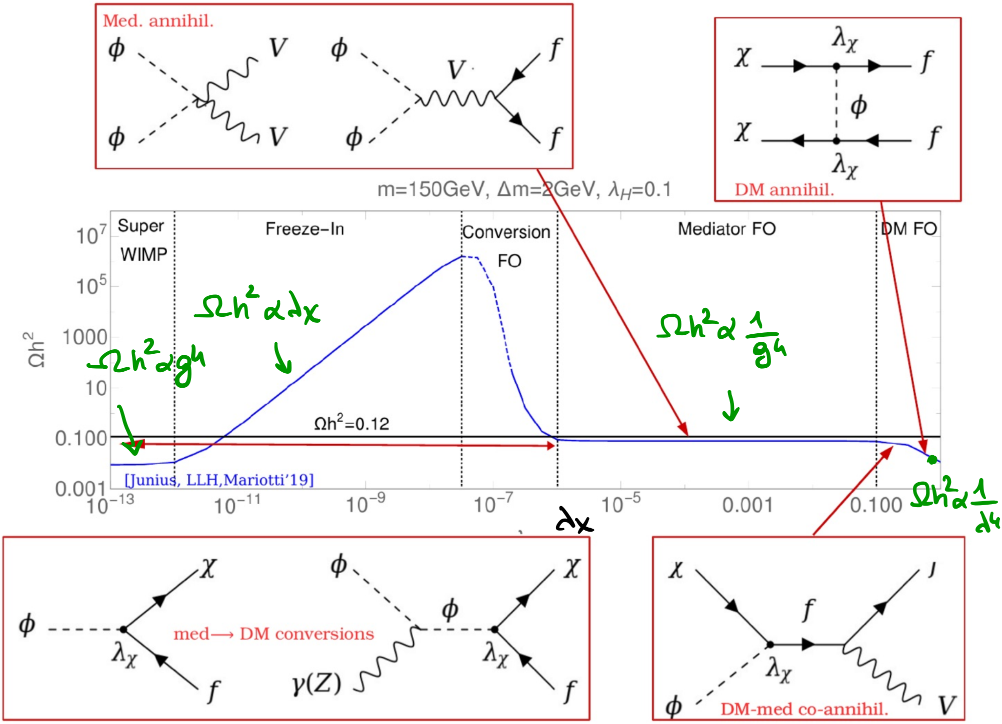
For more details see e.g. 2102.06221.

Here, for definitiveness, I will take the case of fermionic DM Majorana fermion χ, coupling through "Yukawa" like interactions to a charged scalar ϕ and a right handed lepton

$$\mathcal{L} \supset \lambda_\chi \, \bar{\ell}_R \chi \phi$$

For "WIMP" behaviour see e.g. 1307.6480, 1503.01500
For "FIMP" see e.g. 2102.06221, 1904.07513

At the end of the day, our (enthusiastic) goal is to go through all the DM production mechanisms represented in the Fig below



see 1904.07513 for details.